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Abstract

The authors examine the effect of a trade-off between shared credit risk and liquidity efficiency, among participants in Tranche 2 of the Large Value Transfer System (LVTS T2), on their decisions to leave open, or close, their bilateral credit limits (BCLs) to a participant at risk of imminent closure. The authors' analysis considers a network of three banks, in a settlement system similar to the LVTS T2. Although it is widely believed that closure of one bank is imminent, the exact timing of the closure – during or after the settlement cycle – is uncertain. The other two banks face an "open or close" choice regarding their BCLs to the problem participant. Based on the expected net payoff of each choice, which includes the value of network externalities, the analysis shows that, when the expected credit loss is sufficiently low, an open-BCL pure-strategy Nash equilibrium can exist and can be Pareto efficient. This result dispels the generality of the frequent assertion that participants in the LVTS T2 will close their BCLs to a participant that is subject to imminent closure.

JEL classification: G21, L13, L14
Bank classification: Financial institutions; Financial services; Payment, clearing, and settlement systems

Résumé

Les auteurs examinent comment un arbitrage entre le partage du risque de crédit et l'efficience de la gestion de la liquidité amène les participants au Système de transfert de paiements de grande valeur (STPGV) ayant opté pour des paiements de tranche 2 soit à continuer d'octroyer une limite de crédit bilatérale à un participant menacé par une fermeture imminente, soit à clore cette ligne de crédit. Leur analyse porte sur un réseau bancaire tripartite dont le système de règlement s'apparente au mécanisme de tranche 2 du STPGV. La fermeture d'une des trois banques considérées est globalement jugée comme imminente, même si on en ignore le moment précis, c'est-à-dire si elle surviendra pendant ou après le cycle de règlement. Les deux autres banques sont placées devant une alternative : maintenir la limite de crédit qu'elles accordent au participant problématique ou lui en clore l'accès. Les avantages nets escomptés de chaque option, notamment la valeur des externalités du réseau, permettent de montrer que, dans l'éventualité où les créances irrécouvrables anticipées sont suffisamment faibles, un équilibre de Nash en stratégies pures est possible et peut être efficient selon le critère de Pareto si les limites de crédit sont maintenues. Cette conclusion vient réfuter l'idée reçue – souvent avancée – selon laquelle les participants au mécanisme de tranche 2 du STPGV cloront l'accès de leur limite de crédit bilatérale à un participant confronté à l'imminence d'une fermeture.

Classification JEL : G21, L13, L14
Classification de la Banque : Institutions financières; Services financiers; Systèmes de paiement, de compensation et de règlement

1 Introduction and Overview

The Large Value Transfer System (LVTS) operated by the Canadian Payments Association is considered the most systemically important payment settlement system in Canada. The LVTS is a dual-stream system: one stream is essentially an RTGS stream (Tranche 1), and the other involves collateralized continuous netting with real-time finality of individual payments (Tranche 2). It is the Tranche 2 (T2) stream in which the vast majority of payments are settled each day. The principal control variables available to the participants in the LVTS T2 are the bilateral credit limits (BCLs) that they grant to one another. The participants' BCLs can either 'stand' at the previous day's level, or be adjusted at the beginning of each settlement day.[1] These variables help determine the maximum credit-loss exposure that each participant may have to others in the network; the contribution that each participant must provide to the T2 collateral pool used to access intraday credit from the Bank of Canada for out-payments; and, the maximum amount of in-payment value that each participant can receive from others in the network, prior to making an out-payment. To determine the levels at which it will set its BCLs to other participants, each participant in the LVTS T2 will evaluate the potential credit loss, the cost of borrowed liquidity, and the likely level of liquidity inflow through receipt of payments from other network participants.

Other factors will also condition this decision. Notable among these is the likelihood that a particular counterparty may be closed before interbank settlement is completed for the day. Even more critical to the decision is the counterparty's likely net funds position upon closure. When network participants anticipate the closure of another, but are uncertain as to the timing of that closure, as well as its net funds position upon closure, the level at which they set their BCLs to that participant will reflect a joint probability assessment of these two factors. Because of the multilateral effects arising from network interdependency among participants, each individual participant will, in setting its own BCL with the problem participant, take account of how other participants might react in setting their BCLs. Thus, the shared risk of credit loss relating to the bilateral relationship with the problem participant, versus the risk of liquidity loss relating to the multilateral network relationship among participants, will be reflected in each participant's BCL decision regarding the problem institution. Participants will, therefore, need to coordinate, individually, on an equilibrium, defined in accordance with their BCLs, to achieve their optimal payoffs from network participation.

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1. Standing BCLs can also be adjusted on an intraday basis. They revert to their standing value at the start of the following day unless the standing BCL is changed at that time. See Arjani and McVanel (2006).

Our analysis has two objectives: first, to identify plausible conditions under which participants in the LVTS T2 will individually coordinate on a network equilibrium that leaves open their BCLs to another participant likely to be closed, and second, to evaluate the welfare-efficiency of the potential network equilibria derived from the analysis and model that underpins it.

Our model proposes that the conditions for an open BCL network equilibrium are characterized by (i) uncertainty about the timing of closure around the settlement cycle, (ii) the expected credit loss, given closure, relative to the expected payoff from maintaining in-payment flows as low-cost funding of out-payments, and (iii) the degree of symmetry or homogeneity of network participants. Under some plausible configurations of these conditions, an open BCL network equilibrium is found to exist, which can also be Pareto efficient. That said, the model can produce a closed BCL equilibrium that might also be Pareto efficient, but under different expected cost-benefit conditions. Even so, the possibility of multiple equilibria, more than one of which can be welfare efficient, is sufficient to rule out the proposition that one strategic equilibrium, whether it be a closed or open BCL equilibrium, is the only rational welfare-efficient equilibrium for the LVTS T2 when the closure of a participant is anticipated.

Section 2 of the paper describes the closure scenarios and their related credit exposures, to help define the decision space in which setting BCLs becomes problematic for network participants. Section 3 outlines a simplified network arrangement and the payoff structure for participants with respect to their BCL decisions. Section 4 defines the likelihood conditions governing the decisions of individual participants to either close or leave open their BCLs with the problem institution, and the properties of the resulting network equilibria. Our analysis in this section also describes how these network equilibria may change when the degree of network asymmetry changes. In Section 6, we draw some conclusions and policy lessons.

2 Closure Conditions for an LVTS Participant

In much of the past work on the LVTS T2 (Engert 1993; Dingle 1998; CPSS 2005; Arjani and McVanel 2006), the BCLs granted to, and by, individual participants were defined and analyzed in terms of their roles in controlling credit risk and systemic risk in the system. Our analyses focus on the roles of BCLs in the risk controls for accepting payments for settlement in the LVTS T2, the funding of the T2 collateral pool, and the loss-allocation rules for collateral in the event of a participant default that, together with the Bank of Canada's residual guarantee, support the certainty and finality of payment settlement in the LVTS. From this perspective, individual participants concerned with possible closure of another network participant could protect themselves from credit risk exposure by closing their BCLs (i.e., setting the BCL to zero) with the problem participant (CPA 2000; Freedman and Goodlet 1998).

