On the Value of Virtual Currencies*

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Abstract

This paper develops an economic framework to analyze the exchange rate of virtual currency. Three components are important. First, the actual use of virtual currency to make payments. Second, the decision of forward-looking investors to buy virtual currency (thereby effectively regulating its supply). Third, the elements that jointly drive future consumer adoption and merchant acceptance of virtual currency. The model predicts that, as virtual currency becomes more established, the exchange rate becomes less sensitive to the impact of shocks to speculators' beliefs. This undermines the notion that excessive exchange rate volatility will prohibit widespread usage of virtual currency.

Keywords: Virtual currencies, exchange rates, payment systems, speculation, bitcoin, blockchain, cryptocurrencies.

JEL Classification Numbers: E42, E51, F31, G1.

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"You have to really stretch your imagination to infer what the intrinsic value of bitcoin is. I haven't been able to do it. Maybe somebody else can."
– Alan Greenspan, Bloomberg Interview, 4 December 2013

1 Introduction

This quote from the former Federal Reserve chairman could hardly be more accurate in describing the aim of the present study, in which we attempt to answer the broad question of what drives the value of virtual currencies. Virtual currencies such as bitcoin represent both the emergence of a new form of currency and a new payment technology to purchase goods and services. These currencies may move outside the scope of the current financial institutions. That is, their supply is not necessarily controlled by central banks and they allow distant payments to be made directly between consumers and merchants without the use of any financial intermediaries. The key innovation is the implementation of cryptographic identification techniques into a "distributed ledger", i.e., a digital record that allows to track and validate all payments made. This allows virtual currencies to be used in a decentralized payment system while avoiding the possibility of "double spending".

Bitcoin is the most well-known virtual currency.[1] For primers on the economics behind bitcoin; see, e.g., Dwyer (2015) and Böhme et al. (2015). Bitcoin was launched in 2009 and attracted attention from the financial press, economists, central banks and governments. This attention was fueled by the sudden "explosion" and volatility in the exchange rate of bitcoin by the end of 2013. During the month of November 2013, the U.S. dollar exchange rate for one unit of bitcoin increased more than fivefold, and its value, which had begun trading at less than five dollar cents in 2010, exceeded $1,100. During 2014, however, bitcoin lost ground again fast, settling at around $250 per bitcoin in March 2015. While the supply of bitcoin units over time is mathematically specified with an upper limit of 21 million units,

[1]See, e.g., Ong et al. (2015) and Tarasiewicz and Newman (2015) for a description of alternative virtual currencies and their designs.

its current supply amounts to approximately 14 million units (in March 2015). Bitcoin's usage is still limited but rising: from around 20,000 daily transactions on average in 2012, to over 50,000 daily transactions in 2013, and reaching 100,000 daily transactions in March 2014. All in all, compared to volume and value of other existing currencies, bitcoin is still a relatively small monetary phenomenon, but it has been growing.

This paper develops an economic framework that analyzes the exchange of a virtual currency in its early stage and its main drivers. Some unique properties of virtual currencies – at least in their current early adopters stage – play a role in our model. First, virtual currency prices of products and services are perfectly flexible with respect to changes in the exchange rate, since merchants tend to instantly adjust price quotes in virtual currency to the latest available exchange rate. In the model, this property is key in providing a direct link between the exchange rate and virtual currency demand. Second, the choice for making payments with virtual currency is simultaneously also a choice for an alternative transaction technology, since these payments are settled and processed through a peer-to-peer payment network associated with that virtual currency. Network economies affecting payment choice play an important role in determining the ultimate demand for virtual currency. Third, the growth of the supply of virtual currency is to a large extent predetermined. In line with this latter property, future demand for virtual currency to execute payments is one of the main sources of uncertainty in our model.

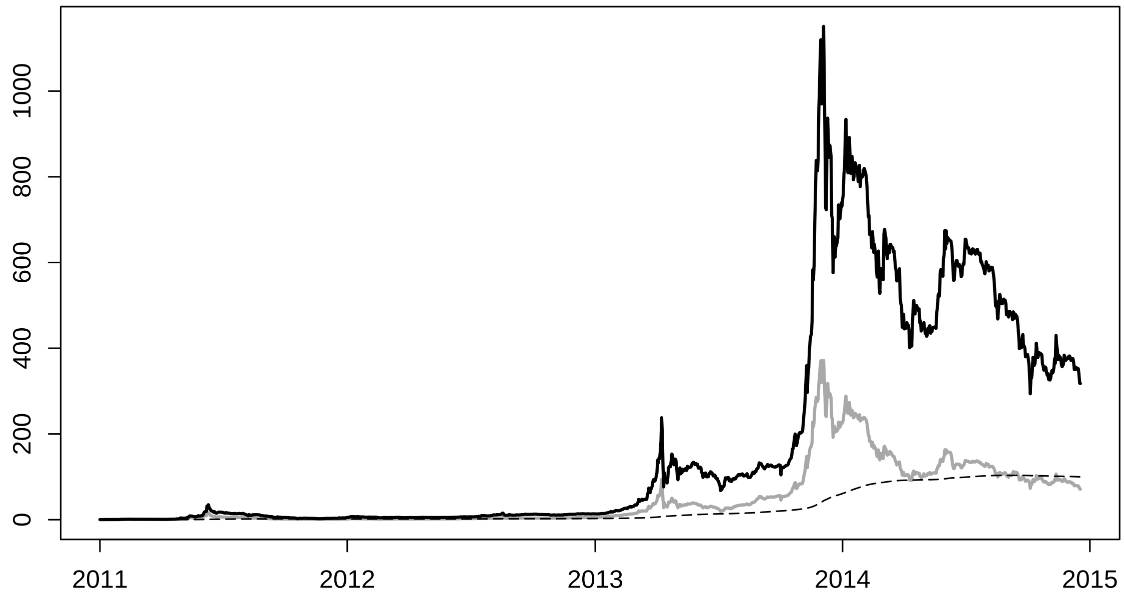
Our framework combines an investor's portfolio model with a payment network model, while adding a flavor of monetary economics. In our framework, three components are important for the exchange rate: First, the actual use of virtual currency to execute real payments. Second, the decision of forward-looking investors to buy virtual currency (thereby effectively regulating its supply). Third, the elements that jointly drive future consumer adoption and merchant acceptance of virtual currency. These latter elements determine the expected long-term growth in virtual currency usage. We show that the equilibrium exchange rate depends both on a "purely speculative" component, that depends on the hypothetical price speculators would offer if not a single real transaction is settled using virtual currency, and a transaction component, that depends on the actual amount necessary to facilitate real payments.

Speculating on the value of currencies is not new. Already Fisher (1911) argued that, in certain situations, speculators may effectively regulate the money supply by withdrawing money from circulation by betting on their future value. We apply this "old" notion in a formal way by showing that the exchange rate of a virtual currency immediately responds to changes in the speculative position of investors. Our model predicts that, as a virtual currency becomes more established, the exchange rate becomes less sensitive to the impact of shocks to speculators' beliefs and their inflow and outflow into the virtual currency market. This prediction undermines the notion that the current high volatility of the exchange rates of virtual currencies will prohibit their widespread usage in the long run.

Additionally, we borrow from the so-called "two-sided markets" literature to explain the main factors that drive future consumer adoption and merchant acceptance of virtual currency as a payment instrument; see, e.g., Armstrong (2006) and Rochet and Tirole (2006). It is shown that private benefits and cross-group externalities among merchants and consumers affect the joint demand for virtual currency to make payments for real goods and services. On one side of the market, private benefits may be large for consumers who frequently execute cross-border payments, such as remittances, that usually carry high fees. Face-to-face payments may also become easier and cheaper via application-based "wallets" on the smartphone. Consumers who value privacy and anonymity more, and those who are technologically more apt are likely to gain from using virtual currencies. On the other side of the market, large merchants may experience considerable private benefits from avoiding high fees charged by traditional payment providers. Internet stores may gain as well as they face relatively low implementation costs of accepting virtual currencies. Our model tries to explain how the resulting joint demand for virtual currency affects its exchange rate.

