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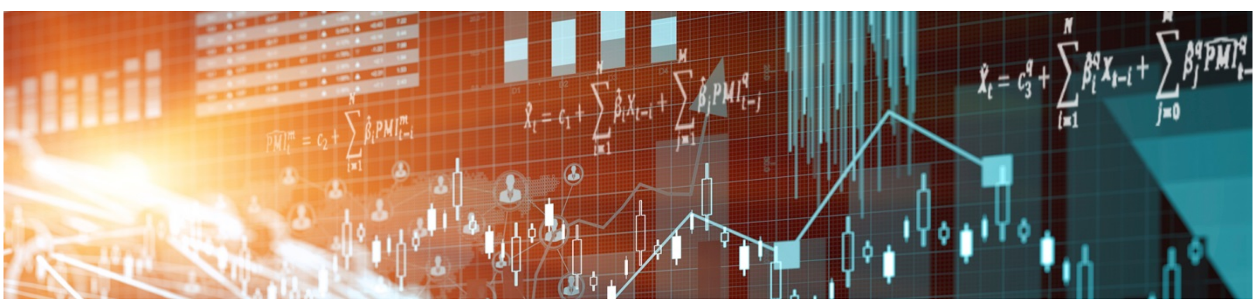

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Early Warning of Financial Stress Events: A Credit-Regime-Switching Approach

by

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Abstract

We propose an early warning model for predicting the likelihood of a financial stress event for a given future time, and examine whether credit plays an important role in the model as a non-linear propagator of shocks. This propagation takes the form of a threshold regression in which a regime change occurs if credit conditions cross a critical threshold. The in-sample and out-of-sample forecasting performances are encouraging. In particular, the out-of-sample forecasting results suggest that the model based on the credit-regime-switching approach outperforms the benchmark models based on a linear regression and signal extraction approach across all forecasting horizons and all criteria considered.

JEL classification: C12, C14, G01, G17
Bank classification: Financial stability; Econometric and statistical methods

Résumé

Nous proposons un modèle d'alerte précoce visant à prévoir la probabilité que survienne un épisode de tensions financières à un moment futur et examinons si le crédit joue un rôle important dans le modèle en tant que facteur de propagation non linéaire des chocs. La propagation est représentée par un modèle à seuil dans lequel un changement de régime se produit quand les conditions du crédit franchissent un seuil critique. La qualité des prévisions effectuées sur l'échantillon et hors échantillon est encourageante. Plus précisément, les résultats hors échantillon portent à croire que le modèle basé sur l'approche à changement de régime se montre supérieur aux modèles de référence fondés sur une régression linéaire et l'approche d'extraction des signaux, pour tous les horizons de prévision et tous critères d'évaluation confondus.

Classification JEL : C12, C14, G01, G17
Classification de la Banque : Stabilité financière; Méthodes économétriques et statistiques

Non-Technical Summary

Financial stress is characterized as a situation in which large parts of the financial sector face the prospects of large financial losses. This stress can lead to financial crises and inflict severe damage on the economy. It is therefore of crucial importance to regularly assess the financial stress in the financial system. A critical question of assessing financial stress is: how will the financial stress evolve in the future? To answer this question, we need an effective early warning model to predict the development of financial stress in the future. Such a model has substantial value to policy-makers by allowing them to detect the weaknesses and vulnerabilities of the financial system in future, and possibly take pre-emptive policy actions to avoid the financial stress or limit its effects.

Using the financial stress index (FSI) developed by Illing and Liu (2006) as the measure of financial stress, we propose an early warning model to predict the likelihood of a financial stress event by taking a credit condition as a non-linear propagator of shocks. Specifically, we model the relationship between the FSI and a set of explanatory variables via a threshold regression model that changes regime if credit conditions cross a critical threshold, where we consider three alternative measures of credit conditions, namely, the growth rate of the ratio between credit and GDP, the growth rate of business credit, and the growth rate of household credit.

The in-sample forecasting results suggest that our model is a useful tool for predicting the likelihood of a financial stress event at a given future time. The out-of-sample forecasting results indicate that our model outperforms the two benchmark models based on a linear regression and signal extraction approach (Kaminsky, Lizondo and Reinhart, 1998) across all forecasting horizons and criteria considered. Our model is applied to predict the subprime crisis that occurred in 2007Q3: the model is able to issue a warning signal of the subprime crisis at both the short horizons (one quarter and two quarters) and the long horizon (two years).

1 Introduction

Financial stress is characterized as a situation in which large parts of the financial sector face the prospects of large financial losses. This stress can lead to financial crises and inflict severe damage on the economy.[1] It is therefore of crucial importance to regularly assess the financial stress in the financial system. Two main questions of assessing financial stress are: (i) what is the current status of the financial stress in a financial system? and (ii) how will the financial stress event evolve in the future?

The starting point to answer (i) is to develop a formal measure of financial stress to observe the risk of financial stress. Such a measure of financial stress for monitoring financial vulnerability in the financial system is constructed as a financial stress index (FSI). Among the recent contributions to constructing FSIs are studies by Hakkio and Keeton (2009), Hatzius et al. (2010), Oet et al. (2012), and Carlson et al. (2012). Compared with the literature on developing an FSI to measure financial stress in a financial system, there has been relatively little effort on prediction analysis for financial stress. An effective prediction tool has substantial value to policy-makers by allowing them to detect the future potential weaknesses and vulnerabilities in the financial system, and possibly take pre-emptive policy actions to avoid financial stress or limit its effects.

To answer (ii), Slingenberg and de Haan (2011) use the FSIs for 13 OECD countries to examine whether a list of variables can help predict financial stress. Misina and Tkacz (2009) investigate whether credit and asset price movements can help to predict financial stress in Canada. However, their work only provides point predictions of future financial stress. Point predictions can at most convey some notion of the central tendency of future financial stress, but they cannot provide

[1] Using the data in 17 advanced economies from 1980 to 2007, Cardarelli et al. (2011) find that financial stress is often a precursor to an economic slowdown or recession. A rapid expansion of credit, a run-up in house prices, and large borrowings by the corporate and household sectors all contribute to a higher likelihood that stress in the financial system will lead to more severe economic downturns.

the possible uncertainty of the future financial stress. For most decision issues, reliance on point forecasts will not be sufficient and probability forecasts will be needed to provide insights on the likelihood of financial stress for a given period of time (Gneiting and Ranjan, 2011).[2]

Christensen and Li (2014) propose an early warning model that can predict the likelihood of financial stress events at a given period of time.[3] The out-of-sample forecasting results suggest that their early warning model is a useful tool for predicting financial stress events. The restriction of their model is that it can only predict the probability that financial stress events will occur within a given period of time, but it cannot predict the probability that a financial stress event will occur at a given future time.

The objective of this paper is that by using an FSI as the measure of financial stress, we predict the likelihood of a financial stress event at a given future time. To achieve this goal, it is important to note that despite the apparent uniqueness of each financial cycle, from the conditions that lead to boom times to the triggers that result in reversals, history suggests that most financial cycles share common features: boom times are typically associated with periods of credit expansion, often followed by rapid reversals. These commonalities, confirmed by empirical work (Misina and Tkacz 2009, Borio and Lowe 2002, Kaminsky and Reinhart 1999), suggest that development in the credit markets may provide an early warning indicator of financial stress in the financial system. In particular, due to the existence of frictions arising from informational asymmetries and contractual rigidities, credit markets may act as non-linear propagators of the impact of aggregate disturbances to the financial system (Balke, 2000). This motivates us to propose an early warning model of financial stress events by using credit as a non-linear propagator of shocks. We model

[2] The detailed definition of a financial stress event will be given in section 2.

[3] Letting FSI_t be the value of the financial stress index at time t, Christensen and Li (2014) define that a financial stress event occurs if $FSI_t > \mu_{FSI} + 1.5\sigma_{FSI}$, where μ_{FSI} and σ_{FSI} are the sample mean and sample standard deviation of the FSI.

the relationship between the FSI and the explanatory variables by a threshold regression model that changes regime if credit conditions cross a critical threshold. A non-parametric bootstrap procedure is used to simulate the future paths of the FSI in the threshold regression model. The simulated future values of the FSI are used to predict the probability of a financial stress event at a given future time.

The in-sample forecasting results suggest that the threshold model is a useful tool for predicting the likelihood of a financial stress event at a given future time. The out-of-sample forecasting results indicate that the threshold model provides informative help for predicting the likelihood of a financial stress event and outperforms two benchmark models based on a linear regression and signal extraction approach across all forecasting horizons and criteria considered,[4] providing empirical evidence that incorporating the credit-regime switching improves the predictive ability of the financial stress events.

To highlight how this model performs in practice, we apply the threshold model to predict the subprime crisis that occurred in 2007Q3. The results show that the threshold model is able to issue a warning signal of the subprime crisis at both the short horizons (one quarter ahead and two quarters ahead) and the long horizon (two years ahead), while both the linear regression model and the signal extraction approach are unable to detect the subprime crisis for all of these horizons.