More recent research focuses on the transfer of network liquidity involved in the LVTS T2 payment settlement. Bech, Chapman, and Garratt (2007) and O'Connor, Chapman, and Millar (2008) consider the role of the BCLs in funding the T2 collateral pool, and in the acquisition of in-payments by participants in the network. Participants use the in-payments from others in the network to fund their own out-payments. These analyses focus on the cost of liquidity in funding out-payments relative to the penalty costs of delaying such payments. The BCLs granted by an individual participant to others in the network directly affect the granting participant's liquidity costs. If the participant grants low BCLs, it could delay in-payment receipts and, thus, may not be able to avoid costly out-payment delays without adding more liquidity to their existing positions. If others in the network react the same way, the liquidity costs and out-payment delays could both increase for all participants in the network.

In this analysis, we extend the recent research to consider the relationship between the shared gains in liquidity efficiency from in-payments and the credit risk of participating in the LVTS T2 in setting BCLs. In setting their BCLs, the participants in the LVTS T2 are, generally, balancing the potential credit risk of maintaining open BCLs with another participant against the risk of incurring higher liquidity costs to fund their out-payments by closing their BCLs. The BCLs granted to one participant by others partly reflect the probability of that participant's closure. If a participant that is granted open BCLs by others in the network is widely believed to be at risk of closure, the assessments of the granting participants must reflect some uncertainty about either the timing of the possible closure, or the size of the loss upon closure. The prospect that a particular participant may face imminent closure may, therefore, not be incentive enough for other participants to close their BCLs to it. Information as to the timing of closure, the potential size of the credit loss upon closure, and conjectures regarding the BCL-setting actions of other participants will also affect the decision, when liquidity efficiency is also considered.

To illustrate the uncertainty about whether to close, or leave open, a BCL with a problem participant in the network, consider the following likelihood rankings based on the basic range of closure events. The two factors of interest in this illustration are (i) the relative timing of the closure of an LVTS participant – within the LVTS settlement cycle or between LVTS settlement cycles (i.e., after the end of today's cycle and before the start of tomorrow's), and (ii) the net settlement position of the institution at the time of its closure. We consider two types of closure events. The first is an "early intervention" closure, in which the Office of the Superintendent of Financial Institutions (OSFI) closes a problem institution, in the absence of a default or insolvency, to avoid the destabilizing effect on the overall financial system of a highly probable failure in the near future.[2] The second is a payment default. The default event can have three possible triggers: (i) default on a creditor payment that results in a court-ordered transaction freeze, windup, and liquidation of the problem institution; (ii) default by the institution on the settlement of its net debit position at the end of the daily settlement cycle because of either insufficient eligible collateral for overnight credit from the Bank of Canada or an insolvency assessment from OSFI; or (iii) a voluntary closure by the problem institution before completion of LVTS settlement when it is in a net debit position.[3]

The most likely timing for the closure of the problem institution in relation to its net LVTS settlement position – the joint probability ranking – is based on the following conditions.

1. The stated purpose of the "early intervention" policy is to allow OSFI (and the Canadian Deposit Insurance Corporation, if its member institutions are involved) to intervene effectively, at an early stage, with a problem institution under its jurisdiction, which includes most LVTS T2 members, to minimize losses to depositors. In practice, the regulators would most likely close a problem institution that participates in the LVTS T2 between LVTS settlement cycles to limit market disruptions and losses. At that point, the net settlement position of the closed participant is zero. Moreover, a policy of early intervention indicates that closure is more likely to occur because of early intervention by regulators than because of a default event.

2. Court-ordered freezes on transactions may occur, but with a lower probability than an early intervention. They can come into force during an LVTS settlement cycle but with a timing that is presumed to be (statistically) independent of the closed participant's net settlement position. Thus, the joint probability of the court-ordered closure of an institution with a net debit position is even lower than the likelihood of either of these events occurring independently.

3. The probability of the voluntary closure of a problem institution during an LVTS settlement cycle is lower than between LVTS settlement cycles, and it is generally

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2. See OSFI (2008) for the intervention policy.
3. We assume that an institution that closes voluntarily will do so at a point that limits its owners' risk to civil actions by creditors and to financial loss. In other words, it would not voluntarily close within an LVTS settlement cycle in a net debit position, which would leave it open to civil suits, unless it fully anticipates an ever-increasing net debit position (up to the level of its net debit cap) by cycle end and has insufficient eligible collateral to cover it. To be in this situation, the institution would have had to grossly underestimate the magnitude of its net liquidity flows for the day.

perceived to be lower than either an early intervention closure or a court-order closure.[4] However, the joint probability that the institution closes during an LVTS settlement cycle, when it is in a net debit position, would be higher than the marginal probability of voluntary closure itself. This marginal probability is not, however, considered higher than the probability of an early intervention closure, or court-ordered closure, with a net debit position.

4. The joint probability of closure in a net debit position because of a decision by the Bank of Canada to withhold access to overnight credit is certainly much lower than an early intervention closure or court-order closure. Since no information or policy statements are available to indicate that it would be higher or lower than the joint probability of voluntary closure in a net debit position during an LVTS settlement cycle, it is assumed to rank about the same.

A relative ordering of the joint probability of closure timing by the type of closure event would likely rank an early intervention closure between LVTS settlement cycles ahead of any closure during an LVTS settlement cycle resulting from a default event. In absolute terms, the probability of the closure of any LVTS participant is extremely low, so that even the highest-ranked closure event is an extreme tail event, in probability terms. Hence, the joint probability of closure of an LVTS participant during an LVTS settlement cycle at a point in time when the closed institution is in a net debit position (thus exposing other participants to a credit loss) is considered an even more extreme tail event.

How large might the credit loss be if such an event did occur? Research on unanticipated defaults in the LVTS, even when the largest participants are at their maximum possible intraday net debit position (McVanel 2005; Ball and Engert 2007), involves a manageable and, in most cases, relatively low, credit loss for surviving participants. The average daily net debit positions of most participants are well within the value limits of their own contributions to the T2 collateral pool, with the possible exception of the very largest participants. But, even then, Northcott (2002), using CDIC data for part of her work on systemic risk in the ACSS, found recovery rates of about 75 per cent of initial credit-loss exposure relating to banking failures in Canada. The introduction of the structural early intervention framework (and other improvements in the

4. Early intervention is designed to close an institution even earlier than the institution itself might perceive strategic closure to be the best strategy. And an institution that continues to operate in the face of a possible court-ordered closure has already demonstrated that voluntary closure is very unlikely.

financial safety net), since that time, would likely raise the recovery rate even further.[5] Thus, the combination of the low probability of a closure event involving a net debit position with the limited size of the credit loss (especially when asset-recovery rates are considered) yields a very low expected credit loss from leaving BCLs open. Indeed, the more certain that participants are that the closure will be between settlement cycles, which would be consistent with the likelihood ranking of early intervention, the less likely it will be that participants will close their BCLs.