A first impression of what the exchange rate of bitcoin might have looked like in the

Figure 1: Speculation and the exchange rate of bitcoin, USD



The figure presents the value of a unit of the virtual currency bitcoin. The solid black line shows the exchange rate. The grey line shows a rough estimate of the hypothetical exchange rate if no units of the virtual currency were held for speculation. The dashed line provides an impression of the potential exchange rate in the absence of speculation and is a strongly smoothed average of the grey line. For technical details; see Appendix A. Source: www.blockchain.info and authors' calculations.

absence of speculation is presented by the dashed line in Figure 1. The solid black line shows daily data of the actual exchange rate. Based on our framework, the grey line shows a downward correction of the actual exchange rate to account for the virtual currency positions held by speculators. Here, the size of the speculative position is roughly proxied by the number of bitcoins that will remain for a longer period in dormant accounts. Economic theory, such as buffer stock models, suggest that consumers and merchants do not instantly adjust their positions to their actual liquidity needs in response to fluctuations; see, e.g., Laidler (1984) and Mizen (1997). This suggests that only a strongly smoothed version of the grey line, such as the dashed line, may provide some impression of the exchange rate in the absence of speculation.

The paper is set up as follows. In section 2 we briefly review the literature. As a preliminary, section 3 analyzes the exchange rate of virtual currency using Fisher's quantity equation. Section 4 described the model, building on a investor's portfolio model and a two-sided payment network model. In section 5 the equilibrium results are presented and explained. Various extensions are discussed in section 6. Finally, section 7 provides some discussion and concluding remarks.

2 Literature

While most papers focus on the technical and computational aspects of creating virtual currencies, the literature on the economics of virtual currencies is still developing.

In a recent paper, Dwyer (2015) provides an overview of private virtual currencies with a special focus on bitcoin (being the most prominent one). Supply and demand conditions are discussed as a first basis for understanding the underlying economics of bitcoin. The behavior of bitcoin's price since bitcoin began trading on electronic exchanges are also analyzed and the author provides a comparison of the volatility of bitcoin's price on those exchanges compared to gold and foreign exchange. As one of the main conclusions, Dwyer (2015) argues that there are no formal results (yet) which characterize a dynamic reputational equilibrium for the central mechanism of bitcoin's functioning. Developing such a theoretic result is important for a deeper fundamental understanding of bitcoin and similar currencies.

In another study, Böhme et al. (2015) present bitcoin's design principles and properties, and review its past, present, and future uses. The authors point out risks and regulatory issues as bitcoin interacts with the conventional financial system and the real economy. bitcoin is of interest to economists as a virtual currency with potential to disrupt existing payment systems and perhaps even monetary systems. The authors acknowledge the deflationary risk of the fixed money growth rule inherent in bitcoin's design, and point to the difficulty whether decentralized virtual currencies can be designed with monetary policies that include feedback or discretion. Whether or not bitcoin expands as a virtual currency, the authors conclude it offers a remarkable experiment with new technology – a lab for researchers – and an attractive means of exchange for a subset of merchants and consumers.

The rapid appreciation of the bitcoin rate and the high volatility of its exchange rate and has been expressed as a major concern on the viability of its use as a currency. For example, Yermack (2015) examines its historical trading behavior to analyze whether it behaves like a traditional existing currency. Bitcoin suffers from much higher exchange rate volatility than the exchange rates of tradional currencies. Moreover, based on a purely statistical analysis of the bitcoin exchange rate, Cheung et al. (2015) and Cheah and Fry (2015) express their concerns on its "bubble-like" behaviour. In general, it is argued that the high volatility of its exchange rates undermines bitcoin's usefulness as a unit of account or as a store of value; see also Weber (forthcoming).

Moreover, Yermack (2015) documents bitcoin's exchange rate to exhibit virtually zero correlation with other existing currencies, reducing bitcoin's use for risk management purposes and making it exceedingly difficult for its owners to hedge. Bitcoin also lacks access to a banking system with deposit insurance, and it is not used to denominate consumer credit or loan contracts. Moreover, it is argued that bitcoin faces a structural economic problem related to the absolute limit of 21 million units that can ever be issued, exerting a deflationary force – if bitcoin becomes widely adopted – on the economy since the "money supply" would not increase in concert with economic growth. As a conclusion of those authors, bitcoin appears to behave more like a highly speculative investment than like a currency.

Ali et al. (2014) examine the economic incentives of adopting virtual currencies and assess potential risks to monetary and financial stability. They argue that the incentives embedded in the current design of virtual currencies restrict their widespread usage. A key attraction of such schemes at present is their low transaction fees. But these fees may increase as usage grows and may eventually be higher than those charged by incumbent payment systems. Without specifying a theoretic framework, they also discuss various factors that influence the value of virtual currencies, such as return-risk tradeoffs, transaction cost and relative benefits, and habit formation. In their paper, the authors conclude that virtual currencies have currently no sufficient mass to pose a real risk to the financial system, but this could conceivably change, if these currencies were to grow significantly.[2]

Velde (2013) reviews the mechanics and characteristics of virtual currencies. Velde defends bitcoin as an elegant implementation of a virtual currency – controlling its creation and avoiding its duplication simultaneously – but questions whether it can truly rival or replace existing currencies. The paper concludes that the use of bitcoin as a medium of exchange has so far been limited. It has been used as a means to transfer funds outside of traditional and regulated channels and, mainly, as a speculative investment opportunity. Investors bet on bitcoin because it may develop into a full-fledged currency that generates real transactions. Should bitcoin become widely accepted, it is unlikely that it will remain free of intervention by public authorities, if only because the governance of the bitcoin computational code and protocol is opaque and vulnerable. However, virtual currencies do represent a clever technical financial innovation that ultimately may be used by existing financial institutions or even governments.

Network effects may play an important role in the adoption of a virtual currency. In the context of competition among virtual currencies, Gandal and Hałaburda (2014) empirically investigate the presence of a "winner-takes-it-all" effect for the most popular virtual currency. Their data suggest that this effect is dominant only early in the market, but this trend is reversed in a later period which may be consistent with the use of virtual currencies as financial assets. Moreover, Sauer (2015) also discusses the demand for a virtual currency in the context of (one-sided) network effects. Our framework supplements these studies by providing a formal treatment of the positive relation between network adoption and the exchange rate.

Finally, in a more technical paper, Kroll et al. (2013) argue that virtual currencies depend for its viability and stability on a combination of cryptography, distributed algorithms, and

[2]Similarly, in an earlier report, the European Central Bank (2012) assessed the potential economic impact of virtual currency schemes for central banks, covering price stability of prices, financial stability and risk to payment systems. The risks were deemed low because of their limited connection with the real economy, their low volume traded and current lack of wide user acceptance. Other publications in outlets of central banks commenting on the potential risks and use of virtual currencies include, e.g., Arias and Shin (2013), Beer and Weber (2014), Segendorf (2014), Tasca (2015) and Young (2015).

incentive-driven behavior. An important aspect of bitcoin's design is the "mining" mechanism, in which participants spend resources on solving computational puzzles in order to collect rewards. Modeled as a game between bitcoin miners and holders, Kroll et al. (2013) analyze the optimal incentives of mining, and whether the Bitcoin protocol can survive attacks. It is shown that there is a Nash equilibrium in which all players behave consistently with bitcoin's reference implementation avoiding disruptive attacks, along with infinitely many other equilibria in which they behave otherwise. They also show how a "Goldfinger-style" attacker might be able to disrupt the bitcoin system, "crashing" the currency and destroying its value.[3] Finally, the authors argue that bitcoin has already been exposed to some *de facto* governance structures through changes in the rules of the (open source) bitcoin reference software, contrary to the commonly held view in the bitcoin community that the currency is ungovernable.

Most papers lack a formal treatment of the economics behind the exchange rate of virtual currencies, unveiling the links between speculative behavior, currency creation and real growth in transactions for goods and services. This paper tries to bridge that gap.

3 Model preliminaries: Fisher's quantity equation

In the model of section 4 we consider two currencies: An established currency, say €, and a virtual currency, say B. But before we proceed, we first turn to the well-known transactions version of the quantity equation popularized by Fisher (1911), i.e.,

$$P_t^B T_t^B = M_t^B V_t^B, \tag{3.1}$$

where V_t^B denotes the velocity of virtual currency B, defined as the average number of times each unit of virtual currency is used to purchase real goods and services within period t. T_t^B

[3]Auric Goldfinger – the villain in the famous James Bond (1964) movie – wants to increase the value of his own gold holdings by making the gold in Fort Knox radioactive and worthless, thereby undermining the (gold backed) U.S. currency.

is the quantity of real goods and services purchased with virtual currency B during period t, and P_t^B is the weighted average price. M_t^B denotes the (nominal) quantity of money defined as the number of units of virtual currency B. Eq. (3.1) holds by definition within any given period t; see, e.g., Friedman (1970). Essentially, the expression follows directly from defining the concept of the velocity of virtual currency B. It does not depend on any behavioral assumptions.