The paper is organized as follows. Section 2 presents the definition of a financial stress event, and introduces how to use a threshold regression (credit-regime-switching) model to predict the likelihood of a financial stress event for a given future time. Section 3 evaluates the performance of our early warning model, and section 4 concludes.

[4]The signal extraction approach used in Christensen and Li (2014) can only be used to predict the likelihood of financial stress events within a given period of time, but it cannot be directly used to predict the likelihood of a financial stress event at a given future time. As a result, we have to modify the signal extraction approach proposed by Christensen and Li (2014) to predict the likelihood of a financial stress event at a given future time. The predicted probability of a financial stress event at a given future time will be estimated in Eq. (10).

2 Model Specification

In this section, we propose a new early warning model based on a threshold regression to predict the likelihood of a financial stress event for a given future time. For comparison, we also consider two benchmark models based on a linear regression and signal extraction approach.

2.1 The definition of a financial stress event

Financial stress can be characterized as a situation in which large parts of the financial sector face the prospects of large financial losses. In general, financial stress is unobservable, but some key features are frequently associated with an increased degree of perceived risk and uncertainty. To capture these features of financial stress, Illing and Liu (2006) propose an FSI as a measure of financial stress in Canada, which is a weighted average of various indicators of expected loss, risk and uncertainty in the financial sector. In constructing the FSI, Illing and Liu consider several weighting options and settle on weights that reflect relative shares of credit for particular sectors in the economy. The resulting financial stress index is a continuous, broad-based measure that includes the following indicators from equity, bond and foreign exchange markets, as well as indicators of banking-sector performance:

a. the spread between the yields on bonds issued by Canadian financial institutions and yields on government bonds of comparable duration;

b. the spread between yields on Canadian non-financial corporate bonds and government bonds;

c. the inverted term spread (i.e., the 90-day treasury bill rate minus the 10-year government yield);

d. the beta derived from the total return index for Canadian financial institutions;

e. Canadian trade-weighted dollar GARCH volatility;

f. Canadian stock market GARCH volatility;

g. the difference between Canadian and US government short-term borrowing rates;

h. the average bid-ask spread on Canadian treasury bills;

i. the spread between Canadian commercial paper rates and treasury bill rates of comparable duration.

Given the measure of financial stress and the sample period from 1981Q2 to 2009Q4, Li and St-Amant (2010) find that the Canadian economy can be characterized by normal and distressed regimes.[5] The Canadian economy is in the normal regime if the value of the financial stress index is lower than 49.96. Otherwise, the Canadian economy is in the distressed regime. In the normal regime, the Canadian economy has low financial stress and high economic activity, such as high economic growth, a low interest rate and low inflation. The distressed regime has low economic activity, a high interest rate and high inflation.

As a first step, we define a financial stress event as occurring when the FSI rises above an extreme value, 49.96.[6] Figure 1 shows the FSI for Canada for the sample period starting from 1982Q1 to 2013Q4. The horizontal line is the threshold value, 49.96. The higher values of the index indicate the higher financial stress. When the FSI exceeds this threshold value, it indicates the

[5]Using a threshold vector autoregression of output growth, short-term interest rate, inflation, and FSI, Li and St-Amant (2010) find that the Canadian economy can be characterized by regimes of low and high financial stress, that monetary policy actions have stronger effects when financial stress is high, and that a tightening of monetary policy tends to have more impact than an easing.

[6]We conduct a robustness check for the threshold value. The result indicates that the threshold value is quite stable when the sample period starts from 1982Q1 to 2005Q1, 1982Q1 to 2005Q2, ..., or 1982Q1 to 2013Q4. Some authors define a financial stress event as follows:

$$hfs_{j,t} = \begin{cases} 1 & \text{if } FSI_t > \mu_{FSI} + k\sigma_{FSI} \\ 0 & \text{otherwise,} \end{cases}$$

where μ_{FSI} and σ_{FSI} are the sample mean of the FSI and the sample standard deviation. Christensen and Li (2014), Cardarelli et al. (2011), and Illing and Liu (2006) use $k = 1.5, 1$ and 2 in the above equation, respectively, to identify financial stress events. Our exercises show that the different definitions of a financial stress event do not impact the results of the comparison of the forecasting performance.

occurrence of a financial stress event. The cataloguing of the financial stress events obtained by this choice tends to follow closely the chronology of financial market stress described in the literature, which suggests that the FSI was most effective in correctly signalling financial stress events that are widely associated with high financial stress (e.g., the stock market crash in October 1987, the peso crisis in 1994 and the long-term capital management crisis in 1998). This is not surprising, given that Canada is a small open economy whose markets are well integrated internationally. As such, it is not insulated from international financial developments. Turmoil in international financial markets will be reflected in increased stress in Canadian markets. This does not mean that financial stress is not or cannot be domestically generated, but it may indicate that the level of external financial stress can spill over into Canadian financial markets.

2.2 Data

As a highly open economy, the Canadian financial system is necessarily exposed to global financial stresses such as the 1994 peso crisis, the 1997 Asian crisis and the 2007 subprime crisis. For this reason, the data set incorporates, in addition to a broad set of domestic variables, several foreign variables. The explanatory variables are divided into the following four categories:

(i) credit measures: the ratio between credit and GDP, household credit, business credit;

(ii) asset prices: the stock market index, the ratio between housing prices and personal disposable income, new house prices;

(iii) macroeconomic variables: the ratio between investment and GDP, CPI core inflation, exchange rate depreciation, real GDP, real investment, M2++, household disposable income, FSI;

(iv) foreign variables: the US federal funds rate, crude oil prices, world gold prices, US commercial bank credit.

Since the values of the FSI are very volatile, in terms of their persistence, implying that frequently switching between a financial stress event and a normal state of financial stress is implausible, we construct a quarterly data-based early warning model of financial stress events. As well, some important indicators (for example, GDP and the ratio between credit and GDP) are available only at quarterly frequencies. The data are converted into quarterly format and span from 1982Q1 to 2013Q4. The variables are all transformed into annual growth rates, since it is possible that longer-run cumulative growth rates in the explanatory variables may contain more information about financial stress than quarterly growth rates.

2.3 A credit-regime-switching approach

2.3.1 Threshold regression model

There is evidence that, due to the existence of frictions arising from informational asymmetries and contractual rigidities, unusually large movements in credit may lead to greater financial uncertainty (Borio and Lowe 2002; Kaminsky et al. 1998). Consequently, the developments in the credit markets may provide an early warning indicator of financial stress in the financial system. In particular, if the herding mentality replaces rational financial decisions, then the relationship between the FSI and some of its explanatory variables may display non-linearity. Within the class of possible non-linear models for the transformed data, we concentrate on a threshold regression model that changes regime if credit conditions cross a critical threshold.[7] The threshold regression model is specified as

$$\begin{aligned} FSI_t = \quad & \alpha_1 \quad + \beta_1 FSI_{t-k} + \gamma_1 x_{t-k} + \delta_1 z_{t-k} \\ + \quad & [\alpha_2 + \beta_2 FSI_{t-k} + \gamma_2 x_{t-k} + \delta_2 z_{t-k}] I(z_{t-k} > \tau) + \varepsilon_t, \end{aligned} \tag{1}$$

[7] This type of threshold regression model provides a relatively simple and intuitive way to model non-linearity such as regime switching, asymmetry, and multiple equilibria implied by the theoretical models of financial and macroeconomic activity (Balke, 2000).

where k is the forecasting horizon of a financial stress event, x_{t-k} is a vector of explanatory variables, z_{t-k} is a credit variable extracted from the vector x_{t-k}, $I(\cdot)$ is the indicator function ($I(z_{t-k} > \tau) = 1$ if $z_{t-k} > \tau$. Otherwise, $I(z_{t-k} > \tau) = 0$), and τ is the threshold value that triggers a regime change.

Choosing the length of the forecasting horizon requires a balance between two opposite requirements. On the one hand, a financial stress event can be anticipated more reliably the closer the financial stress event. On the other hand, from a policy-maker's perspective it is desirable to have an indication of a financial stress event as early as possible in order to be able to take pre-emptive policy measures. In this paper, to evaluate the forecasting performance for different forecasting horizons, the forecasting horizons are taken as one quarter, two quarters, four quarters and eight quarters.