Even so, there is a range of uncertainty about closure timing and the magnitude of credit loss that, together with considerations about bilateral and multilateral liquidity cost and the penalty cost of out-payment delays, will require individual participants to analyze the strategic payoff of leaving their BCLs open to the problem participant or closing them. The next section formulates a simple network model that will help identify the underlying conditions for a network equilibrium in which BCLs remain open versus those for which the equilibrium is characterized by closed BCLs.

3 The Model and the Payoff Structure

Consider a network structure of three banks in a settlement system similar to the LVTS T2. Suppose that Bank 3 is a problem bank for which closure is widely anticipated.[6] Since Bank 3 can participate in the LVTS T2 only if at least one of the other banks provides it with an open BCL, then Bank 3 could be in a net debit position when it closes. Consequently, any potential credit losses resulting from the closure of Bank 3 during the settlement cycle, when it is in a multilateral net debit position, would either be shared by Banks 1 and 2 or borne only by one if the other closes its BCL.[7]

Suppose that Bank $i = \{1, 2\}$ has a binary strategy with respect to setting its BCL for Bank 3. It provides a BCL of zero or a positive fixed finite value (i.e., strategy space: $s_i = \{0 , C_i\}$ for $C_i > 0$). The set of strategic profiles are defined as: $s_i \times s_j \equiv \{(C_i , C_j) , (C_i , 0) , (0 , C_j) , (0 , 0)\}$. The gross benefit to Bank i under the set of strategic profiles in the LVTS T2 network is defined as $D_i (s_i , s_j)$. It depends on the BCL it grants to Bank 3, as well as on the BCL that the other healthy bank grants to Bank 3. This interdependency of Bank i's action with those of other banks in the network represents the indirect network effects of their actions on Bank i's well-being.

5. See Engert (2005) for information on the evolution of the financial safety net in Canada, including the early intervention program, as well as the framework, process, and objectives for the overall financial safety net.
6. The analysis extends to $N > 3$ banks where $N - 1$ are healthy banks.
7. See Arjani and McVanel (2006) for details on the role of BCLs in a loss-allocation formula for the LVTS.

If Bank 3 is closed in a net debit position, the potential credit loss will be the value of Bank 3's multilateral net debit position less its collateral contributions to the LVTS. This will be termed Bank 3's collateral shortfall position. Let the value of the shortfall be L, where $L < \sum_{i \neq 3} V_i$ and V_i is the value of Bank i's contribution to the collateral pool.[8] Bank i's share of the shortfall is defined as: $\alpha_i = (s_i / \sum_i s_i)$; $i = 1, 2$ (which is low in value relative to the absorption capacity of Banks 1 and 2). The maximum possible loss from the intraday closure of Bank 3 is defined by the product of the systemwide parameter and the sum of the BCLs, granted to it, by Banks 1 and 2 (i.e., its net debit cap). Although the maximum possible loss is clearly greater when both banks maintain open BCLs instead of just one of them, the actual loss upon closure of Bank 3 will generally be significantly lower than its maximum possible value.

Define $1 >> \pi > 0$ as the joint probability of Bank 3 closing during the settlement cycle, while in a shortfall position. Although essentially a subjective private assessment, this probability distribution is assumed to be the same for all participants, since all generally have the same information. The distribution is defined on information available at the start of the settlement cycle when the BCLs are determined.[9] It represents the degree of anticipation of a credit loss for Banks 1 and 2 by leaving their BCLs with Bank 3 open.

The payoff to Bank i of its BCL strategy combines its gross benefit net of expected loss from leaving open or closing its BCL with Bank 3, conditioned on the other bank's BCL strategy. The 2 x 2 normal form of the payoff structure for the surviving banks in the potential network equilibria is as follows:

	C_2	0
C_1	$[D_1(C_1, C_2) - \pi\,\alpha_1 L]$, $[D_2(C_1, C_2) - \pi\,\alpha_2 L]$	$[D_1(C_1, 0) - \pi L]$, $[D_2(C_1, 0)]$
0	$[D_1(0, C_2)]$, $[D_2(0, C_2) - \pi L]$	$[D_1(0, 0)]$, $[D_2(0, 0)]$

8. Note that $L = NDP_3 - V_3$ for NDP_3 is the net debit position of Bank 3 upon its closure.
9. The subjective probability distribution is conditioned on available prior information on the financial and regulatory status of Bank 3. It is independent of the magnitude of possible gross benefits or losses. Indeed, it is derived to evaluate the expected net payoffs of the open versus closed BCL strategies and, hence, is a determining factor in the individual strategic choice of Banks 1 and 2.

The potential equilibrium payoffs for the BCL decisions of Banks 1 and 2 with respect to Bank 3, defined in each cell of the payoff matrix, depend on the following assumption.

Assumption 1 *Relative (gross) benefit structure*

Assume that (i) gross benefits are linear homogeneous in (C_1, C_2)*, (ii)* $D_1(C_1, C_2) > D_1(0, C_2)$ *and* $D_1(C_1, 0) > D_1(0, 0)$ *so that* $[D_1(C_1, C_2) - D_1(0, C_2)] > [D_1(C_1, 0) - D_1(0, 0)]$*, and (iii)* $D_2(C_1, C_2) > D_2(C_1, 0)$ *and* $D_2(0, C_2) > D_2(0, 0)$ *so that* $[D_2(C_1, C_2) - D_2(C_1, 0)] > [D_2(0, C_2) - D_2(0, 0)]$.

Linear homogeneity states that an equiproportionate increase in (C_1, C_2) will increase related gross benefits by the same proportion. It simplifies the analysis without much risk to the generality of its results.

The first point in the second part of this assumption states that, for any given strategy by Bank 2, Bank 1 benefits more in terms of gross benefit from leaving its BCL with Bank 3 open than from closing it (i.e., before taking account of expected credit losses). This follows from the liquidity-efficiency proposition of BCL-setting; namely, by extending a BCL to another network participant, the extender can obtain in-payment flows that are used to fund its out-payments. Sometimes called an in-payment (or liquidity) pooling effect, the sum of the BCLs granted by a participant caps the level of in-payment liquidity that it can accumulate and "trap" before making an out-payment. The cap is made effective by the fact that the extender is required to contribute an amount of collateral to the T2 collateral pool based on the maximum BCL that it grants. As a result, greater in-payment liquidity pooling requires, at some point, a higher contribution to the collateral pool. The last point in the assumption simply mimics the first point, with the two banks in reverse positions.

The first and second elements of the assumption are non-controversial. If there were no individual benefits to leaving a BCL open with a network counterparty (given the BCL decisions of other network participants regarding the same counterparty), every participant would close its BCL. The last element of the assumption is, however, more technical and is critical to the equilibrium outcome. It states that a bank gains at least as much benefit from leaving its BCL open, when another does so, as it does when the others close their BCLs. This is the essential strategic complementarity property of settlement networks that drives common action. It indicates that every individual participant benefits more from a complete system (i.e., from maintaining open BCLs with each other) than from an incomplete system in terms of the efficiency of liquidity transfer.