Some manipulation of the left hand side of Eq. (3.1) gives

$$\frac{P_t^B}{P_t^{€}}(P_t^{€} T_t^B) = M_t^B V_t^B, \text{ or, } \frac{P_t^B}{P_t^{€}} T_t^{B*} = M_t^B V_t^B, \tag{3.2}$$

where $P_t^{€}$ denotes the weighted average price of the goods and services purchased with virtual currency when quoted in the established currency, and where T_t^{B*} denotes the volume of trade in goods and services with payments settled in virtual currency B, where the asterisk in the superscript signifies that this quantity is now measured in terms of the established currency.[4] Although (3.2) is for simplicity derived for a single established currency, it can easily be extended to cover multiple established currencies instead.

In the model, we essentially assume the established currency to be the unit of account.[5] Let $S_t^{€/B}$ denote the exchange rate, i.e., the number of units of the established currency one pays to obtain a single unit of virtual currency in period t. We assume that the price of goods and services expressed in virtual currency are completely determined by the exchange rate and their price level in the traditional currency as $P_t^B = P_t^{€}/S_t^{€/B}$. This interpretation connects well to the practice of many electronic stores to adjust prices quoted in virtual currencies instantly to the latest available exchange rate. The prices in these stores expressed in virtual currency are perfectly flexible with respect to changes in the exchange rate. Using

[4] Note that it is a simplification to interpret the index of the general price level as the empirical counterpart of $P_t^{€}$. It is easily verified that this would hold if the basket of goods used to calculate the index of the general price level is the same as the basket of goods for which payments using virtual currency are made. The differences between the baskets may be more pronounced if virtual currency is used for certain niche products and services, such as the market for electronics.

[5] This is an unrealistic assumption for an economy where virtual currency would take the role of the established currency.

this assumption in Eq. (3.2) yields the exchange rate for any t as

$$S_t^{€/B} = \frac{T_t^{B*}}{M_t^B V_t^B}. \tag{3.3}$$

Moreover, without loss of generality, the average velocity, the V_t^B, can be rewritten as a weighted average between the average velocity of the units of virtual currency used to settle payments for goods and services, V_t^{B*}, and the velocity of those not used to settle the payments for goods and services. Note that the latter equals zero, since velocity was defined as the average number of times each unit of virtual currency is used to purchase real goods and services. Formally,

$$V_t^B = \frac{M_t^B - Z_t^B}{M_t^B} V_t^{B*} + \frac{Z_t^B}{M_t^B} 0, \tag{3.4}$$

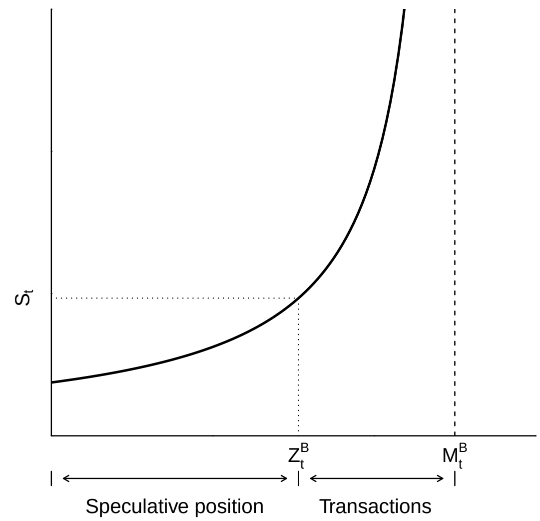
where $Z_t^B \geq 0$ is the number of units of virtual currency not used to settle the payments for goods and services during period t. Essentially, Z_t^B units are "stored value" and we suggestively refer to Z_t^B as the "speculative position" in virtual currency. After all, speculators buy units of virtual currency in the hope to make a profit by selling it in the future. Such a strategy only involves the exchange of the established currency against virtual currency and does not involve the use of virtual coins for the payment of real goods or services. These speculators may include both "pure" speculators, who do not use virtual currency for any real transaction, as well as merchants and consumers who hold larger positions in virtual currency than is demanded to execute the payment transactions for goods and services.

Combining Eqs. (3.3) and (3.4) gives the level of the exchange rate as

$$S_t^{€/B} = \frac{T_t^{B*}}{(M_t^B - Z_t^B) V_t^{B*}}. \tag{3.5}$$

This equation describes the effect of three factors affecting the exchange rate of virtual currency which are common in the context of the value of money: The exchange rate increases in the volume of the payments for goods and services with virtual currency, T_t^{B*}; it decreases

Figure 2: Speculation and the value of virtual currency



The figure shows the value of a unit of virtual currency, S_t^B, as a function of the speculative position, Z_t^B, for some fixed level of T_t^{B*}/V_t^{B*}, such as described by Eq. (3.5). The size of the speculative position is limited from above by the total number of units of virtual currency, M_t^B, as indicated by the dashed vertical line. The units of virtual currency not held for speculative investment, i.e., $M_t^B - Z_t^B$, are in circulation to accommodate payments.

in the velocity of virtual currency, V_t^{B*}, and the total quantity of virtual currency, M_t^B.

The expression in (3.5) also includes a fourth factor affecting the exchange rate of virtual currency, which is the quantity of virtual currency held in the speculative position, Z_t^B. Eq. (3.5) shows that the speculative position effectively reduces the quantity of virtual currency available to facilitate real payments, and, therefore, increases the value of virtual currency. The impact of speculation on the exchange rate is graphically shown in Figure 2. In the absence of speculation, the exchange rate for virtual currency would only be determined by the intersection between the curve and vertical axis in Figure 2. This point on the curve corresponds to the case in which all units of virtual currency are used to facilitate payments for real goods and services. In the presence of speculation, the exchange rate is higher because fewer units of virtual currency are available to facilitate real payments.

Of course, assigning a role to speculation in determining the value of money is an old notion. For example, Fisher (1911) mentioned the relation between speculation and the effective quantity of money in the context of the redemption of greenbacks by the US government from 1879 onwards. In the run-up of the promised redemption of greenbacks, he wrote, "*[s]ome of them were withdrawn from circulation to be held for the rise. (...) Thus speculation acted as a regulator of the quantity of money.*" (p. 261). This role of speculation is formalized in Eq. (3.5). It shows how the exchange rate of virtual currency responds immediately to changes in the magnitude of the speculative position, as a consequence of assuming that prices quoted in virtual currency are perfectly flexible with respect to the exchange rate. With fewer units of virtual currency available because of speculation, an immediate rise in the exchange rate is necessary to facilitate a given volume of transactions in goods and services in terms of the established currency.

Figure 1 in the introduction, which provides some impression of the exchange rate of bitcoin in the absence of speculation, is based on the expression in (3.5). The observed exchange rate $S_t^{€/B}$ is presumed to be the result of both real transactions and speculation, where the latter is measured by a rough proxy, i.e., the units of virtual currency that will remain for an extended period in dormant accounts. For each point in time, it is subsequently possible to determine hypothetical exchange in the absence of speculation by the intersection of the curve and the vertical axis in Figure 2. The intersection provides a rough estimate of the exchange rate if all units of virtual currency were in use to facilitate payments of goods and services, and is presented for each point in time by the solid grey line.

4 Model

The model contains two building blocks. The first building block is based on two-sided market theory that tries to identify which factors drive the uptake of virtual currency as a payment instrument.[6] The more consumers are willing to transact using virtual currency, the

[6]See, e.g., Armstrong (2006) and Rochet and Tirole (2006) for a deeper understanding of two-sided markets.

more merchants may wish to accept this type of currency for payment. And, vice versa, the more merchants are willing to accept virtual currency, the more likely that consumers will start using it. These two-sided cross-group externalities play an important role for optimal pricing and therefore the use of virtual currency to make payments.

The second building block concerns the behavior of speculators. The investment decision of a speculator is modeled as a trade-off between investing in a risk-free bond denominated in the established currency or speculation on the future value of virtual currency.