The model is estimated by least squares (LS).[8] By definition, the LS estimators minimize jointly the sum of the squared errors, $S_n(\tau)$. For this minimization, τ is assumed to be restricted to a bounded set $\Gamma = [\tau_1, \tau_2]$, where Γ is an interval covering the sample range of the threshold variables. The computationally easiest method to obtain the LS estimators is through concentration. Conditional on τ, the estimation yields the conditional estimators. The concentrated sum of squared residuals is a function of τ. $\hat{\tau}$ is the value that minimizes $S_n(\tau)$. Since $S_n(\tau)$ can take on less than n (the time span of the time series) distinct values, τ can be identified uniquely as

$$\hat{\tau} = argmin_{\tau \in \Gamma} S_n(\tau). \tag{2}$$

We consider three alternative credit variables as threshold variables: the growth rate of the ratio between total credit and GDP, the growth rate of business credit, and the growth rate of household credit. Corresponding to the three different threshold variables, the estimated parameters of the

[8] See Hansen (2000) for more details on the estimation of threshold models. Note that the estimators from LS are the maximum likelihood estimators when the errors are $i.i.d.N(0, \sigma^2)$.

threshold regression models at each horizon ($k = 1, 2, 4$ and 8) are reported in Table 1, Table 2 and Table 3, respectively.

We note that, in general, the US federal funds rate, US credit growth, oil price growth and gold price growth do not impact the financial stress significantly. This result is consistent with Misina and Tkacz (2009), who find that international variables play a smaller role than one would expect in a small open economy.

Given the information set I_t at time t, the probability that a financial stress event will occur at time $t+k$ can be expressed as

$$P[FSI_{t+k} \geq 49.96 | I_t]. \tag{3}$$

Since we do not know the distribution function of FSI_t, we resample the residuals, which are the estimations of the errors in (1) to non-parametrically estimate this probability in (3). Since the errors in (1) are correlated, the block bootstrapping method is used to resample the residuals. The residuals are split into $n-b+1$ overlapping blocks of length b: b observations starting from 1 to b will be block 1, b observations from 2 to $b+1$ will be block 2, etc. From the $n-b+1$ block, n/b blocks will be drawn at random with replacement. Then, aligning these n/b blocks in the order they were picked will give the bootstrap samples, which are used to estimate the probability of a financial stress event at time $t+k$. The block bootstrap procedure used to estimate the probability forecast comprises four steps, as described below.

Step 1: Use the original sample to estimate the unknown parameters in (1), and obtain the residual:

$$\begin{aligned} \hat{\varepsilon}_t = \quad & FSI_t \quad -\hat{\alpha}_1 - \hat{\beta}_1 FSI_{t-k} - \hat{\gamma}_1 x_{t-k} - \hat{\delta}_1 z_{t-k} \\ & - \quad \hat{\alpha}_2 - \hat{\beta}_2 FSI_{t-k} - \hat{\gamma}_2 x_{t-k} - \hat{\delta}_2 z_{t-k}. \end{aligned} \tag{4}$$

Step 2: Obtain the bootstrapping residuals $\{\varepsilon_t^*\}$ by the block bootstrap method as introduced above.

Step 3: Use the bootstrapping residuals ε_i^* to compute

$$
\begin{aligned}
FSI_i^{*j} = \quad & \hat{\alpha}_1 + \hat{\beta}_1 FSI_{t-k} + \hat{\gamma}_1 x_{t-k} + \hat{\delta}_1 z_{t-k} + [\hat{\alpha}_2 + \hat{\beta}_2 FSI_{t-k} \\
& + \hat{\gamma}_2 x_{t-k} + \hat{\delta}_2 z_{t-k}] I(z_{t-k} > \hat{\tau}) + \varepsilon_i^*,
\end{aligned} \tag{5}
$$

and obtain the bootstrapping estimation of the probability of a financial stress event:

$$
B^j = \frac{1}{n} \sum_{i=1}^{n} I[FSI_i^{*j} \geq 49.96]. \tag{6}
$$

Step 4: Repeat Step 2-Step 3 R times; the probability of a financial stress event at time t is computed as

$$
\frac{1}{R} \sum_{j=1}^{R} B^j. \tag{7}
$$

2.3.2 Cut-off probability

Given a forecasting horizon k, we can predict the probability of a financial stress event at time $t+k$. For a given predicted probability, the decision maker must decide whether the predicted probability is large enough to issue a warning, because taking no action is costly when a financial stress is nearing, but so is taking action when a financial event is not impending.[9] Since the predicted probability is a continuous variable, one needs to specify a threshold probability above which the predicted probability can be interpreted as a signal of an impending financial stress event. It should be noted that the lower the chosen threshold probability, the more signals the model will send, with

[9] An efficient warning model should minimize false alarms, since issuing a warning will lead to some sort of preventive actions, which are usually costly. For instance, the decision maker may invest in gathering further information, such as holding discussions with senior managers of the financial markets, bank supervisory agencies, or other market participants. Alternatively, the decision maker may use the monitoring model to decide whether to take preventive policy measures, such as tightening prudential capital or liquidity requirements for banks, or reducing interest rates to ease pressures on bank balance sheets.

the drawback that the number of wrong signals will increase. By contrast, raising the threshold probability reduces the number of wrong signals, but at the expense of increasing the number of missed financial stress events. Thus, a decision maker needs to choose a threshold probability that minimizes a loss function.

For a given threshold probability chosen by the decision maker, if the forecasted probability of a financial stress event at time $t+k$ exceeds the probability threshold, the model will signal a warning. Otherwise, the model will be silent. Thus, for a given threshold probability we can obtain a binary time series of signal or no-signal observations, which is then checked against actual events to construct the optimal probability threshold. For this purpose, we need to use the following two-dimensional matrix with four possible scenarios to build up a measure of predictive accuracy:

	Financial stress event	No financial stress event
Signal issued	A	B
No signal issued	C	D

In this matrix, A is the number of quarters in which the model issued correct signals; B is the number of quarters in which the model issued wrong signals; C is the number of quarters in which the model failed to issue a signal; and D is the number of quarters in which the model refrained from issuing signals. If the early warning model issues a signal that is followed by a financial stress event within the next k quarters, then $A > 0$ and $C = 0$, and if it does not issue a signal that is not followed by a financial stress event, then $B = 0$ and $D > 0$. A perfect model would only produce A and D, and $B = 0$ and $C = 0$.

The higher the threshold probability setting for calling a financial stress event, the higher will be the probability of a type I error (failure to call a financial stress event) and the lower will be the probability of a type II error (false alarm). We obtain the optimal threshold probability at which the noise-to-signal ratio, which is defined as $[B/(B+D)]/[A/(A+C)]$, is minimized. To obtain the optimal threshold probability, a grid search is performed over the range of potential threshold probabilities from 0.15 to 0.50. The probability value where the noise-to-signal ratio is at a minimum is chosen and is called the cut-off probability.

In order to highlight the importance that credit plays as a non-linear propagator of shocks in early warning of a financial stress event, we compare the forecasting performance between the early warning model based on the threshold regression model and the early warning model based on a linear regression model:

$$FSI_t = \alpha + \beta FSI_{t-k} + \gamma x_{t-k} + \varepsilon_t, \tag{8}$$

where the FSI is a linear function of the k-quarter lagged FSI and the k-quarter lagged explanatory variables x. Given the residuals from the linear least squares regression, we use the same bootstrapping method as for the threshold regression model to non-parametrically estimate the forecast probability of a financial stress event at a given future time.

2.4 Signal extraction approach

For a comparative study of the forecasting performance across different models, we expand the signal extraction approach proposed by Kaminsky, Lizondo and Reinhart (1998) to predict the likelihood of financial stress events at a given future time. To achieve this goal, we monitor the evolution of a number of indicators (the explanatory variables in (1)) that tend to show unusual behaviour in the period preceding a financial stress event. At time t, an indicator j is denoted by X_t^j. A signal variable relating to j is denoted by S_t^j constructed as a binary variable: $S_t^j = \{0,1\}$. If the indicator crosses the threshold denoted by X^{*j}, a signal is issued and $S_t^j = 1$. If the indicator remains within its threshold boundary, it behaves normally and does not issue a signal: thus, $S_t^j = 0$.

The optimal threshold for indicator j is calculated to minimize $[B/(B+D)]/[A/(A+C)]$, where A is the number of quarters in which the indicator signals a financial stress event and a financial stress event occurs at time $t+k$; B is the number of quarters in which the indicator issued a signal but a financial stress event did not occur in reality; C is the number of quarters in which the indicator failed to issue a signal; and D is the number of quarters in which the indicator refrained from issuing a signal.

We combine information provided by all indicators to build the composite indicator by weighting the signal of each of the indicators with the inverse of its noise-to-signal ratio. Given the forecast horizon level k ($k = 1, 2, 4, 8$), the composite indicator is defined as

$$I_t^k = \sum_{j=1}^{n} \frac{S_t^j}{\omega^j(k)}, \tag{9}$$

where $\omega^j(k)$ is the noise-to-signal ratio of indicator j for the given forecast horizon level k.