The next step is to determine the (relative) probability conditions for the closure of Bank 3, with a collateral shortfall, during a settlement cycle. These probability conditions help determine which of the strategic BCLs set by Banks 1 and 2 formulate an equilibrium.

4 The Analysis of the Potential Equilibria

By definition, the joint probability of closure of Bank 3 within a settlement cycle and with a collateral shortfall is in the extreme low end of the range $\pi = [0 , 1]$. To narrow the range around relative threshold levels that support the possible equilibria, the following definition is imposed.

Definition 1 *Probability supports for potential equilibria:*

(a) $\pi_c > 0$ *is the maximum probability threshold capable of supporting* (C_1 , C_2) *as a Nash equilibrium;*

(b) $\pi_o > 0$ *is the minimum probability threshold capable of supporting* $(0 , 0)$ *as a Nash equilibrium;*

(c) $\pi_p > 0$ *is the minimum probability threshold for which* $(0 , 0)$ *Pareto dominates* (C_1 , C_2) *as the network equilibrium.*

The elements of the definition reflect the following logic. Given the extremely low probability of an intracycle closure of Bank 3 when it is in a shortfall position, and the limited credit exposure given this event, the participants are likely to leave their BCLs with Bank 3 open, since the gross benefit of doing so is at least as great as closing the BCL, under Assumption 1. Beyond the probability threshold of π_c, this is no longer the case, and maintaining an open BCL is no longer optimal. Consequently, there is some probability level, π_o, above which closing BCLs is optimal. The first issue to consider is the level of π_o relative to π_c.

The last part of the definition will not affect the BCL decisions of the individual participants, but will help indicate whether the equilibrium outcome of those decisions is welfare-optimal, systemwide.[10] The issue here is the relation between π_c and π_p.

4.1 Open BCL equilibria under symmetry

To consider these probability relationships, assume that Banks 1 and 2 are homogeneous in business lines and symmetric in their network properties and their BCL decisions.

10. In effect, this would be a network or social planner's equilibrium-support level in probability for open BCLs by all network participants.

Assumption 2 *Symmetry*

$C_1 = C_2$; $D_1(C_1, C_2) = D_2(C_1\ C_2)$; $D_1(0, 0) = D_2(0, 0)$; *and* $D_1(0, C_2) = D_2(C_1, 0)$.

Symmetry restricts the network equilibria to the pure-strategy Nash equilibria $\{(C_1, C_2), (0, 0)\}$, but simplifies the analysis somewhat. The symmetry condition will be relaxed later in the analysis, to show what the comparative effects of asymmetry could imply.

The following proposition addresses the first issue on relative probability supports.[11]

Proposition 1 *Range of indeterminacy*

Under Assumptions 1 and 2 and Definition 1, $\pi_c > \pi_o$

This proposition implies that there are three regions in the probability range $\pi = [0, 1]$. In the subrange $\pi < \pi_c$, (C_1, C_2) is the potential Nash equilibrium. For $\pi > \pi_o$, $(0, 0)$ is the potential Nash equilibrium. But, since the proposition states that $\pi_c > \pi_o$, there is an intermediate subrange, in which $\pi_o < \pi < \pi_c$, where both equilibria can exist. For a given range of possible shortfall losses upon closure of Bank 3 (zero to the maximum possible loss as defined by Bank 3's net debit cap), the uncertainty subrange in probability is narrower and further out in the tail of the marginal probability distribution $\pi \in [\pi_o, \pi_c]$, with more of the mass of the distribution of losses concentrated near zero. In other words, the distribution has very 'thin' tails. This leads to the following corollary to Proposition 1.

Corollary 1 *Uniqueness of* (C_1, C_2) *under certainty of no loss*

If $\pi = 0$, (C_1, C_2) *is the unique Nash equilibrium in the intermediate subrange.*

Simply stated, if there is a commonly assessed zero probability that Bank 3 will be closed during the upcoming settlement cycle (i.e., certainty of no closure or, more exactly, that it will not be closed in a shortfall position), then other network participants will leave their BCLs with Bank 3 open. There is no credit risk to consider, and the liquidity-transfer role of BCLs dominates the decision. Taken together, these results indicate that, even as $\pi \to \pi_c$, there is not necessarily a sufficient incentive for an individual participant to close its BCL with Bank 3. The symmetry assumption also suggests that all participants will individually make the same BCL decision at the same level of $\pi \in [\pi_o, \pi_c]$.

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11. The proofs for the propositions are in the appendix. The model set-up, the propositions, and their proofs are drawn from Fudenberg and Tirole (1991), Fudenberg and Levine (1998), and Weibull (1997).

A more practical expression of the argument is the following: the higher is the gross payoff from leaving a BCL open relative to closing it, the greater will be the tolerable loss when it is left open. Accordingly, the probability levels defining the uncertainty range for the BCL choice are further out in the tail of a given distribution, and the intervals are narrower. The results do not indicate with certainty, however, that network participants will leave their BCLs with Bank 3 open for any specific range of potential losses. The decision is dependent on the subjective probability distribution and the associated expected net payoff from doing so. Even so, the results do suggest that there is some subinterval in the uncertainty range, $\pi \in [\pi_o, \pi_c]$, in which the open BCL equilibrium (C_1, C_2) will be the risk-dominant equilibrium.

Proposition 2 *Risk dominance of* (C_1, C_2)

Under Assumptions 1 and 2 and Definition 1, there exists a $\pi_d \in [\pi_o, \pi_c]$ *such that, for* $\pi \leq \pi_d$, (C_1, C_2) *is the risk-dominant equilibrium.*

The proposition asserts that, in the intermediate subrange, there is a probability threshold for the closure of Bank 3 in a shortfall position that is low enough for other participants in the network to keep their BCLs open. In this case, the benefits of open BCLs, in terms of liquidity transfer, dominate the expected credit-loss exposure for each of the individual participants. The further out in the tail of the marginal distribution that a loss (given closure) is, the more likely it is that open BCLs will be chosen by Banks 1 and 2. Outside that subinterval (i.e., in the $\pi \in [\pi_d, \pi_c]$, where $\pi_o \leq \pi_d$), the open BCL equilibrium will not be risk dominant, and a closed BCL equilibrium can arise.

Even though the symmetry assumption implies that Banks *i* and *j* make the same individual decision, Proposition 2 indicates that Bank *i* will make its BCL decision independently of Bank *j*, but in anticipation of what action Bank *j* is likely to take. Assumption 1 is critical to this result, since it states that there is still a gross benefit of Bank *i* leaving open its BCL, whether or not Bank *j* does so as well. Hence, for $\pi \leq \pi_d$, if Bank *i* chooses to leave its BCL with Bank 3 open, it would not need to believe that the other participants in the network would do the same. If, on the other hand, it chooses to close its BCL on its belief that $\pi > \pi_d$, Bank *i* will need to strongly believe that other participants will do the same, or it would be worse off than if it left its BCL open. Since it is riskier for a participant in the network to unilaterally close its BCL with Bank 3 than to unilaterally leave it open, the participant is more likely to leave it open. Thus, (C_1, C_2) is the risk-dominant Nash equilibrium for $\pi \leq \pi_d$.