4.1 Model setup

Essentially, the model is a one-period model. Period t refers to the initial state; period $t+1$ refers to the final state. Regarding the final state, for simplicity, we consider the following scenario. Given technological uncertainties, potentially adverse regulatory policies, or successful introductions of other virtual currencies, two extreme events may occur at $t+1$. With probability q, the virtual currency payment network will end up in its stationary equilibrium. Alternatively, with probability $1-q$, the payment network will be abandoned, in which case the units of virtual currency will be worthless. In case of success with probability q, the stationary equilibrium is such that the frequencies of virtual currency users (i.e. consumers and merchants) are equal to their equilibrium values.[7]

Lastly, similar to the bitcoin supply process in practise, we assume that the number of virtual currency units at $t+1$ follows a predetermined growth rule, such that $M^B_{t+1} = (1+m^B_{t+1})M^B_t$.

[7] In their seminal paper, Kareken and Wallace (1981) show that the nominal exchange rate between two perfectly substitutable "fiat" currencies is indeterminate. This indeterminacy result would not hold in our case since the two currencies are different in terms of their risk profiles as well as in terms of the liquidity services provided by the virtual currency's payment network.

4.2 Two-sided virtual currency payment network

Organized as an electronic payment network, the underlying idea is that virtual currency creates economic value by enabling transactions between merchants and consumers. Assume that consumers and merchants derive net utility from using a monopolistic virtual currency network

$$U_{i,t} = \alpha_i N_{j,t} + \beta_i - p_{i,t}, \quad i, j = c, m, \ i \neq j, \tag{4.1}$$

where $N_{j,t}$ is the number of users from the other side who use the virtual currency network in period t, $\alpha_i > 0$ is the benefit that user i enjoys from transacting with each user on the other side, β_i is the fixed benefit the user obtains from connecting to the network, and $p_{i,t}$ is the fixed "membership" fee that is levied by the network as a lump-sum charge. The users of virtual currency are assumed to be heterogenous in their fixed benefit for network services. This heterogeneity is described by a cumulative density function $F_i(\cdot)$ with probability density function $f_i(\cdot)$, $i = m, c$.

On the consumer side, the benefits from joining a virtual currency network may be larger for more technologically adaptive consumers. Such consumers may incur a lower cost when adopting new payment technologies. Benefits may also be larger for consumers who face higher cost from traditional payment networks for a variety of reasons. Costly cross-border payments, such as sending remittances, may be one such reason. Privacy and anonymity may also increase private benefits from using virtual currencies. On the merchant side, heterogeneity in benefits from joining a virtual currency network may depend on various factors, such as merchant size and distribution channels. It is well documented that large retailers pay high merchant service fees for accepting certain credit and debit cards. Regarding distribution and business models, online stores may face lower implementation costs from accepting virtual currency than traditional stores.

Based on utility maximizing behavior of potential users, the number of users i that connects to the network is given by

$$N_{i,t} = \overline{N}_i D_i(u_{i,t} \geq 0) = \overline{N}_i \Pr(\beta_i \geq p_{i,t} - \alpha_i N_{j,t}) = \overline{N}_i \left(1 - F_i(\underline{\beta}_{i,t})\right), \tag{4.2}$$

where $\overline{N}_i$ is the maximum number of potential users of type i, and where $\underline{\beta}_{i,t} = p_{i,t} - \alpha_i N_{j,t}$.

Given costs C_i that are incurred by the network when users join the virtual currency network, (per-period) total network profits are given by

$$\pi_t(p_{c,t}, p_{m,t}) = N_{c,t}(p_{c,t} - C_c) + N_{m,t}(p_{m,t} - C_m). \tag{4.3}$$

Substituting $p_{i,t} = \alpha_i N_{j,t} + \underline{\beta}_{i,t}$ in (4.3), we write for network profits:

$$\begin{aligned} \pi(\underline{\beta}_{c,t}, \underline{\beta}_{m,t}) =& \overline{N}_c D_c(\underline{\beta}_{c,t}) \left(\alpha_c \overline{N}_m D_m(\underline{\beta}_{m,t}) \underline{\beta}_{c,t} - C_c\right) + \\ & \overline{N}_m D_m(\underline{\beta}_{m,t}) \left(\alpha_m \overline{N}_c D_c(\underline{\beta}_{c,t}) + \underline{\beta}_{m,t} - C_m\right). \end{aligned} \tag{4.4}$$

Thus, network profits can be expressed as a function of only $(\underline{\beta}_{c,t}, \underline{\beta}_{m,t})$.

The interior solution $(\underline{\beta}_c^*, \underline{\beta}_m^*)$ that solves the first-order conditions in turn determines profit-maximizing numbers (N_c^*, N_m^*) of virtual currency users and profit-maximizing fees (p_c^*, p_m^*). Profit-maximizing network fees satisfy

$$p_c^* = C_c - \alpha_m N_m^* + \frac{1 - F_c(\underline{\beta}_c^*)}{f_c(\underline{\beta}_c^*)}; \quad p_m^* = C_m - \alpha_c N_c^* + \frac{1 - F_m(\underline{\beta}_m^*)}{f_m(\underline{\beta}_m^*)}. \tag{4.5}$$

Note that $D_i'(x) = f_i(x),\ i = c, m$. This pricing rule shows that profit-maximizing fees for each side of the market are equal to the cost of providing the service (C_i), adjusted downwards by the external benefit to the other side ($\alpha_j N_j^*$), and adjusted upwards by a factor ($(1 - F_i)/f_i$) that is related to the elasticity of participation.

To pin down the total number of transactions, the payment literature often assumes a multiplicative relation $N_{c,t} \cdot N_{m,t}$, relying on an "independence" assumption. For the key implications of our model, it is not essential to assume a specific functional form to specify the number of transactions. What is essential, however, is the relatively weak assumption that the value of virtual currency units necessary to make real transactions, i.e., T_t^{B*}/V_t^{B*}, increases in the adoption of virtual currency by consumers and merchants.[8]

For convenience, we follow the convention regarding the multiplicative relation for the number of transactions using virtual currency. Moreover, while assuming that the unit average value per transaction in terms of the traditional currency is unaffected by the number of transactions, we obtain $T_t^{B*} = T(N_{c,t}, N_{m,t}) = N_{c,t} \cdot N_{m,t}$. Additionally, we assume that the velocity of virtual currency is proportional to the fraction of merchants who accepts virtual currency, i.e., $V_t^{B*} = \phi^{-1} N_{m,t}$, for some constant $\phi > 0$. In essence, this last assumption captures the notion that consumers face more opportunities to spend their virtual currency units if more merchants accept virtual currency. In other words, as the probability of a merchant accepting payments in virtual currency increases, consumers may use their buffers of virtual currency more efficiently. This effectively increases the velocity of virtual currency. Taken together, $T_t^{B*} = N_{c,t} \cdot N_{m,t}$ and $V_t^{B*} = \phi^{-1} N_{m,t}$, this yields the total value of the units of virtual currency necessary to make real payments, that is

$$\frac{T_t^{B*}}{V_t^{B*}} = \phi N_{c,t}. \tag{4.6}$$

Using this expression in Eq. (3.5) gives the level of the exchange rate as an increasing function of network adoption at any particular point in time as

$$S_t^{€/B} = \frac{\phi N_{c,t}}{M_t^B - Z_t^B}. \tag{4.7}$$

[8] Formally, it is sufficient to assume $T_t^{B*}/V_t^{B*} = f(N_{c,t}, N_{m,t})$, with either $\partial f/\partial N_{c,t} > 0$ and $\partial f/\partial N_{m,t} \geq 0$, or $\partial f/\partial N_{c,t} \geq 0$ and $\partial f/\partial N_{m,t} > 0$. This generalization can be implemented by replacing $\phi N_{c,t}$ by $f(N_{c,t}, N_{m,t})$ and ϕN_c^* by $f(N_c^*, N_m^*)$ in subsequent equations.