When the composite indicator I_t^k lies in the interval $(a,b]$, the probability of a financial stress event at a given future time $t+k$, expressed by $P[C_{t,t+k}|a < I_t^k \leq b]$, can be estimated by

$$\begin{aligned} &\hat{P}[C_{t,t+k}|a < I_t^k \leq b] = \\ &\frac{\text{Quarters with } a < I_t^k \leq b \text{ and a financial stress event at } t+k}{\text{Quarters with } a < I_t^k \leq b}, \end{aligned} \tag{10}$$

where $C_{t,t+k}$ is the occurrence of a financial stress event at time $t+k$. In this paper, we use the approach proposed by Diebold and Rudebusch (1989) to choose the intervals.[10]

Using information on the quarterly values of the composite indicators, and the probabilities of financial stress events, we can construct series of probabilities of financial stress events both in-sample and out-of-sample.

[10] Details on how to use the approach proposed by Diebold and Rudebusch (1989) to choose the interval are introduced in Christensen and Li (2014).

3 Predictive Ability

For a given forecasting horizon k ($k = 1, 2, 4, 8$), we divide our data into two subsamples. The first subsample, from 1982Q1 to k quarters before 2007Q1, is used to estimate the model parameters. The second subsample, from 2007Q1 to 2013Q4, is used to evaluate the out-of-sample performance.[11] We take $R = 100$ in the bootstrapping method to estimate the probability of a financial stress event at $t + k$. The block lengths of $b = \{2, 4, 5\}$ are tried. We only report the results from $b = 5$, because the results seem to be quite robust to the choice of block length, b.

Let A, B, C and D be the same as in the matrix used to build up the cut-off probability. The probability forecast evaluation is based on five different criteria, namely, the signal-noise ratio (the inverse of the noise-signal ratio), the probability of financial stress events being correctly called, the probability of false alarms in total alarms, the conditional probability of financial stress events given an alarm, and the conditional probability of financial stress given no alarm. The probability of financial stress events being correctly called is defined as the percentage of the correct alarms out of the total number of financial stress events, $A/(A+C)$. The probability of false alarms in total alarms is defined as the percentage of the false alarms out of the total alarms, $B/(A+B)$. The conditional probability of financial stress events given an alarm is defined as the percentage of correct alarms out of total alarms, $A/(A+B)$. The conditional probability of financial stress events given no alarm is defined as the probability that an alarm does not issue but a financial stress event occurs, $C/(C+D)$.

Corresponding to the three credit conditions, the growth rate of the ratio between credit and GDP, the growth rate of business credit, and the growth rate of household credit, the early warn-

[11] For a robustness check, we also use the data from 1981Q2 to k quarters before 2007Q3 to estimate the model parameters, and 2007Q3 to 2013Q4 are used to evaluate out-of-sample performance. The results are qualitatively similar to those for observations from 1981Q2 to k quarters before 2007Q1.

ing models based on the threshold regression approach are denoted as threshold regression model 1 (Threshold 1), threshold regression model 2 (Threshold 2), and threshold regression model 3 (Threshold 3), respectively. Table 4 reports the in-sample forecasting performance of the threshold regression models under different forecasting horizons. For comparison, the forecasting performance of the linear model and the signal extraction model is also reported in Table 4. Table 4 shows that the signal-to-noise ratios for all models, with the exceptions of the linear model with $k = 8$ and Threshold 2 with $k = 8$, are higher than one, indicating that the in-sample forecasting performance of the three types of models is better than random guesses, suggesting that these models are useful tools for predicting financial stress events. We also note that at least one of the threshold regression models has better performance at correctly calling financial stress events and non-financial stress events, although none of these models outperforms the others across all criteria considered.

The in-sample predictive ability is important and can reveal useful information, but there is no guarantee that a model that fits historical data well will also perform well out-of-sample. It is important to note that the value of an early warning model of financial stress events lies in its ability to provide policy-makers early warning of impending financial stress events, which depends on its out-of-sample predictive ability.

Table 5 reports the out-of-sample forecasting accuracy for different forecasting horizons. For Threshold 1 with $k = 1$ and Threshold 3 with $k = 1$ and $k = 2$, since the number of models that issue wrong signals is zero, the signal-noise ratios are infinity. The out-of-sample values of the signal-noise ratios of all these models, with the exception of the signal extraction model with $k = 4$ and the linear model with $k = 8$, are greater than one, suggesting that these models provide helpful information for predicting the likelihood of a financial stress event. The performance of the threshold regression models is notably better than other models across all criteria and forecasting horizons considered. For example, under forecasting horizon $k = 4$, moving the signal extraction model to Threshold 1 or Threshold 2 increases the probability of financial stress events correctly called from 0.23 to 0.92, while reducing false alarms from 0.11 to 0.08. Similarly, the conditional probability of experiencing a financial stress event when an alarm is issued rises from 0.89 to 0.95, and the conditional probability of correctly calling non-financial stress events increases from 0.10 to 0.33. Overall, the out-of-sample forecasting results show that the threshold regression models perform better than the two others across all criteria and all forecasting horizons considered, providing empirical evidence of the importance of credit as a non-linear propagator of shocks, and the non-linearity implied by the regime switching is an important contributor to predicting the likelihood of a financial stress event at a given future time.

Figures 2 to 5 plot the predicted probabilities of the threshold regression model (Threshold 3), the linear model and the signal extraction model for different forecasting horizons. The shaded regions represent the time spans of the well-known financial crises. In particular, for a given forecasting horizon k, the out-of-sample performance of these models for predicting the 2007 subprime crisis can be outlined as follows. In 2007Q1 and 2007Q2, the predicted probabilities of the threshold model for two-quarter and one-quarter horizons are, respectively, 0.32 and 0.28, which are higher than their respective cut-off probabilities for issuing a warning signal, indicating that the threshold model is able to detect the impending subprime crisis in 2007Q3 for both the one-quarter and two-quarter horizons. Notably, neither the linear model nor the signal extraction model is able to issue a warning signal of the subprime crisis for the one-quarter or the two-quarter horizons. For $k = 3$, in 2006Q4 all three models are unable to predict the subprime crisis in 2007Q3. For $k = 4$, in 2005Q3 only the threshold model issues a warning signal of the subprime crisis. Additionally, a noticeable feature of these figures is that the predicted probabilities are significantly higher than their cut-off probabilities for most of the duration of the subprime crisis. This result is consistent with the fact that Canada experienced financial stress events during the subprime crisis. Overall, the empirical results show that the threshold model performs better than the linear model and the signal extraction model, which indicates that incorporating credit-regime switching improves the predictive ability of the financial stress events.

4 Conclusion

This paper proposes an early warning model to predict a financial stress event at a given future time. In this model, the non-linear relationship between the FSI and the explanatory variables is captured by a threshold regression model in which a regime change occurs if credit conditions cross a critical threshold. A non-parametric bootstrap procedure is used to simulate the future paths of the FSI in the threshold regression model. The simulated future values of the FSI are used to predict the likelihood of a financial stress event for a given future time.

The in-sample and out-of-sample forecasting results are encouraging. In particular, the out-of-sample forecasting results suggest that the model based on the credit-regime-switching approach outperforms the benchmark models based on a linear regression and signal extraction approach across all forecasting horizons and all criteria considered.

In future research, additional explanatory variables should be considered, such as the volatility of the stock market return and the explanatory variables that capture financial contagion across different financial sectors. Finally, it should be stressed that although the early warning model proposed is useful as a diagnostic tool, it must be complemented by more standard surveillance and cannot substitute for such surveillance.