Propositions 1 and 2 focus on the probability-support levels and the relative gross benefits to (symmetric) individual participants to achieve an equilibrium in which all BCLs with Bank 3 remain open. However, the actual decision process, and hence the location of $\pi_d \in [\pi_o , \pi_c]$, depends on the net payoffs associated with this equilibrium, not just the gross benefits. The expected individual loss, which reflects both the size of the loss given a closure in a shortfall position and the loss-sharing arrangement, matters. Assumption 2 is helpful in this regard, since it implies that $\forall\ i = 1, 2$, $C_i = C$; $D_i (s_1 , s_2) = D(s_1 , s_2)$; and $\alpha_i = \alpha = 0.5$. Starting with an extreme case in terms of the probability-support levels, under what conditions might (C , C) be the unique Nash equilibrium?

Proposition 3 *Uniqueness of* (C_i , C_j) *under uncertainty*

Under Assumptions 1 and 2 and Definition 1, network participants will not close their BCLs with Bank 3 so that (C , C) *is the unique pure-strategy Nash equilibrium if* $L < [D (C , 0) – D (0 , 0)]$.

Proposition 3 follows from Proposition 2. As proposed, (0, 0) can exist as an equilibrium only if $\pi_o \geq 1$. By definition, the inequality cannot hold strictly. Moreover, even if $\pi_o = 1$, so that the participants are certain that Bank 3 will be closed during the settlement cycle in a shortfall position, an individual participant will still leave its BCL open, since the loss-given-default is less than the marginal gain from doing so. This decision will be taken even if other participants are expected to close their BCLs with Bank 3. An open BCL would, therefore, always be the preferred strategy, when an individual participant's share of the loss from Bank 3's collateral shortfall is less than the gross marginal gain from leaving the BCL open. Even if π_o falls sufficiently relative to π_d and π_c as L rises, an open BCL could still remain the preferred strategy. It might even still be the risk-dominant strategy.

4.2 The welfare properties of equilibria under symmetry

Proposition 3 is unambiguous about the network preference for the (C, C) Nash equilibrium, but it does not imply that (C , C) is always the welfare-efficient equilibrium. The last part of Definition 1 helps determine, along with the following assumption, the conditions for the probability supports under which the (C , C) Nash equilibrium is also the Pareto-efficient equilibrium for the network. Since (C , C) is welfare-enhancing for Bank 3, which would otherwise be forced out of the network and into costlier settlement or default, Pareto optimality depends on the welfare properties of an open BCL equilibrium for both Banks 1 and 2.

Assumption 3 *Dominance in relative (gross) benefits*

In addition to Assumption 1, suppose that
$D(C\,,\,C) > D\,(C\,,\,0)$ *and* $D(0\,,\,C) > D(0\,,\,0)$ *so that* $D(C\,,\,C) > D\,(C\,,\,0) \lesseqgtr D(0\,,\,C) > D(0\,,\,0)$.

Under this assumption, the equilibrium $(C\,,\,C)$ dominates equilibrium $(0\,,\,0)$ in terms of gross benefits (and net payoffs for given $\{\pi, \alpha, L\}$ and Assumption 2). Also, even if Bank i closes its BCL, it still receives gross benefits from indirect liquidity transfers as long as another participant leaves its BCL open. Note that, under Assumption 2, $D(C\,,\,0) = D(0\,,\,C)$. This leads to the next proposition.

Proposition 4 *Pareto efficiency of the* $(C\,,\,C)$ *network equilibrium*

Under Assumptions 1, 2, and 3 and Definition 1, then $\pi_p > \pi_c$ *and, if* $(C\,,\,C)$ *is a Nash equilibrium (i.e.,* $\pi < \pi_c$ *), it is also a Pareto-efficient equilibrium.*

When the conditions hold for an open BCL with Bank 3 to be the preferred payoff strategy of each participant, the $(C\,,\,C)$ equilibrium, as the risk-dominant equilibrium, will also be the Pareto-efficient equilibrium. Therefore, where π_p is the threshold probability level for Pareto optimality, it cannot be that $\pi_p \leq \pi_c$. The probability support for the Pareto-efficient equilibrium encompasses that for $(C\,,\,C)$ as the Nash equilibrium.

Simply stated, $(C\,,\,C)$ is the "first-best" equilibrium among all feasible equilibria. It also implies that, as long as $\pi_o < \pi_c < \pi_p$ and Assumptions 1 and 3 hold, $(C\,,\,C)$ would still be the Pareto-efficient equilibrium, even if it were no longer strictly risk dominant and did not emerge as the equilibrium outcome for the network. This result holds because the expected credit loss is low relative to the liquidity-efficiency gains, network-wide, from maintaining open BCLs. Moreover, the expected losses and liquidity-efficiency gains are shared among the participants such that no individual participants would bear more of the expected loss than the gain from an open BCL equilibrium, under Assumption 2 (symmetry).

This Pareto-efficiency result also depends critically on Assumption 3. For example, if $D(0\,,\,0) > D(0\,,\,C)$, so that Assumption 3 were violated, and there were implicit penalties for closing BCLs when others do not, it may pay Bank i to close its BCL when Bank j does. In this case, the probability support for the Pareto-efficient equilibrium is below that for the open BCL equilibrium ($\pi_p < \pi_c$).[12]

This result is not unreasonable. O'Connor, Chapman, and Millar (2008), for example, suggest that if, by closing its BCL unilaterally, a participant delays receiving in-payments for its customers, or making out-payments to other network participants, these customers and network counterparties might retaliate. Participants in the network could delay their out-payments to the "non-compliant" participant, and its customers could shift their funds to another bank. Retaliation is more likely for corporate and institutional customers that have accounts with multiple banks than for customers that concentrate their deposit and payments business with a single bank. In terms of our model, if a customer tries to shift funds from Bank 3 to its account at Bank 2, only to find that the absence of an open BCL from Bank 2 slows the process, the customer may decide to shift the funds from Bank 3 to its corporate account at Bank 1, if the latter has kept its BCL open with Bank 3.

4.3 Equilibria under asymmetry

The symmetry assumption for Banks 1 and 2 ensures that the direct and indirect gross benefits and the potential credit losses, from open BCLs with Bank 3, are equally shared by Banks 1 and 2. Such symmetry is not the case for the LVTS. In fact, the LVTS T2 has substantial asymmetry among its participants. Such asymmetry can affect a participant's BCL strategies and the network equilibria.

Assumption 4 Asymmetric banks

Assume that the banks are asymmetric in terms of bilateral payment flows and BCLs so that (i) $max\ C_{1,2} > max\ C_{21} > max\ C_{3,2}$ *and (ii)* $C_{31} < C_{13} \leq C_{23}$.