4.3 Speculative behavior

We presume that the central bank is successful in the sense that changes in the price level $P^{€}$ in terms of the traditional currency are anticipated with certainty. Speculators are therefore assumed to choose their holdings of units of virtual currency while maximizing a second order approximation of their utility with respect to their next period wealth, W_{t+1}, where wealth is expressed in the established currency as

$$U_{s,t} = \mathbb{E}(W_{t+1}) - \frac{\gamma}{2}\sigma^2(W_{t+1}), \tag{4.8}$$

where $\mathbb{E}(W_{t+1})$ and $\sigma^2(W_{t+1})$ denote period's t expectation and variance of wealth at time $t+1$, respectively. For simplicity we assume the same positive coefficient of risk aversion γ across all speculators.[9] The investment decision of the speculator is modeled as a trade-off between investing in a risk-free bond denominated in the established currency and speculation on virtual currency.[10] Under the assumption of the absence of lending and borrowing in virtual currency, we have that the return on a position in virtual currency in terms of the established currency is only determined by the change in the exchange rate. Hence, next period, the wealth of a speculator investing in z_t^B units of virtual currency is

$$W_{t+1} = R(W_t - S_t^{€/B} z_t^B) + \tilde{S}_{t+1}^{€/B} z_t^B, \tag{4.9}$$

where W_t is wealth at time t, R denotes the gross return on bonds denominated in the traditional currency and where $\tilde{S}_{t+1}^{€/B}$ denotes the uncertainty about the future exchange

[9]This assumption can easily be relaxed by replacing the expression $\gamma/N_{s,t}$ in the solution with the expression $\left(\sum_i 1/\gamma_i\right)^{-1} > 0$, where $\gamma_i \neq 0$ denotes the coefficient of risk aversion of speculator i; see, e.g., Viaene and De Vries (1992). This modification does not change the essence of the model much. Both expressions measure more or less the 'risk aversion aggregated across speculators'. We opt for the simplest expression.

[10]We refer to Hirshleifer (1988) for a more sophisticated model of speculation with multiple investment opportunities in a mean-variance framework. The results of Hirshleifer (1988) show that, given some fixed costs for entering into a speculative position, the size of the speculative position depends on systematic risk and residual risk. In our study, the size of the speculative position will depend on the total risk.

rate $S_{t+1}^{€/B}$. We assume that individual speculators take the current exchange rate S_t as given. In summary, the investor's optimization problem is the maximization of (4.8) with respect to z_t^B.

Solving the speculator's optimization problem gives the optimal number of units of virtual currency as

$$z_t^B = \frac{\mathbb{E}(\tilde{S}_{t+1}^{€/B}) - RS_t^{€/B}}{\gamma \sigma^2(\tilde{S}_{t+1}^{€/B})}. \tag{4.10}$$

The expression in Eq. (4.10) follows the standard solution in the literature for this type of models, in which the expected additional return earned by a marginal increase in the speculative position in the numerator equates the marginal decrease in utility due to additional risk taking. Summing the positions of $N_{s,t}$ speculators in Eq. (4.10) gives the aggregate speculative position in period t as

$$Z_t^B = N_{s,t} z_t^B = \frac{\mathbb{E}(\tilde{S}_{t+1}^{€/B}) - RS_t^{€/B}}{\frac{\gamma}{N_{s,t}} \sigma^2(\tilde{S}_{t+1}^{€/B})}. \tag{4.11}$$

This equation gives the speculative demand for virtual currency as a function of the exchange rate. Note that the aggregate speculative position cannot be negative (which would imply money creation). Hence, Eq. (4.11) holds for $\mathbb{E}(\tilde{S}_{t+1}^{€/B}) \geq RS_t^{€/B}$, or,

$$R^{-1}\mathbb{E}(\tilde{S}_{t+1}^{€/B}) \geq \frac{T_t^{B*}}{M_t^B V_t^{B*}}.[11] \tag{4.12}$$

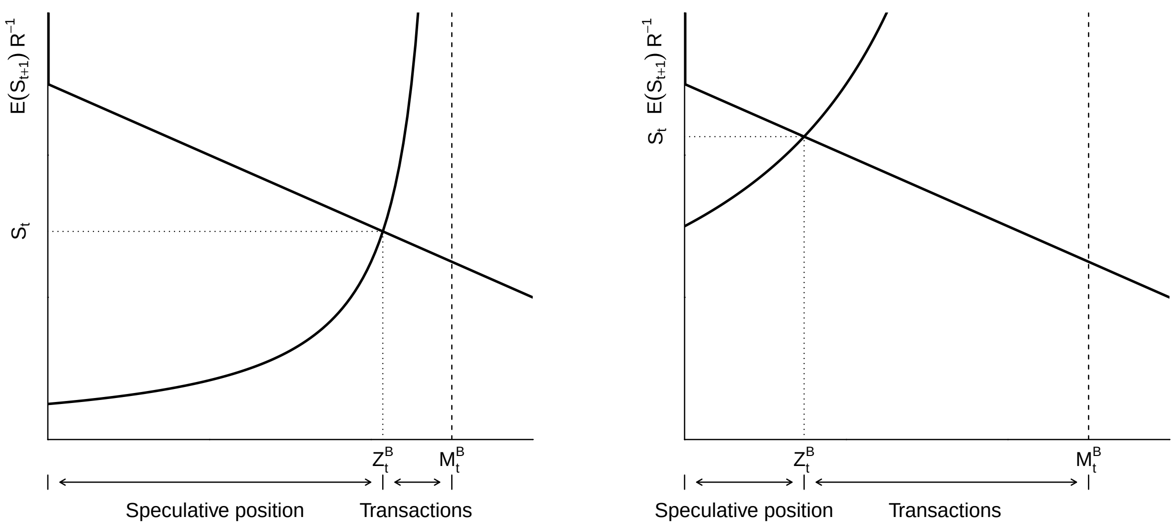
In other words, in the aggregate, speculators choose to take a positive position in virtual currency only if the discounted expected value exceeds the hypothetical value of the current exchange rate in the absence of speculation. Otherwise, they prefer to short-sell the currency, which they cannot do collectively, and their optimal aggregate position equals zero.

In summary, rewriting Eq. (4.11) gives the exchange rate at which speculators absorb

[11] This is implied by $Z_t^B \geq 0$ and Eq. (3.5).

Figure 3: Equilibrium exchange rate for virtual currency

(a) Low volume of real transactions (b) High volume of real transactions



The figure shows the demand of speculators for units of virtual currency in Eq. (4.11) and the available units of virtual currency for speculators in Eq. (3.5) for different levels of the exchange rate, $S_t^{€/B}$. The equilibrium exchange rate follows from the intersection between the two lines and corresponds to the solution in Eq. (4.13).

$Z_t^B > 0$ units of virtual currency as

$$S_t^{€/B} = \left(\mathbb{E}(\tilde{S}_{t+1}^{€/B}) - \frac{\gamma}{N_{s,t}} Z_t^B \sigma^2(\tilde{S}_{t+1}^{€/B}) \right) R^{-1}. \tag{4.13}$$

The price speculators are willing to pay for virtual currency equals the (discounted) expected future exchange rate minus a risk premium for the uncertainty in the future value of the speculative position in virtual currency.

4.3.1 Partial equilibrium

The level of the exchange rate is pinned down by Eqs. (3.5) and (4.13). These two equations act respectively as supply and demand schedules for units of virtual currency. Figure 3, panel (a) shows the two curves in a single diagram.

The upward sloping curve of Eq. (3.5), derived from the price-quantity relation, shows the price speculators would have to pay to invest in an additional unit of virtual currency. The larger the speculative position, the more units that are effectively withdrawn from circulation in the payment system. Withdrawing units from circulation in the payment system results in a price increase via the quantity relation. Hence, the larger the speculative position, the higher the price to withdraw an additional unit from the virtual currency payment system.

The downward sloping curve of Eq. (4.13) shows the price speculators would be willing to pay, given their expectations, for a unit of virtual currency while investing in Z_t^B units. The larger the speculative position Z_t^B, the more units they absorb, the larger their risk-taking, and the lower the price they are willing to pay to absorb an additional unit of virtual currency.

The equilibrium exchange rate, $S_t^{€/B}$, is determined by the intersection between the two curves. Only at this point on the upward sloping curve speculators have no incentives to adjust the size of their positions.