Table 1: **Estimation result for threshold regression model**

Threshold variable	The growth rate of the ratio between credit and GDP							
	$k=1$		$k=2$		$k=4$		$k=8$	
Threshold value	$\hat{\tau}_1=0.5641$		$\hat{\tau}_2=0.5625$		$\hat{\tau}_3=0.5681$		$\hat{\tau}_4=0.5655$	
Regime	$\leq\hat{\tau}_1$	$>\hat{\tau}_1$	$\leq\hat{\tau}_2$	$>\hat{\tau}_2$	$\leq\hat{\tau}_3$	$>\hat{\tau}_3$	$\leq\hat{\tau}_4$	$>\hat{\tau}_4$
Constant	58.44	175.99	63.01	75.77	62.11	35.22	47.60	-844.67
	(0.24)	(0.12)	(4.76)	(17.24)	(3.09)	(17.66)	(2.98)	(10.02)
FSI_{t-k}	0.24	0.63	-0.31	-0.13	0.01	-0.12	-0.03	0.02
	(0.12)	(0.20)	(0.15)	(0.21)	(-0.03)	(0.17)	(-0.18)	(0.79)
GDP_{t-k}	-2.41	-1.98	-3.83	1.91	-2.86	5.92	-0.43	1.15
	(0.97)	(2.76)	(1.13)	(2.98)	(0.94)	(6.48)	(1.33)	(3.67)
House price$_{t-k}$	-0.16	2.16	0.45	1.00	-1.38	-1.23	1.04	-6.67
	(0.53)	(1.43)	(0.60)	(2.46)	(0.52)	(5.79)	(0.73)	(3.50)
Household income$_{t-k}$	-0.52	0.84	-1.86	1.76	1.55	-3.58	0.56	-7.18
	(0.61)	(1.34)	(0.69)	(1.09)	(0.60)	(2.35)	(0.56)	(3.53)
CPI_{t-k}	1.26	2.46	-0.25	-9.13	-3.72	0.79	-0.10	0.34
	(0.75)	(1.67)	(2.01)	(4.63)	(1.62)	(1.68)	(2.12)	(4.29)
Stock index$_{t-k}$	-0.05	-0.17	-0.20	0.05	0.33	0.04	-0.10	0.34
	(0.09)	(0.17)	(0.10)	(0.17)	(0.33)	(0.04)	(0.11)	(0.48)
Exchange rate$_{t-k}$	0.27	0.02	-0.60	1.52	0.37	-1.01	-0.52	2.63
	(0.31)	(0.59)	(0.39)	(0.72)	(0.29)	(1.46)	(0.41)	(2.36)
Fed fund rate$_{t-k}$	1.67	5.72	1.42	1.78	-0.44	-3.47	-3.78	-4.62
	(1.02)	(2.31)	(1.15)	(2.70)	(1.01)	(6.70)	(1.42)	(1.48)
M2++$_{t-k}$	3.66	-5.84	4.95	-5.57	-1.61	4.91	0.47	1.76
	(1.16)	(2.01)	(1.37)	(2.10)	(1.12)	(3.90)	(1.56)	(7.57)
Credit/GDP$_{t-k}$	111.12	-320.18	103.09	21.84	-86.07	-285.66	-35.33	1065.91
	(28.84)	(267.74)	(37.00)	(183.01)	(73.73)	(328.92)	(38.33)	(1527.78)
Investment$_{t-k}$	0.91	-0.37	0.97	0.49	0.02	-1.37	-0.10	0.19
	(0.26)	(0.55)	(0.97)	(0.49)	(0.33)	(0.38)	(1.04)	(0.42)
Investment/GDP$_{t-k}$	-679.01	77.54	-529.35	-30.89	184.15	739.41	182.37	1573.73
	(139.35)	(349.17)	(172.42)	(411.74)	(132.63)	(504.83)	(182.37)	(1088.93)
Oil price$_{t-k}$	0.12	-0.03	0.16	-0.04	-0.12	0.02	0.13	0.18
	(0.05)	(0.06)	(0.06)	(0.06)	(0.05)	(0.15)	(0.07)	(0.17)
Household credit$_{t-k}$	1.77	6.89	0.86	-0.91	2.52	-1.03	1.14	-2.37
	(0.59)	(2.76)	(0.70)	(2.99)	(0.54)	(4.25)	(0.74)	(8.20)
US credit$_{t-k}$	1.06	-3.72	1.88	-0.97	-2.75	1.99	0.97	4.01
	(0.70)	(1.23)	(0.86)	(1.31)	(0.70)	(2.13)	(0.97)	(4.01)
Business credit$_{t-k}$	0.92	1.03	0.60	0.23	1.73	1.23	-0.53	-1.19
	(0.46)	(1.08)	(0.53)	(1.26)	(0.45)	(2.28)	(0.62)	(4.89)
Gold price$_{t-k}$	-0.21	-0.18	-0.42	0.08	0.24	-0.39	-0.40	0.25
	(0.15)	(0.21)	(0.17)	(0.24)	(0.15)	(0.54)	(0.21)	(0.96)
House price/	0.05	-1.54	0.04	1.14	0.72	1.79	-0.86	-2.90
person income$_{t-k}$	(0.33)	(0.91)	(0.37)	(0.96)	(0.32)	(1.67)	(0.46)	(3.07)

This table reports the estimations of the coefficients in Eq. (1) when the growth rate of credit/GDP is taken as the threshold variable. The estimation period starts from 1982Q1 to k quarters before 2007Q1. The standard errors are reported in parentheses.

Table 2: **Estimation result for threshold regression model**

Threshold variable	The growth rate of business credit							
	$k = 1$		$k = 2$		$k = 4$		$k = 8$	
Threshold value	$\hat{\tau}_1 = 3.3140$		$\hat{\tau}_2 = 5.3969$		$\hat{\tau}_3 = 8.3732$		$\hat{\tau}_4 = 6.7490$	
Regime	$\leq \hat{\tau}_1$	$> \hat{\tau}_1$	$\leq \hat{\tau}_2$	$> \hat{\tau}_2$	$\leq \hat{\tau}_3$	$> \hat{\tau}_3$	$\leq \hat{\tau}_4$	$> \hat{\tau}_4$
Constant	58.09	20.77	63.08	61.01	62.24	-49.73	47.52	247.56
	(6.18)	(34.82)	(4.81)	(9.98)	(3.17)	(20.61)	(3.35)	(40.50)
FSI_{t-k}	-0.38	0.53	-0.24	0.07	0.33	-0.44	-0.53	0.14
	(0.46)	(0.08)	(0.19)	(0.13)	(0.10)	(0.33)	(0.22)	(0.31)
GDP_{t-k}	-3.95	0.24	0.94	0.67	1.35	5.47	-6.18	-3.01
	(4.28)	(1.02)	(2.26)	(1.51)	(1.44)	(3.92)	(2.53)	(2.95)
House price_{t-k}	5.50	0.25	-0.63	0.28	-2.54	-1.34	3.54	0.83
	(3.59)	(0.58)	(1.92)	(0.77)	(1.44)	(5.79)	(2.08)	(1.61)
Household income_{t-k}	-1.40	0.25	-2.00	-1.45	1.18	1.76	-0.43	1.76
	(1.89)	(0.52)	(1.19)	(0.78)	(0.67)	(1.55)	(1.32)	(1.46)
CPI_{t-k}	9.15	1.07	-3.66	-6.25	-1.95	-1.00	-1.74	-6.09
	(6.13)	(1.63)	(3.10)	(3.36)	(1.73)	(7.71)	(3.49)	(7.63)
Stock index_{t-k}	-1.44	0.03	-0.41	0.06	-0.03	-0.16	0.37	-0.11
	(0.41)	(0.07)	(0.17)	(0.11)	(0.09)	(0.25)	(0.29)	(0.26)
Exchange rate_{t-k}	-0.67	0.68	-0.56	-0.78	-1.12	1.08	1.04	-0.32
	(0.98)	(0.32)	(0.67)	(0.65)	(0.41)	(1.41)	(0.94)	(1.31)
Fed fund rate_{t-k}	-25.54	-0.23	-2.69	0.41	-2.26	-6.48	1.95	-5.96
	(9.40)	(0.69)	(3.04)	(1.22)	(0.99)	(6.39)	(3.00)	(4.08)
M2++_{t-k}	24.32	-0.91	1.99	0.40	3.93	-1.13	-1.30	4.94
	(6.48)	(0.94)	(2.00)	(1.43)	(1.41)	(3.26)	(2.42)	(3.25)
Credit/GDP_{t-k}	-473.55	13.36	-111.00	-62.01	-36.34	7.49	-25.11	-62.82
	(256.01)	(32.18)	(75.51)	(47.75)	(31.76)	(100.31)	(73.79)	(108.79)
Investment_{t-k}	-1.72	-0.06	0.36	-0.83	0.30	0.02	-0.11	0.82
	(0.84)	(0.23)	(0.35)	(0.31)	(0.24)	(0.65)	(0.48)	(0.58)
$\text{Investment/GDP}_{t-k}$	1360.40	-130.25	307.45	70.59	-194.84	700.59	128.64	-878.28
	(800.47)	(143.97)	(314.59)	(206.78)	(148.61)	(460.34)	(338.78)	(477.61)
Oil price_{t-k}	-0.22	0.15	0.12	-0.01	-0.05	0.03	0.28	-0.06
	(0.15)	(0.03)	(0.08)	(0.05)	(0.05)	(0.14)	(0.10)	(0.13)
Household credit_{t-k}	13.09	1.27	2.24	-0.02	1.27	2.18	1.70	0.70
	(5.61)	(0.45)	(1.47)	(0.82)	(0.60)	(2.35)	(1.29)	(1.73)
US credit_{t-k}	-1.10	-0.39	-1.96	-1.09	-1.55	-0.04	-0.39	-0.14
	(1.43)	(0.56)	(0.95)	(0.90)	(0.66)	(3.29)	(1.33)	(3.00)
Business credit_{t-k}	-8.47	2.05	-1.38	3.74	1.76	1.02	2.14	1.74
	(5.66)	(0.58)	(1.15)	(0.84)	(0.57)	(2.09)	(1.56)	(1.86)
Gold price_{t-k}	1.13	0.05	-0.12	0.24	0.14	-0.47	-0.60	-0.66
	(0.65)	(0.10)	(0.29)	(0.15)	(0.17)	(0.76)	(0.38)	(0.57)
House price/	-2.34	-0.25	-1.81	0.00	0.82	1.59	-0.88	0.13
personal income_{t-k}	(0.33)	(0.91)	(0.37)	(0.96)	(0.32)	(1.67)	(0.46)	(3.07)

This table reports the estimations of the coefficients in Eq. (1) when the growth rate of the business credit is taken as the threshold variable. The estimation period starts from 1982Q1 to k quarters before 2007Q1. The standard errors are reported in parentheses.