Under this assumption, Bank 2 is the largest participant, Bank 3 is the smallest, and the BCL relationships are such that ($max\ C_{12} - C_{13}$) > ($max\ C_{21} - C_{23}$). Note that the BCLs granted to

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12. There are other cases in which $\pi_p \leq \pi_c$ for violations of Assumptions 1 and 3, which relate to the relative size of indirect network effects. Recall that under the symmetry condition $D(C , 0) = D(0 , C)$ so that if $D(0 , 0) > (C , C)$, then $\pi_p < \pi_c$. The (0 , 0) Nash equilibrium is first-best. So, while the (C , C) equilibrium can still exist, it would not be Pareto efficient. If there are no indirect benefits or penalties from closing a BCL when others do not, $D(0 , 0) = D(0 , C)$, then $\pi_p = \boldsymbol{\pi_c}$ and under Assumption 1, $D(C , C) > D(0 , 0)$. In this case, the (C , C) Nash equilibrium is Pareto efficient only when it is risk-dominant. When $D(0 , 0) < D(C , C) = D(C , 0)$, then $\pi_p < \pi_o < \pi_c$, and a bank that leaves its BCL open when all other participants close theirs will receive indirect benefits from network liquidity transfers from Bank 3 in a value sufficient to compensate for the expected credit losses from doing so. In this case, (0 , 0) would never be a Pareto-efficient equilibrium, although it may exist as a Nash equilibrium.

Bank 3 are not the maximum BCLs granted by either of the other two banks. Bank 1, and to a lesser extent Bank 2, could raise their BCLs with Bank 3 and still not be required to contribute more value to the collateral pool.

With an asymmetry specified as $C_{13} < C_{23}$, if Bank 3 is closed with a shortfall during the settlement cycle, Bank 2's share of the loss will exceed that of Bank 1. But, if the anticipated closure of Bank 3 is expected to generate a rise in its out-payments, Bank 1 would benefit less from the additional liquidity flow than would Bank 2 by maintaining the existing BCL structure. This leads to the following definition and proposition.

Definition 2 *Marginal benefits and losses*

For $\triangle C_i = C_i^* - C_i$, *let* $C_i^* = (1 + \delta) C_i$ *and* $L^* = [1 + ((\delta C_i) / L)] L$ *for* $0 < \delta < 1$.

The definition indicates that, by raising its BCL by proportion δ, Bank i can internalize the marginal benefit, if Bank j maintains its BCL, but share the additional risk of credit loss. Specifically, if Bank 1 raises its BCL to Bank 3, the marginal gross benefit can be fully internalized only if Bank 2 maintains its existing BCL. If the probability of closure in a shortfall position π does not change, the size of the loss (given closure) might still increase when Bank 1 raises its BCL to Bank 3. The loss shares will also change in favour of Bank 2 if it does not increase its BCL when Bank 1 does. Bank 1's share of the expected loss rises, while Bank 2's falls.

Proposition 5 *Asymmetric BCL adjustment*

Under Assumptions 1 and 4 and Definitions 1 and 2, then (C_1^*, C_2) *is a pure-strategy Nash equilibrium.*

When Bank 1 raises its BCL with Bank 3, it takes on a larger share of the credit risk exposure, which rises with Bank 3's higher net debit cap.[13] However, it will still pay Bank 1 to do so, if its expected share of the loss ($\alpha_1 * \pi$) at the new (C_1^*, C_2) equilibrium, is still less than the ratio of its gross payoff to the shortfall loss, at that equilibrium. In this equilibrium, there is no expected positive payoff to Bank 2 for following suit. The lower share of the expected loss from Bank 3's closure outweighs any likely gross benefit gain to Bank 2 from following Bank 1's lead. As for the welfare properties of this equilibrium, when Banks 1 and 2 are asymmetric, the adjustment to

13. A participant's net debit cap is the product of the sum of the BCLs granted to it by other participants times the systemwide parameter. See Arjani and McVanel (2006) for more detail.

a new Nash equilibrium (C_1*, C_2) may not always be a Pareto-efficient change. By raising its BCL to Bank 3, Bank 1 internalizes all of the potential gain, but it may impose an additional marginal loss on Bank 2 by increasing Bank 3's potential shortfall, upon closure.

This proposition is conditioned on a specific form of asymmetry among network participants, in which Bank 2 is a larger participant than Banks 1 and 3. Even so, it demonstrates that asymmetry among network participants can yield equilibria that reflect different BCL adjustment strategies for large and small participants. It also demonstrates that asymmetry can affect the Pareto efficiency of network equilibria.

5 Conclusion and Policy Implications

The complexity of the network interdependencies and pooling effects in the LVTS T2 requires a fair degree of simplification of the system, and its equilibrium payoff structure, to allow tractability in the analysis of participant behaviour. The risk of such simplification is that the general validity of the analytical results and conclusions might be affected. We believe that this risk is low since, despite the model simplifications, the logic of the main propositions seems reasonable. First, even when network participants widely anticipate the closure of another participant, they may rationally choose not to close their BCLs with that participant if they (i) are uncertain as to the timing of the closure, (ii) perceive direct liquidity-efficiency gains from keeping their BCLs open, and (iii) expect any loss (given closure) to be small relative to the potential gains from leaving their BCLs open. Second, the decision of participants not to close their BCLs may, under appropriate risk conditions, be the "first-best solution" for the network, yielding a welfare-efficient equilibrium.

The analysis also supports the findings in other research that asymmetry among network participants can produce strategic equilibria for a network, additional to those found under symmetry. In this case, the additional BCL equilibria under asymmetry are generally consistent with the underlying conditions for an open BCL equilibrium under symmetry. These additional equilibria are not, however, Pareto efficient. Indeed, even without asymmetry among the participants, their BCL decisions might not always prove to be a first-best solution. Even symmetric participants can choose to maintain their open BCLs too long, or be too quick to close them, because of uncertainty about the timing of a closure.[14]

14. This outcome will arise in the probability subrange $\pi \in [\pi_o , \pi_d]$ in which (C_1 , C_2) is a possible equilibrium but not a risk-dominant equilibrium. Conversely, in the subrange $\pi \in [0 , \pi_o]$, (0 , 0) is a possible equilibrium but not the first-best among all possible equilibria.

The results of our analysis are broadly consistent with previous experience of the behaviour of participants, when the closure of a network participant was widely anticipated but its timing was uncertain. In the mid-1980s, participants in the Canadian Payments Association's Automated Clearing Settlement System actively sought information from official sources about the timing of an anticipated closure of another participant, yet continued to take payments from, and make payments to, that institution. More recently, some LVTS participants have indicated their reluctance to close their BCLs to a problem participant without information about the circumstance and timing of its closure.

Since our research question is very specific, there are really only two main policy implications to be drawn from the analysis. The first is that overseers and regulators need to better understand the complexity of the network effects and the liquidity-transfer properties on the full range of performance objectives, constraints, and incentives for individual participants in payment networks, before underpinning their policy decisions on postulated behaviour. In general, the behaviour of market participants is highly conditional. In markets for network services, this conditionality is even more complicated than in more basic market structures because of the network interdependency among individual participants. The pooling and sharing of liquidity and credit risk in systems such as the LVTS T2, as well as asymmetries in the degree and value of connectivity among participants, add even more complexity to the strategic decision making of network participants. Behavioural conjectures for payments-policy decisions that do not adequately take account of the complex conditions motivating participants' behaviours can lead to unanticipated, and unintended, outcomes. Therefore, regulators should intervene in the network markets for payment services cautiously and on the basis of firmly grounded, credible analysis not only of the "impact" effects, but of the ultimate equilibrium effects as well.