If the condition in (4.12) is not satisfied, i.e., if current usage is sufficiently high, or, if speculators are sufficiently pessimistic, then the intersection coincides with the vertical axis. The equilibrium exchange rate follows directly from Eq. (3.5) as speculators optimally choose $Z_t = 0$.[12]

Per contrast, if the condition in (4.12) holds because speculators are optimistic, or, because virtual currency usage is still low, then the analytical solution of the equilibrium exchange rate is

$$S_t^{€/B} = \frac{1}{2} S^{€/B}_{t|T_t^{B*}=0} + \sqrt{\left(\frac{1}{2} S^{€/B}_{t|T_t^{B*}=0}\right)^2 + \frac{T_t^{B*}}{V_t^{B*}} \frac{\gamma}{N_{s,t}} \sigma^2(\tilde{S}_{t+1}^{€/B}) R^{-1}}, \tag{4.14}$$

[12]This may also reflect the exchange rate of a traditional currency in a country facing extremely high and volatile inflation. In such conditions, price setting usually occurs in a foreign currency (such as the dollar); see, e.g., Dornbusch et al. (1990). The assumed perfect flexibility of prices with respect to the exchange rate is a reasonable characterization of such a situation, while the extremely inflationary nature eliminates the incentives of investors to hold such a currency as a store of value because of the low expected value.

where

$$S^{€/B}_{t|T^{B*}_t=0} = \left(\mathbb{E}(\tilde{S}^{€/B}_{t+1}) - \frac{\gamma}{N_{s,t}} M^B_t \sigma^2(\tilde{S}^{€/B}_{t+1})\right) R^{-1}. \tag{4.15}$$

The equilibrium exchange rate in Eq. (4.14) is determined by two main components. The first component, $S_{t|T^{B*}_t=0}$, represents the "purely speculative" exchange rate. It is the exchange rate obtained from Eq. (4.13) when fixing $Z^B_t = M^B_t$. This hypothetical exchange rate is indicated in Figure 3, panel (a) by the intersection between the solid downward sloping line and the dashed vertical line, which specifies the total amount of available units of virtual currency. We call it the purely speculative exchange rate, because it is the hypothetical price speculators would be willing to pay for virtual currency if not a single real transaction is currently settled using virtual currency, i.e., if $T^{B*}_t = 0$.

The second component is the current value of virtual currency that is necessary to facilitate payments, i.e., T^{B*}_t / V^{B*}_t. The larger the amount necessary for payments using virtual currency, the higher the exchange rate. The more value of virtual currency is absorbed by payments, the smaller the exchange rate risk that has to be absorbed by speculators, and thus the higher the level of the exchange rate of virtual currency. Figure 2, panel (b) reports the upward pressure on the current exchange rate from more payments using virtual currency by showing the new equilibrium after an increase in T^{B*}_t / V^{B*}_t causing a shift of the upward sloping curve.

4.3.2 Impact of speculative environment

Virtual currency has suffered from highly volatile exchange rates compared to the exchange rates of established currencies; see, e.g., Yermack (2015). The high level of volatility is often attributed to the behavior of speculators: Speculators herding for into a new opportunity and erratic changes in their beliefs may cause large swings in the exchange rate. Of course, the mere presence of speculators cannot explain the higher level of volatility, since speculators may invest in any currency. Nevertheless, it is possible that the exchange rate is especially sensitive to changes in speculators' beliefs in the early adoption phase, when still few real payments are settled in virtual currency.

The impact of speculators' beliefs regarding the exchange rate can be assessed by taking the derivative of Eq. (4.14) with respect to the speculators' expectations of the future exchange rate as

$$\frac{\partial S_t^{€/B}}{\partial \mathbb{E}(\tilde{S}_{t+1}^{€/B})} = \frac{R^{-1}}{2} + \frac{R^{-1}}{2} \frac{\frac{1}{2} S_{t|T_t^{B*}=0}^{€/B}}{\sqrt{\left(\frac{1}{2} S_{t|T_t^{B*}=0}^{€/B}\right)^2 + \frac{T_t^{B*}}{V_t^{B*}} \frac{\gamma}{N_{s,t}} \sigma^2(\tilde{S}_{t+1}^{€/B}) R^{-1}}}. \tag{4.16}$$

This equation shows which determinants affect the change in the exchange rate following a shock to speculators' beliefs about the future value of virtual currency.

Shocks in the expectations of speculators on the future value have a larger impact on the exchange rate in the early phase of a virtual currency. From Eq. (4.16), we obtain $\partial S_t^{€/B} / \partial \mathbb{E}(\tilde{S}_{t+1}^{€/B}) = R^{-1}$ if $T_t^{B*} = 0$. Hence, in the absence of any real transactions, changes in the beliefs of speculators on the future exchange rate translate one-to-one into discounted changes in the current exchange rate. The impact of changes in the beliefs of speculators is a strictly decreasing function in the volume of the real transactions, i.e., in T_t^{B*}. Hence, the impact of actions based on changes in speculators' beliefs on the future value of virtual currency is expected to become smaller once a virtual currency is widely used. This can also be observed from Figure 3. An improvement in speculators' beliefs on the future value of the exchange rate is represented by an upward shift in the downward sloping curve. The resulting change in the equilibrium exchange rate is larger in the low transaction volume environment, panel (a), then in the high transaction volume environment, panel (b).

The larger impact of beliefs of speculators about the exchange rate in the early phase of a virtual currency can intuitively explained as follows. In the absence of real transactions, any adjustment in the exchange rate to the new expectations is completely in the price domain. In the presence of real transactions, the adjustment towards the new equilibrium will partly be in the quantity domain because of the following reason. A more pessimistic view on the exchange rate by speculators will reduce the exchange rate. Given the value of real transactions, such a reduction in the exchange rate will simultaneously require an increase in the number of the units of virtual currency used to facilitate real transactions. This implies a lower number of units of virtual currency in the hands of speculators in the new equilibrium. The larger the value of real transactions using virtual currency, the larger the quantity effect and hence, the smaller the price change.

In a similar way it can be shown that the entry of speculators drives up the current exchange rate. In the model, this is equivalent to have the entry of new speculators or having current speculators taking on more risk: both correspond to a decrease in the risk aversion aggregated across speculators, i.e., $\gamma/N_{s,t}$. Given the expectations, the entry of new speculators turns the downward-sloping line in Figure 3 counter-clockwise in the intersection with the y-axis. Taking the derivative of Eq. (4.14) with respect to the speculators' aggregated risk aversion yields

$$\frac{\partial S_t^{€/B}}{\partial \frac{\gamma}{N_{s,t}}} = -\frac{\sigma^2(\tilde{S}_{t+1}^{€/B})}{2R}\left(M_t^B - \frac{\frac{T_t^{B*}}{V_t^{B*}} - \frac{1}{2}M_t^B S_{t|T_t^{B*}=0}}{\sqrt{\left(\frac{1}{2}S_{t|T_t^{B*}=0}^{€/B}\right)^2 + \frac{T_t^{B*}}{V_t^{B*}}\frac{\gamma}{N_{s,t}}\sigma^2(\tilde{S}_{t+1}^{€/B})R^{-1}}}\right). \tag{4.17}$$

This expression is negative for any $\gamma/N_{s,t}$: The term in parentheses is positive, because from (4.13) we have that the denominator in the expression is larger than or equal to $S_t^{€/B}$ and from (3.5) we have $M_t^B \geq T_t^{B*}/(V_t^{B*}S_t^{€/B})$. In other words, an inflow of more speculators or an increased risk appetite are both expected to increase the exchange rate of virtual currency.

The inflow and outflow of speculators from the virtual currency market have a larger impact on the exchange rate in the early phase of a virtual currency. Although, the derivative in (4.17) remains negative, it gets closer to zero as the transaction volume T_t^{B*}/V_t^{B*} increases. In other words, the model suggests that the exchange rate becomes less sensitive to the inflow and outflow of speculators as the usage of virtual currency by merchants and consumers increases in intensity.

To summarize, the impact of speculative behavior on the exchange rate of virtual currency becomes smaller as virtual currency matures in its adoption to make real payments.

5 Rational equilibrium exchange rate analysis

In this section we combine the payment network equilibrium outcome with the derived (partial) equilibrium for the exchange rate as specified in Eq. (4.14).