Table 3: **Estimation result for threshold regression model**

Threshold variable	The growth rate of household credit							
	$k=1$		$k=2$		$k=4$		$k=8$	
Threshold value	$\hat{\tau}_1 = 6.5245$		$\hat{\tau}_2 = 6.7790$		$\hat{\tau}_3 = 6.7183$		$\hat{\tau}_4 = 6.3077$	
Regime	$\leq \hat{\tau}_1$	$> \hat{\tau}_1$	$\leq \hat{\tau}_2$	$> \hat{\tau}_2$	$\leq \hat{\tau}_3$	$> \hat{\tau}_3$	$\leq \hat{\tau}_4$	$> \hat{\tau}_4$
Constant	57.64	101.15	62.55	48.40	62.20	76.61	47.58	230.84
	(6.12)	(4.09)	(5.22)	(6.49)	(2.85)	(6.46)	(3.19)	(8.99)
FSI_{t-k}	0.42	0.26	-0.06	-0.07	0.40	-0.06	-0.10	0.21
	(0.16)	(0.13)	(0.17)	(0.19)	(0.14)	(0.14)	(0.40)	(0.21)
GDP_{t-k}	-1.81	-1.42	-0.90	-1.03	1.33	-2.65	-4.66	-2.65
	(1.80)	(1.14)	(1.91)	(1.72)	(1.49)	(1.35)	(1.53)	(1.67)
House price_{t-k}	-2.29	-0.58	-1.65	-0.39	-1.51	-1.84	-4.30	1.50
	(0.95)	(1.00)	(1.02)	(1.04)	(0.83)	(1.18)	(1.08)	(1.50)
Household income_{t-k}	0.33	0.82	0.58	-0.91	1.21	1.35	0.53	2.08
	(0.87)	(0.65)	(1.01)	(0.87)	(0.80)	(0.66)	(0.53)	(1.05)
CPI_{t-k}	-4.21	-0.62	-3.74	-0.24	-5.94	-0.09	-5.30	-0.06
	(2.13)	(1.63)	(1.15)	(3.36)	(1.93)	(1.88)	(1.25)	(0.27)
Stock index_{t-k}	0.07	0.00	0.26	-0.16	0.09	0.03	0.25	0.02
	(0.15)	(0.09)	(0.15)	(0.13)	(0.12)	(0.09)	(0.40)	(0.17)
Exchange rate_{t-k}	1.61	-0.05	1.07	-0.88	-1.49	-0.15	3.01	-0.45
	(0.54)	(0.36)	(0.60)	(0.51)	(0.41)	(0.49)	(0.40)	(0.71)
Fed fund rate_{t-k}	-3.88	1.55	-6.32	2.93	-1.52	-0.85	-3.51	-2.44
	(2.35)	(0.90)	(2.41)	(1.24)	(2.03)	(0.99)	(5.04)	(1.47)
M2++_{t-k}	0.38	-0.25	-0.99	5.11	1.69	0.21	0.96	-0.13
	(1.32)	(1.46)	(1.46)	(2.12)	(1.22)	(1.47)	(1.60)	(2.34)
Credit/GDP_{t-k}	-128.42	18.04	-224.55	91.13	-99.67	-4.79	38.89	-78.65
	(45.71)	(37.29)	(47.24)	(48.47)	(122.32)	(64.98)	(13.79)	(18.79)
Investment_{t-k}	0.67	0.27	-0.49	0.37	-0.12	0.34	1.28	0.79
	(0.43)	(0.28)	(0.35)	(0.31)	(0.24)	(0.65)	(0.48)	(0.58)
$\text{Investment/GDP}_{t-k}$	359.33	-525.38	704.50	-485.40	48.93	-220.86	6.60	-884.48
	(204.30)	(195.30)	(201.34)	(282.99)	(181.14)	(274.50)	(354.96)	(377.18)
Oil price_{t-k}	0.01	0.04	0.08	-0.17	-0.01	0.08	0.00	0.28
	(0.08)	(0.03)	(0.08)	(0.07)	(0.06)	(0.04)	(0.15)	(0.06)
Household credit_{t-k}	-2.13	1.98	0.94	-0.56	3.28	0.55	-3.91	2.08
	(1.64)	(0.89)	(1.58)	(1.48)	(1.34)	(1.13)	(2.85)	(2.10)
US credit_{t-k}	-1.10	-0.39	-1.96	-1.09	-1.55	-0.04	-0.39	-0.14
	(1.43)	(0.56)	(0.95)	(0.90)	(0.66)	(3.29)	(1.33)	(3.00)
Business credit_{t-k}	2.55	1.82	3.00	1.28	2.64	3.30	3.74	3.05
	(0.48)	(0.83)	(0.51)	(1.12)	(0.42)	(0.90)	(1.51)	(1.27)
Gold price_{t-k}	-0.70	0.16	-0.68	0.17	0.55	-0.39	-0.25	-0.65
	(0.30)	(0.11)	(0.32)	(0.18)	(0.26)	(0.23)	(0.75)	(0.29)
House price/_{t-k}	-0.23	-0.26	-2.06	0.61	-3.05	-0.43	0.33	0.65
personal income_{t-k}	(0.47)	(0.64)	(1.32)	(0.91)	(1.05)	(0.69)	(2.33)	(1.14)

This table reports the estimations of the coefficients in Eq. (1) when the growth rate of the household credit is taken as the threshold variable. The estimation period starts from 1982Q1 to k quarters before 2007Q1. The standard errors are reported in parentheses.

Table 4: **In-Sample Predictive Ability from Alternative Models**

Model	Cut-off probability	Signal-noise ratio	Stress events correctly called	False alarms	Correctly call given alarm	Correctly call non-financial stress events
$\underline{k=1}$						
Linear	0.35	3.79	0.71	0.45	0.55	0.90
Threshold 1	0.35	4.09	0.71	0.43	0.57	0.90
Threshold 2	0.35	3.96	0.79	0.44	0.56	0.92
Threshold 3	0.30	3.37	0.58	0.48	0.52	0.86
Signal extraction	0.10	4.99	0.62	0.38	0.62	0.88
$\underline{k=2}$						
Linear	0.35	3.13	0.71	0.50	0.50	0.89
Threshold 1	0.35	3.33	0.67	0.48	0.52	0.88
Threshold 2	0.35	4.17	0.67	0.43	0.57	0.89
Threshold 3	0.25	2.78	0.67	0.53	0.47	0.88
Signal extraction	0.20	3.89	0.54	0.46	0.54	0.86
$\underline{k=4}$						
Linear	0.30	1.69	0.54	0.65	0.35	0.82
Threshold 1	0.30	1.81	0.46	0.63	0.37	0.81
Threshold 2	0.25	1.56	0.46	0.67	0.33	0.80
Threshold 3	0.35	1.56	0.42	0.67	0.33	0.80
Signal extraction	0.35	3.13	0.47	0.53	0.47	0.75
$\underline{k=8}$						
Linear	0.10	0.99	0.75	0.76	0.24	0.75
Threshold 1	0.35	2.01	0.38	0.61	0.39	0.80
Threshold 2	0.20	0.93	0.46	0.77	0.23	0.75
Threshold 3	0.25	1.17	0.50	0.73	0.27	0.88
Signal extraction	0.35	3.10	0.48	0.52	0.48	0.85