The second lesson is specific to the closure policy for participants in the LVTS. In their statements about closure policy, regulators might emphasize more clearly that, in most circumstances, they would not force closure of a participant during a settlement cycle. Exceptional circumstances in which "forced" closures, during a cycle, may occur – notably, court-ordered, creditor-initiated closure, or a closure because of disqualification for Bank of Canada credit – may still arise, but under an effective early intervention policy they are considered to be extremely low-probability events. If the closure policy were clear to participants, the probability supports could shift further out in the tail of the distribution to create a narrower range of uncertainty than otherwise. It could also result in a wider subrange in the tail of the distribution for which an open BCL equilibrium would be risk dominant and Pareto efficient.

Several potential improvements could be made to the analysis in future work. Most notably, in this analysis, we consider binary choices only – open or closed BCLs and, under asymmetry, higher or fixed BCLs. Such choices do not allow participants to adjust the "open" value of their BCLs as new information arrives to alter their subjective assessments, and the expected payoffs, of their BCL strategies. Continuity in BCL adjustment related to new information would require more attention to the shapes of these subjective probability distributions and to the functional forms for the expected net payoffs in terms of continuously valued BCLs.

A second issue for future work relates to the policy implication favouring greater transparency in the closure policy. It has been noted that, because of uncertainty about the timing of the closure of a problem participant, participants can coordinate on network equilibria that are Pareto inefficient. Greater transparency about the closure policy may enhance the likelihood that an open BCL equilibrium for the LVTS T2 would be risk dominant and Pareto efficient. It cannot, however, guarantee Pareto efficiency of an open BCL equilibrium, especially if the potential for moral hazard is considered. If policy transparency is misinterpreted by network participants as a firm guarantee of no closures within an LVTS settlement cycle, BCLs may be left open, even when closed BCLs would be the Pareto-efficient decision. Future work might wish to consider whether the social cost of such a potential moral hazard problem would exceed the social benefit in terms of additional liquidity efficiency in the network for greater transparency about the closure policy. The extremely low credit losses likely to arise under the early-intervention strategy suggest, to us, a very low social cost from the potential moral hazard of greater policy transparency. But confirmation of this prior belief is warranted.

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Appendix
Proofs of Propositions

Proof of Proposition 1 ***Range of indeterminacy***

Let $U_i\ (C_i\,,\ C_j)$ denote the expected net benefit of Bank i in the Nash equilibrium $(C_i\,,\ C_j)$. Under the symmetry assumption (Assumption 2), $U_i = U$, $C_i = C$ and for equilibrium $(C,\ C)$ $\alpha_i = \alpha = 0.5,\ \forall\ i = 1, 2$. Note that, for Nash equilibrium $(0\ ,\ 0)$, $L = 0$ and $\alpha = 0$. For $\{(0,\ C), (C\ ,\ 0)\}$ equilibria, $L \geq 0$ and $\alpha = 1$, indicating that only one of Bank 1 or 2 accepts the credit risk by maintaining an open BCL.

At π_c, the maximum probability threshold for the Nash equilibrium $(C_i\,,\ C_j)$, it must be that the net expected payoff for an open BCL for each of Banks 1 and 2 must be zero so that $[(0.5)\ \pi_c\, L = D(C\ ,\ C) - D(0\ ,\ C)]$. Thus, $U(C,\ C) \geq U(0\ ,\ C)$ implies $\pi \leq \pi_c = \{2[D(C,\ C) - D(0,\ C)] / L\ \}$.

At π_o, the minimum probability threshold for the Nash equilibrium (0, 0), it must be that $\pi_o\, L\ = [D(0,C) - D(0, 0)]$ so that $U(0\ ,\ C) = U(C,\ 0) \geq U(0, 0)$ implies $\pi \geq \pi_o = \{[D(0,\ C) - D(0, 0)] / L\ \}$.

It follows from Assumption 1 that $\{2[D(C,\ C) - D(0,\ C)]\} > \{[D(C,\ 0) - D(0, 0)]\}$ so that $\pi_c > \pi_o$.

Proof of Corollary 1 ***Uniqueness under certainty of no loss***

When $\pi = 0$, then $(C_1\,,\ C_2)$ is a Nash equilibrium when $[D(C,\ C) - D(0,\ C)] > 0$ given π_c from Definition 1. According to π_o from Definition 1, when $[D(C,\ 0) - D(0, 0)] > 0$, then (0, 0) is not a Nash equilibrium. Since both inequalities hold under Assumption 1, then $(C_1,\ C_2)$ is the unique Nash equilibrium.

Proof of Proposition 2 ***Risk dominance of*** (C_i, C_j)

For this proposition to hold, it must be that Bank i believes more firmly that: (i) Bank j will choose $BCL_j = (0)$, when it chooses $BCL_i = (0)$ than (ii) Bank j will choose (C_j) when it chooses (C_i). In this case, (C_i) risk dominates (0) as a strategic choice for Bank i, $i = 1, 2$.

In addition to Assumptions 1 and 2 and Definition 1, let y be the minimum probability that Bank j selects (C_j) when Bank i plays (C_i). Similarly, let x be the minimum probability that Bank j selects (0) when Bank i selects (0). Then, $0 \leq \{x\ , y\} \leq 1$.

To prove Proposition 2, we need to: (i) find values of probabilities $\{x, y\}$ for Bank i from its net expected benefit functions at equilibrium; (ii) demonstrate that $x>y$ from conditions given in Assumption 1; and (iii) using the specifications of $\{x, y\}$, solve for a threshold specification for π_d from the equality $x = y$ and show from Proposition 1 that it must be that $\pi_o < \pi_d < \pi_c$.

Probability y

From the symmetry assumption, what is true for Bank 1 also holds for Bank 2 (and $\alpha_i = \alpha = 0.5,\ \forall\ i = 1, 2$). Then,

$U_1 [C , (y (C) + (1-y) (0)] > U_1 [(0) , (y (C) + (1-y) (0))]$ implies in net payoff functions at Nash equilibrium that

$y [D(C , C) - (0.5) \pi L] + (1-y) [D(C , 0) - \pi L] > y [D(0 , C)] + (1-y) [D(0 , 0)]$,

which solves to

$y > \{[D(0 , 0) - D(C , 0)] + \pi L\} / \{[D(C , C) - D(C , 0)] - [D(0 , C) - D(0 , 0)] + (0.5) \pi L\}$,

which clearly holds, since $y \geq 0$ by definition and the numerator of the RHS is negative under Assumption 1 and a sufficiently low value of (πL) while the denominator is strictly positive under Assumption 1.