When the number of transactions has reached its stationary equilibrium (and no more growth can be expected), the condition in (4.12) implies that speculators will no longer have an incentive to invest in virtual currency. Hence, all available units will be used to purchase real goods and services. Based on rational expectations among speculators, Eq. (4.7) implies that the expected future exchange rate in period t equals

$$\mathbb{E}(\tilde{S}_{t+1}^{€/B}) = q\left(\frac{\phi N_c^*}{M_{t+1}^B}\right), \tag{5.1}$$

where $M_{t+1}^B = (1 + m_{t+1}^B)M_t^B$ is the number of virtual currency units at $t+1$ that follows from the predetermined growth rule. Moreover, the volatility of the future exchange rate equals

$$\sigma^2(\tilde{S}_{t+1}^{€/B}) = q(1-q)\left(\frac{\phi N_c^*}{M_{t+1}^B}\right)^2. \tag{5.2}$$

In other words, the larger the success probability q, the larger the expected exchange rate, and, conditional upon $q > 1/2$, the lower the volatility of the future exchange rate. Substitution of Eqs. (4.7), (5.1) and (5.2) into the partial equilibrium for the current exchange rate in Eq. (4.14) gives

$$S_t^{€/B} = q\left(\frac{\phi N_c^*}{M_{t+1}^B}\right) \times \left(\frac{1}{2}\sqrt{\delta_t^2} + \frac{1}{2}\sqrt{\delta_t^2 + 4\gamma\phi\frac{N_{c,t}}{N_{s,t}}\frac{1-q}{q}R^{-1}}\right). \tag{5.3}$$

The first term in Eq. (5.3) is the expected future exchange rate. Its value depends on the success probability q, and on the – conditional upon success – expected long-term growth in the adoption of virtual currency towards N_c^*.

The second term in Eq. (5.3) is the “discount factor” applied by investors. The δ_t represents the *hypothetical* discount factor in case no real transactions are settled using virtual currency, i.e., if $N_{c,t} = 0$. This *hypothetical* discount factor is calculated as

$$\delta_t = \left(1 - \frac{(1-q)}{1+m_{t+1}^B}\gamma\phi\frac{N_c^*}{N_{s,t}}\right)R^{-1}. \tag{5.4}$$

The full equilibrium equation in shows that actual discount factor is an increasing function of the current adoption of virtual currency $N_{c,t}$. Actual adoption increases the exchange rate of virtual currency via this channel. Moreover, the exchange rate of virtual currency does not suffer from “money illusion”. Given all other parameters in Eqs. (5.3) and (5.4), doubling the number of virtual currency units M_{t+1}^B reduces the exchange rate $S_t^{€/B}$ by one half.

6 Discussion

In this section we discuss several extensions of the model.

6.1 Speculating consumers

Speculators and virtual currency users making actual payments are assumed to be different agents in the model. In practice, speculators and users may be the same agents; see, e.g., Johnson (1960). This would not change the essence of the model. We show this by considering the case of speculators on the consumer side. Suppose that the decision to join the network provides the consumers with the opportunity to speculate on the value of virtual currency. Presuming that users face the same investment decision as speculators, then their net utility derived from using the monopolistic virtual currency payment network would be

$$U_{c,t} = \alpha_c N_{m,t} + \beta_c + \Delta_{c,t} - p_{c,t}, \tag{6.1}$$

where $\Delta_{c,t}$ is the additional utility derived from the opportunity to speculate on the value of virtual currency, which equals

$$\Delta_{c,t} = z_t^B \mathbb{E}(\tilde{S}_{t+1}^{€/B} - S_t) - \frac{\gamma}{2}\sigma^2(z_t^B \tilde{S}_{t+1}^{€/B}), \tag{6.2}$$

for the optimal level of the speculative position z_t^B, which is derived in (4.10).[13]

The steady state equilibrium will not be changed by giving consumers the opportunity to speculate on the value of virtual currency. The reason is that the additional utility derived from the opportunity to speculate on the value of virtual currency $\Delta_c^* = 0$ equals zero in the steady state, since no further appreciation of virtual currency is to be expected. In the steady state, both speculators and consumers will reduce their positions held for speculation to zero, and therefore the expression in (6.2) will equal zero. If the utilities of the users are not changed, so will the equilibrium number of users, N_c^* and N_m^*, be unchanged. Therefore, the future exchange rate will be unchanged.

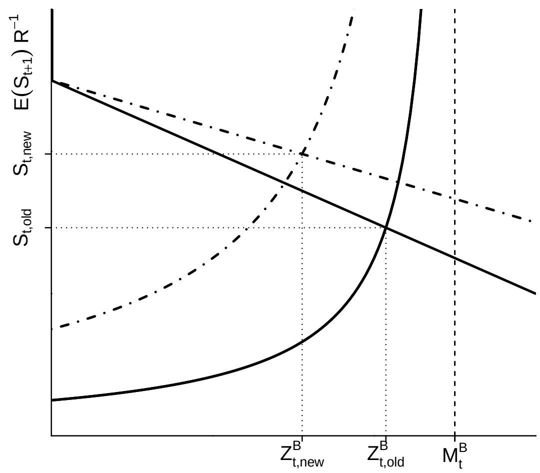
Nevertheless, the opportunity for consumers to speculate on the value of virtual currency will affect the current exchange rate. This is because the aggregate speculative position of both speculators and consumers will be equal to $z_t^B(N_{s,t} + N_{c,t})$. When working through the entire equilibrium solution, this basically means that the $N_{s,t}$ in the denominators in Eqs. (5.3) and (5.4) will be replaced by $N_{s,t} + N_{c,t}$.[14] In other words, the additional speculative demand from the consumer side will increase the discount factor and, therefore, provide upward pressure on the level of the current exchange rate.

The opportunity of consumers to speculate may change the dynamics of the model. In terms of comparative statics for the current exchange rate, an increase in the consumer base of virtual currency network will not only increase the value of the transactions using virtual

[13] Basically, the extension here is that consumers are no longer constrained to $z_t^B = 0$ in their investment decision. The value of $\Delta_{c,t}$ in Eq. 6.2 is derived from the difference between (4.8) for any z_t^B and for $z_t^B = 0$. Differences in the degree of risk aversion, e.g., between speculators and consumers, can be dealt with as described in footnote 9.

[14] The case in which everybody speculating on the value of virtual currency also pays with virtual currency corresponds to $N_{s,t} = 0$.

Figure 4: Equilibrium exchange rate with speculating consumers



The figure shows the shift in the upward-sloping curve representing transactional demand and the shift in the downward-sloping speculative demand schedule as a result of more consumers adopting virtual currency if consumers are allowed to take a speculative position in virtual currency.

currency, but will also increase speculative demand. In other words, both curves in the diagrams in Figure 3 will shift upward as a consequence of more consumers using virtual currency. Such an increase in consumer usage is illustrated in Figure 4. Besides the upward shift of the upward-sloping curve as a result of higher transactional demand stemming from more consumers using virtual currency, the inflow of new speculating consumers also turns the downward-sloping schedule representing speculative demand counter-clockwise. This further increases the current exchange rate. The counter-clockwise turn of the speculative demand schedule shown in Figure 4 does not presume higher expectations regarding the exchange rate as new consumers enter the virtual currency network. Such a change in expectations would shift the speculative demand schedule further upwards, and would result in a reinforced increase in the current exchange rate.

Moreover, allowing speculating consumers results in a higher level of utility derived from joining the network for early adopters, since these consumers do not only benefit from using virtual currency, but also from a nonzero speculative position in virtual currency. Therefore, higher expectations regarding the future value of virtual currency will not only increase the positions of speculators, but also draw more consumers towards the network, which may result in a faster convergence to the steady state use of virtual currency.

6.2 Intra-group benefits

Positive intra-group externalities due to, e.g., peer to peer payments, such as remittances, do not change the essence of the model. To explicitly account for positive intra-group externalities, we may modify the net utility from joining the virtual currency network to

$$U_{i,t} = \alpha_i N_{j,t} + \alpha_{ii} N_{i,t} + \beta_i - p_{i,t}, \quad i, j = c, m, \ i \neq j. \tag{6.3}$$

Here the parameter $\alpha_{ii} > 0$ denotes the benefit that user i enjoys from interacting with other (same type) users on the network. Essentially, given prices $p_{c,t}$ and $p_{m,t}$, the extension in (6.3) yields two demand equations $N_{c,t} = N_c(p_{c,t}, N_{m,t}, N_{c,t})$ and $N_{m,t} = N_m(p_{m,t}, N_{c,t}, N_{m,t})$ in two unknowns $(N_{c,t}, N_{m,t})$. Solving this system gives $N^*_{c,t}(p_{c,t}, p_{m,t})$ and $N^*_{m,t}(p_{c,t}, p_{m,t})$. Substituting these optimal demand functions into the network's profit function (4.3) and maximizing then yields optimal fees $p^*_{c,t}$ and $p^*_{m,t}$. Qualitatively, this extension will not alter our results.