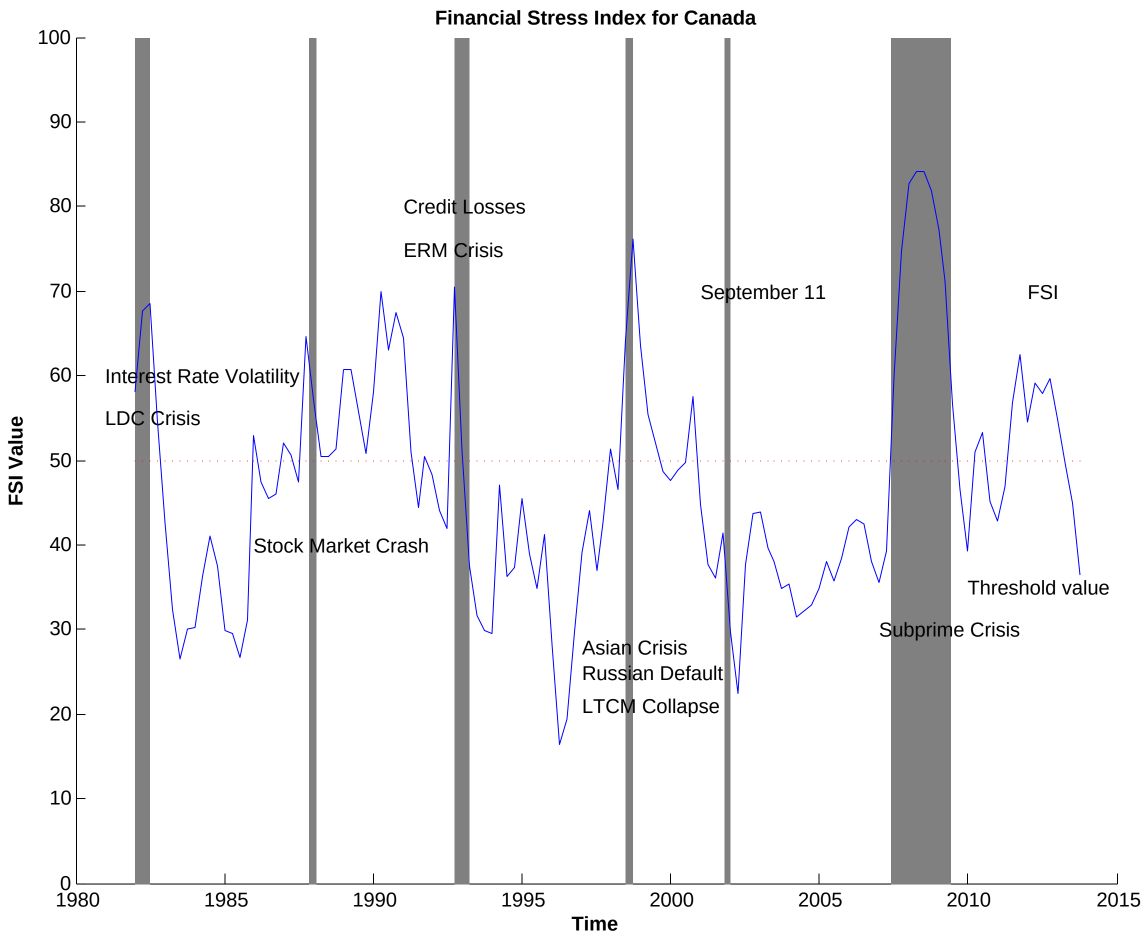
The cut-off probability comes from the in-sample estimation by minimizing the noise-to-signal ratio. Estimation period starts from 1982Q1 to k quarters before 2007Q1. The percentage of financial stress events correctly called is defined as $A/(A+C)$; the percentage of false alarms out of total alarms is defined as $B/(A+B)$; the probability of financial stress events given an alarm is defined as $A/(A+B)$; the probability of financial stress events given no alarm is defined as $C/(C+D)$. For the threshold regression models, the three alternative measures of credit market conditions are: credit/GDP, household credit, and business credit. Corresponding to the three credit conditions and each k, the threshold regression models are expressed as threshold regression model 1 (Threshold 1), threshold regression model 2 (Threshold 2), and threshold regression model 3 (Threshold 3), respectively.

Table 5: **Out-of-Sample Predictive Ability from Alternative Models**

Model	Cut-off probability	Signal-noise ratio	Stress events correctly called	False alarms	Correctly call given alarm	Correctly call non-financial stress events
$k = 1$						
Linear	0.35	2.52	0.84	0.05	0.95	0.33
Threshold 1	0.35	Inf	0.96	0.00	1.00	0.75
Threshold 2	0.35	2.88	0.96	0.04	0.96	0.77
Threshold 3	0.30	Inf	0.92	0.00	1.00	0.60
Signal extraction	0.10	8.59	0.78	0.01	0.99	0.33
$k = 2$						
Linear	0.35	2.40	0.80	0.05	0.95	0.29
Threshold 1	0.35	2.76	0.92	0.04	0.96	0.50
Threshold 2	0.35	2.76	0.92	0.04	0.96	0.50
Threshold 3	0.25	Inf	0.92	0.04	1.00	0.60
Signal extraction	0.20	6.13	0.36	0.02	0.98	0.15
$k = 4$						
Linear	0.30	1.56	0.52	0.07	0.93	0.14
Threshold 1	0.30	1.38	0.92	0.08	0.92	0.33
Threshold 2	0.25	1.38	0.92	0.08	0.92	0.33
Threshold 3	0.35	2.52	0.84	0.05	0.95	0.33
Signal extraction	0.35	0.93	0.23	0.11	0.89	0.10
$k = 8$						
Linear	0.10	0.76	0.76	0.14	0.86	0.00
Threshold 1	0.35	2.88	0.96	0.04	0.96	0.67
Threshold 2	0.20	2.88	0.96	0.04	0.96	0.67
Threshold 3	0.25	1.32	0.88	0.08	0.92	0.25
Signal extraction	0.35	1.10	0.28	0.10	0.90	0.11

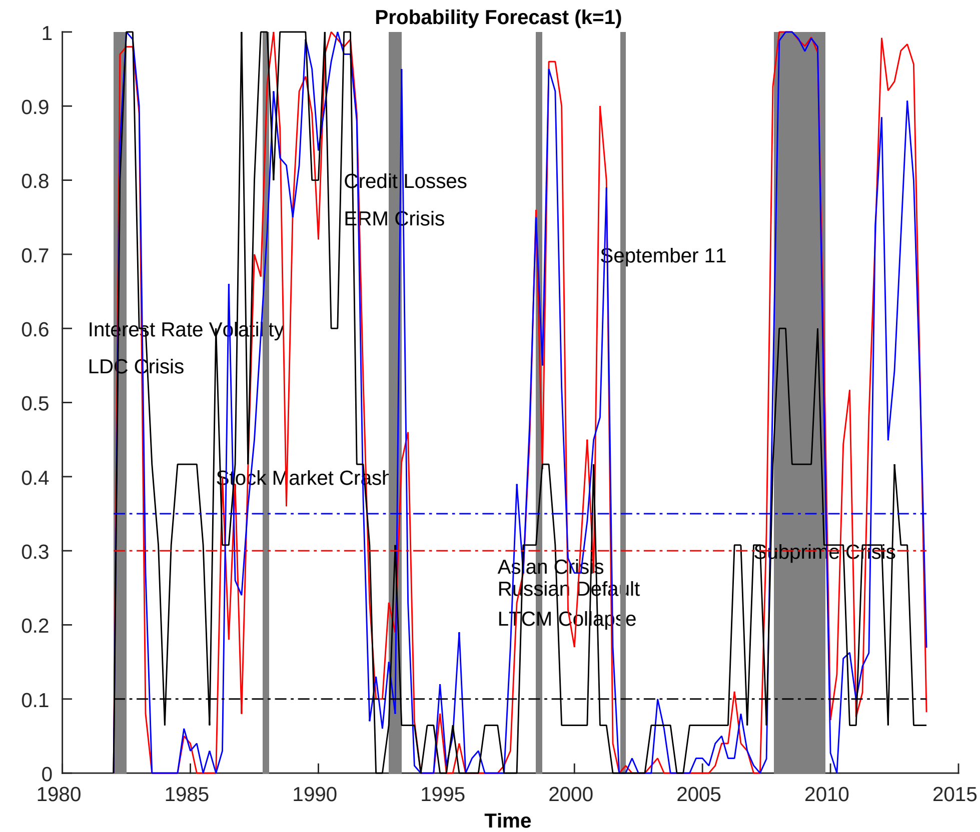
The cut-off probability comes from the in-sample estimation by minimizing the noise-to-signal ratio. The first subsample from 1982Q1 to k quarters before 2007Q1 is used to build up the model parameters, and the second subsample from 2007Q1 to 2013Q4 is used to evaluate out-of-sample early warning of financial stress events. The percentage of financial stress events correctly called is defined as A/(A+C); the percentage of false alarms out of total alarms is defined as B/(A+B); the probability of financial stress events given an alarm is defined as A/(A+B); the probability of financial stress events given no alarm is defined as C/(C+D). For the threshold regression models, the three alternative measures of credit market conditions are: credit/GDP, household credit, and business credit. Corresponding to the three credit conditions and each k, the threshold regression models are expressed as threshold regression model 1 (Threshold 1), threshold regression model 2 (Threshold 2), and threshold regression model 3 (Threshold 3), respectively.