Probability x

$U_1 [0 , ((1 - x) (C) + x (0))] > U_1 [(C) , ((1 - x) (C) + x (0))]$ implies in net payoff functions at Nash equilibrium that

$x > \{[D(C , C) - D(0 , C)] - (0.5) \pi L\} / \{[D(C , C) - D(C , 0)] - [D(0 , C) - D(0 , 0)] + (0.5) \pi L\}$.

Risk dominance of C_i for i =1, 2.

Subtracting y from x and solving, then

$(x- y) > 0$ *if* $\{[D(C, C) - D(0, C)] + [D(C, 0) - D(0, 0)] - (1.5) \pi L\} > 0$, which holds under Assumption 1 for a sufficiently low (πL).

Under Assumption 1 and a sufficiently low expected loss, an open BCL with Bank 3 will be the risk-dominant strategy for Bank i, regardless of Bank j's strategy.

(C, C) as the risk-dominant Nash equilibrium

Solving for π from $x = y$ and defining $\pi = \pi_d$

$$\pi_d = \{[D(C, C) - D(0, C)] - [D(C, 0) - D(0, 0)]\} / \{(1.5) L\}$$
$$= \{2[D(C, C) - D(0, C)] / 3(L)\} - \{2[D(C, 0) - D(0, 0)] / 3(L)\}.$$

From Proposition 1, it is shown that $(\pi_c - \pi_o) = \{2[D(C, C) – D(0, C)]\} – [D(C, 0) – D(0, 0)]\} > 0$, where $\pi_c = \{2[D(C, C) – D(0, C)] / L\}$ and $\pi_o = \{[D(C, 0) – D(0, 0)] / L\}$. Then, $\pi_d = \{[\pi_c – 2(\pi_o)] / 3\} > 0$ so that $\pi_c > \pi_d$. Then, $(\pi_c - \pi_d) = (2/3)(\pi_c - \pi_o)$, implying that $\pi_d > \pi_o$.

Proof of Proposition 3 ***Uniqueness of*** **(C_i, C_j)** ***under uncertainty***

Under Assumption 1 and the definition of π_o in Definition 1, then if $L < [D(C_1, C_2) – D(0, C_2)]$, and even if $\pi = 1$, (0,0) can never be a Nash equilibrium, since it always pays one bank (Bank 2) to have an open BCL. Moreover, (C_1, C_2) is a unique pure-strategy Nash equilibrium and if $\alpha L < [D(C_1, C_2) – D(0, C_2)]$ for $\alpha \leq 1$.

Proof of Proposition 4 ***Pareto efficiency of*** **(C_i, C_j)**

The Nash equilibrium (0, 0) can Pareto dominate the Nash equilibrium (C_i, C_j) only if $[D(C, C) – D(0, 0)] < [(0.5)\pi L]$, which violates Assumptions 1 and 3 when Assumption 2 holds. Defining the threshold value for π_p where $[D(C, C) – D(0, 0)] = [(0.5)\pi_p L]$ and solving, then $\pi_p = \{(2[D(C, C) – D(0, 0)]) / L\} > \pi_c$.

Corollary 2 ***Pareto efficiency of*** **(0 , 0)**

If Assumptions 1 and 3 are replaced by the assumption $D(0, 0) > D(0, C) = D(C, C)$ when Assumption 2 holds, implying no indirect liquidity benefits from open BCLs, only then can (0, 0) be a Pareto-efficient Nash equilibrium.

Proof of Proposition 5 ***Asymmetric BCL adjustment***

With the adoption of Assumption 4 and with Definition 2, we need to show that (C_1^*, C_2) is a Nash equilibrium. To do so, we need to show the conditions individually for which Bank 1 will raise its BCL when Bank 2 selects to maintain its BCL and, then, show the conditions under which Bank 2 chooses to maintain its BCL when Bank 1 raises its.

Bank 1 conditions

Bank 1 will raise its BCL when Bank 2 chooses to hold its constant if $U_1(C_1^*, C_2) \geq U_1(C_1, C_2)$, which is strictly positive and implies

$$D_1[(1 + \delta)C_1, C_2] – \{[(1 + \delta)C_1 / ((1 + \delta)C_1 + C_2)] \cdot [1 + (\delta C_1/ L)] \cdot \pi L\} \geq D_1[C_1, C_2] - \{[(C_1 / (C_1 + C_2)] \cdot \pi L\},$$

which can be re-expressed as

$$D_1[(1 + \delta)C_1, C_2] - D_1[C_1, C_2] \geq \{[((1 + \delta)C_1 / ((1 + \delta)C_1 + C_2) \cdot (1+ (\delta C_1 / L)) - (C_1 / (C_1 + C_2)] \cdot \pi L\}.$$

We need to show that this condition holds for boundary conditions: $C_1^* \in [0\,,\,C_2]$.

For $C_1^* = 0 \Rightarrow C_1 = 0$
$D_1[(0\,,\,C_2\,] - D_1[0\,,\,C_2\,] = 0$ under Assumption 1, which indicates that the condition holds at the lower bound as a strict equality.

For $C_1^* = C_2$, the inequality solves to
$D_1[C_2\,,\,C_2\,] - D_1[C_1\,,\,C_2\,] \geq (\,\pi\,/\,2) \cdot \{[(1+\delta)\cdot L\,/\,(2+\delta)] + [\delta \cdot C_2\,/\,(1+\delta)]\}$ at the boundary by definition. The inequality will hold for a small enough L.

Bank 2 conditions

Bank 2 will maintain its existing BCL when Bank 1 raises its BCL with Bank 3 if
$U_2\,(C_1^*,\,C_2) \geq U_2\,(C_1^*\,,\,C_2^*)$ where $C_2^* = (1+\delta)C_2$, which implies under Assumption 1,
$D_2[(1+\delta)C_1\,,\,C_2\,] - [(1+\delta)\,D_2[C_1\,,\,C_2\,] \geq \{[(C_2\,/\,((1+\delta)C_1 + C_2) \cdot (1 + (\,\delta\,C_1\,/\,L))] -$
$[((C_2\,/\,(C_1 + C_2))\cdot(1+\delta)(C_1 + C_2)/\,L)]\} \cdot \pi\,L.$

For boundary conditions: $C_1^* \in [0\,,\,C_2^*]$:

At $C_1^* = 0 \Rightarrow C_1 = 0$,
the inequality above solves to: $D_2[0\,,\,C_2\,] \leq \{[\,(1+\delta)C_2\,/\,L\,] - 1\} \cdot [\,\pi\,L\,/\,\delta\,]$.

At $C_1^* = C_2^* = (1+\delta)C_2 \Rightarrow C_1 = C_2$, under Assumption 1 (linear homogeneity), the inequality solves to $D_2[\,C_2\,,\,C_2\,] \leq \{[2(1+\delta) \cdot \delta \cdot C_2)]\,/\,L\,\} \cdot [\,\pi\,L\,/\,(2+\delta\,) \cdot \delta\,]$.

The inequality holds at both boundaries for a large enough C_2.