7 Concluding Remarks

This paper proposes an economic framework for analyzing the exchange rate of a virtual currency and its key drivers. The paper unveils three main determinants: First, the current use of virtual currency to make real payments. Second, the decision of forward-looking investors to buy virtual currency (thereby effectively reducing its supply) given expected growth in virtual currency transactions. Third, the elements that jointly drive consumer adoption and merchant acceptance of virtual currency, which determine expected long-term growth in usage. In particular, we show that the equilibrium exchange rate depends both on a pure speculative component pinning down a “floor” under the exchange rate and a transaction component that affects the exchange rate risk absorbed by the speculators. More widespread use of virtual currencies by merchants and consumers lowers the impact of speculative behavior and therefore stabilizes the exchange rate.

Our analysis illustrates that a steep increase in the exchange rate due to speculative motives is exactly what you can expect at the introduction of a potentially successful virtual currency. Moreover, the current high levels of volatility seem to be a childhood disease: Theoretically, volatility is expected to drop if the adoption by consumers and merchants increases. The future will show by how much volatility will drop. The fixed supply of virtual currency may suggest its volatility will reflect that of other commodities (see, e.g., Selgin (2014)) rather than that of traditional currencies whose quantities are managed by central banks. Moreover, the model also shows that, conditional upon survival, deflationary virtual currency prices may be expected during the early adopters stage.

Our study is only a first step in trying to understand the underlying economics of virtual currencies. Empirically, little is known about the actual number of payments in virtual currency for goods and services. Reliable time series of user and acceptance statistics are still lacking and need to be developed, as these will be – without doubt – an important input for studying the behavioral aspects of using virtual currencies. Competition among virtual currencies raises a further issue of whether only a few virtual currencies will ultimately dominate a global market where network effects play an important role. Finally, analyzing empirical determinants of switching behavior from traditional means of payments to virtual currencies may lead to interesting cross-fertilizations between the payment choice literature and the literature on currency substitution. Much more research remains to be done – we are planning to go down this route.

Appendix A: Estimate of the speculative position

The hypothetical exchange rate in the absence of speculation in Figure 1 is based on a very rough estimate of the magnitude of the speculative position. Theoretically, the magnitude of speculative position is defined as the number of units of virtual currency not used to settle payments for goods and services. A rough estimate of this amount is obtained as follows. At each day t, we obtain from the public ledger the number of bitcoins associated with addresses that are not involved in any transactions over the next three months. Given the total number of bitcoins in circulation on day t, the actual exchange rate on day t and the estimated size of the speculative position on day t, it is not difficult to calculate from Eq. (3.5) the hypothetical exchange rate for a speculative position with a zero balance, while assuming a fixed ratio T_t^{B*}/V_t^{B*}. Figure 1 presents this level as the grey line. The dashed line is based on an exponentially weighted average with a daily smoothing parameter of 0.997 (this corresponds to an observation one quarter ago receiving a 75% weight of that of the current observation in the average).

We described our estimate of the size of the speculative position as a *rough* estimate. The reason is that there are several issues with our estimate of the speculative position. Our approach relies on subtracting the bitcoins not involved in transactions for real goods and services from the total amount. Even though all transactions are recorded in the public ledger, this does not directly reveal which units of virtual currency have not been used to settle payments for real goods and services. All recorded transactions represent a larger set than the transactions for processing payments for real goods and services, since they may include, among others, transactions with exchanges, transactions with other speculators, and transactions between addresses without a change in ownership (i.e., someone moving bitcoins from the left to the right pocket). In this respect, our approach is likely to result in overestimating the number of bitcoins involved in processing payments for real goods and services, and, therefore, in underestimating the size of the speculative position. In this context, the level of the grey line in Figure 1 is probably too high.

Moreover, the raw data on the total number of bitcoins in circulation is not corrected downwards for units that are effectively permanently withdrawn from circulation. This refers to units that are permanently lost because users lost access to the cryptographical keys which are necessary to transfer bitcoins from one to another address. Of course, the number of bitcoin units that are truly permanently lost is difficult to measure. However, if such data were available, following Eq. (3.5), a correction based on such data would result in an upward adjustment of the exchange rate in the absence of speculation.

References

R. Ali, J. Barrdear, and R. Clews. The economics of digital currencies. *Bank of England Quarterly Bulletin*, 54(3):276–286, 2014.

M.A. Arias and Y. Shin. There are two sides to every coin. *Federal Reserve Bank of St. Louis Regional Economist*, October, 2013.

M. Armstrong. Competition in two-sided markets. *RAND Journal of Economics*, 37(3): 668–691, 2006.

C. Beer and B. Weber. Bitcoin: The promise and limits of private innovation in monetary and payment systems. *Oesterreichische Nationalbank Monetary Policy & the Economy*, Q4:53–66, 2014.

R. Böhme, N. Christin, B. Edelman, and T. Moore. Bitcoin: Economics, technology, and governance. *Journal of Economic Perspectives*, 29(2):213–238, 2015.

E.-T. Cheah and J. Fry. Speculative bubbles in bitcoin markets? An empirical investigation into the fundamental value of bitcoin. *Economics Letters*, 130:32–36, 2015.

A. Cheung, E. Roca, and J.-J. Su. Crypto-currency bubbles: An application of the Phillips-Shi-Yu (2013) methodology on Mt. Gox bitcoin prices. *Applied Economics*, 47(23):2348–2358, 2015.

R. Dornbusch, F. Sturzenegger, and H. Wolf. Extreme inflation: Dynamics and stabilization. *Brookings Papers on Economic Activity*, 2:1–84, 1990.

G.P. Dwyer. The economics of bitcoin and similar private digital currencies. *Journal of Financial Stability*, 17:81–91, 2015.

European Central Bank. Virtual currency schemes. *Report*, 2012.

I. Fisher. *The purchasing power of money: Its determination and relation to credit, interest and crises*. Macmillan, New York, 1911.

M. Friedman. A theoretical framework for monetary analysis. *Journal of Political Economy*, 78(2):193–238, 1970.

N. Gandal and H. Hałaburda. Competition in the cryptocurrency market. *Bank of Canada Working Paper*, 2014–33, 2014.

D. Hirshleifer. Residual risk, trading costs, and commodity futures risk premia. *Review of Financial Studies*, 1(2):173–193, 1988.

L.L. Johnson. The theory of hedging and speculation in commodity futures. *Review of Economic Studies*, 27(3):139–151, 1960.

J. Kareken and N. Wallace. On the indeterminacy of equilibrium exchange rates. *Quarterly Journal of Economics*, 96(2):207–222, 1981.

F. Kroll, I. Davey, and E. Felten. The economics of bitcoin mining or, bitcoin in the presence of adversaries. *Princeton Working Paper*, 2013.

D. Laidler. The buffer stock notion in monetary economics. *Economic Journal*, 94:17–34, 1984.

P. Mizen. Microfoundations for a stable demand for money function. *Economic Journal*, 107 (443):1202–1212, 1997.

B. Ong, T.M. Lee, G. Li, and D.L.K. Chuen. Evaluating the potential of alternative cryptocurrencies. In D.L.K. Chuen, editor, *Handbook of digital currency*, pages 81–135. Elsevier, 2015.

J. Rochet and J. Tirole. Two-sided markets: A progress report. *RAND Journal of Economics*, 37(3):645–667, 2006.

B. Sauer. Central bank behaviour concerning the level of bitcoin regulation as policy variable. *Working Paper*, 2015.

B. Segendorf. What is bitcoin? *Sveriges Riksbank Economic Review*, 2014(2):71–87, 2014.

G. Selgin. Synthetic commodity money. *Journal of Financial Stability*, 17:92–99, 2014.

M. Tarasiewicz and A. Newman. Cryptocurrencies as distributed community experiments. In D.L.K. Chuen, editor, *Handbook of digital currency*, pages 201–222. Elsevier, 2015.

P. Tasca. Digital currencies: Principles, trends, opportunities, and risks. *Technical Report*, 2015.

F. Velde. Bitcoin: A primer. *Chicago Fed Letter*, December 2013(317), 2013.

J.M. Viaene and C.G. De Vries. International trade and exchange rate volatility. *European Economic Review*, 36(6):1311–1321, 1992.

B. Weber. Bitcoin and the legitimacy crisis of money. *Cambridge Journal of Economics*, forthcoming.

D. Yermack. Is bitcoin a real currency? An economic appraisal. In D.L.K. Chuen, editor, *Handbook of digital currency*, pages 31–43. Elsevier, 2015.

W. Young. What community bankers should know about virtual currencies. *Federal Reserve System Community Banking Connections*, Q2:3, 12–13, 2015.