Figure 1: **Financial Stress Indexes for Canada**



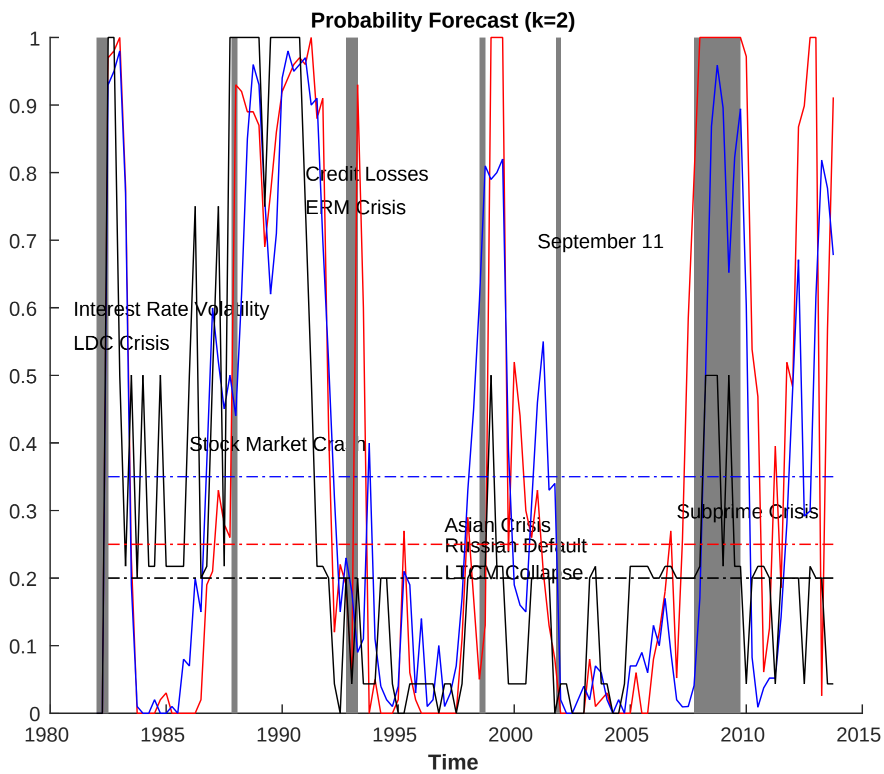
Notes: The shaded regions represent the time spans of financial crises. The solid line is FSI, and the dashed line is the threshold value.

Figure 2: **Probability Forecasts of Financial Stress Events**



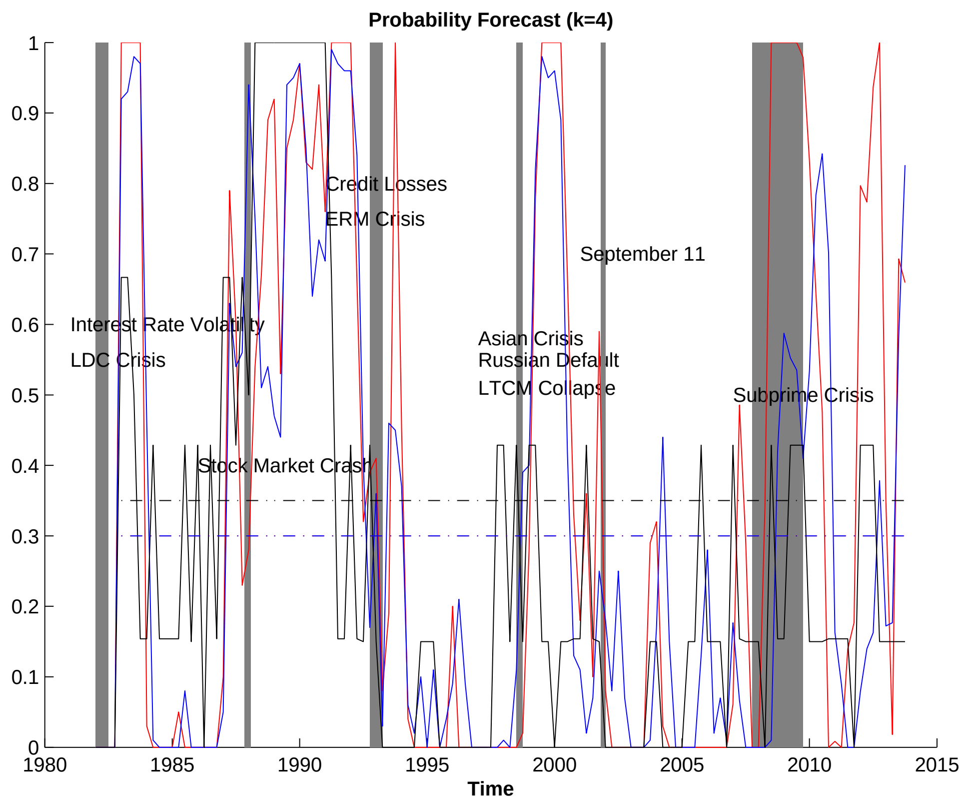
Notes: The shaded regions represent the time spans of financial crises. The solid lines are the forecasted probabilities, and the dashed lines are the cut-off probabilities. In particular, the solid red line is the forecasted probability of the threshold model, the solid blue line is the forecasted probability of the linear model, and the solid black line is the forecasted probability of the signal extraction model. The dashed red line is the cut-off probability of the threshold model, the dashed blue line is the cut-off probability of the linear model, and the dashed black line is the cut-off probability of the signal extraction model. Note that the cut-off probability of the threshold model is equal to the cut-off probability of the line model.

Figure 3: **Probability Forecasts of Financial Stress Events**



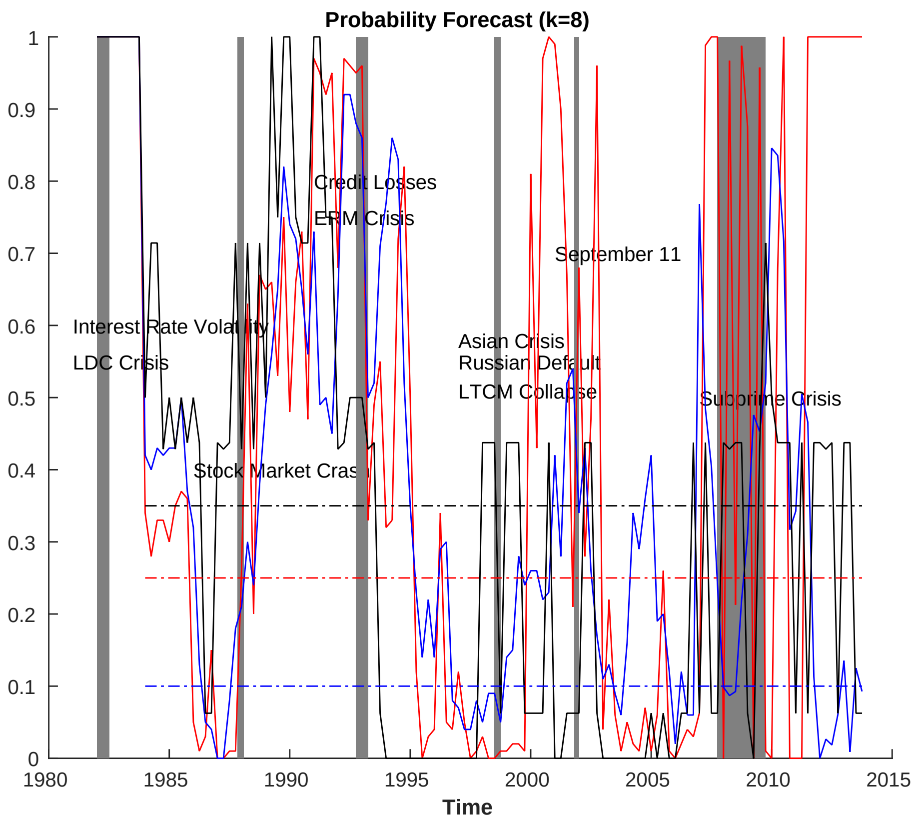
Notes: The shaded regions represent the time spans of financial crises. The solid lines are the forecasted probabilities, and the dashed lines are the cut-off probabilities. In particular, the solid red line is the forecasted probability of the threshold model, the solid blue line is the forecasted probability of the linear model, and the solid black line is the forecasted probability of the signal extraction model. The dashed red line is the cut-off probability of the threshold model, the dashed blue line is the cut-off probability of the linear model, and the dashed black line is the cut-off probability of the signal extraction model. Note that the cut-off probability of the threshold model is equal to the cut-off probability of the linear model.

Figure 4: **Probability Forecasts of Financial Stress Events**



Notes: The shaded regions represent the time spans of financial crises. The solid lines are the forecasted probabilities, and the dashed lines are the threshold values. In particular, the solid red line is the forecasted probability of the threshold model, the solid blue line is the forecasted probability of the linear model, and the solid black line is the forecasted probability of the signal extraction model. The dashed red line is the cut-off probability of the threshold model, the dashed blue line is the cut-off probability of the linear model, and the dashed black line is the cut-off probability of the signal extraction model. Note that the cut-off probability of the threshold model is equal to the cut-off probability of the signal extraction model.

Figure 5: **Probability Forecasts of Financial Stress Events**



Notes: The shaded regions represent the time spans of financial crises. The solid lines are the forecasted probabilities, and the dashed lines are the threshold values. In particular, the solid red line is the forecasted probability of the threshold model, the solid blue line is the forecasted probability of the linear model, and the solid black line is the forecasted probability of the signal extraction model. The dashed red line is the cut-off probability of the threshold model, the dashed blue line is the cut-off probability of the linear model, and the dashed black line is the cut-off probability of the signal extraction model. Note that the cut-off probability of the threshold model is equal to the cut-off probability of the signal extraction model.

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