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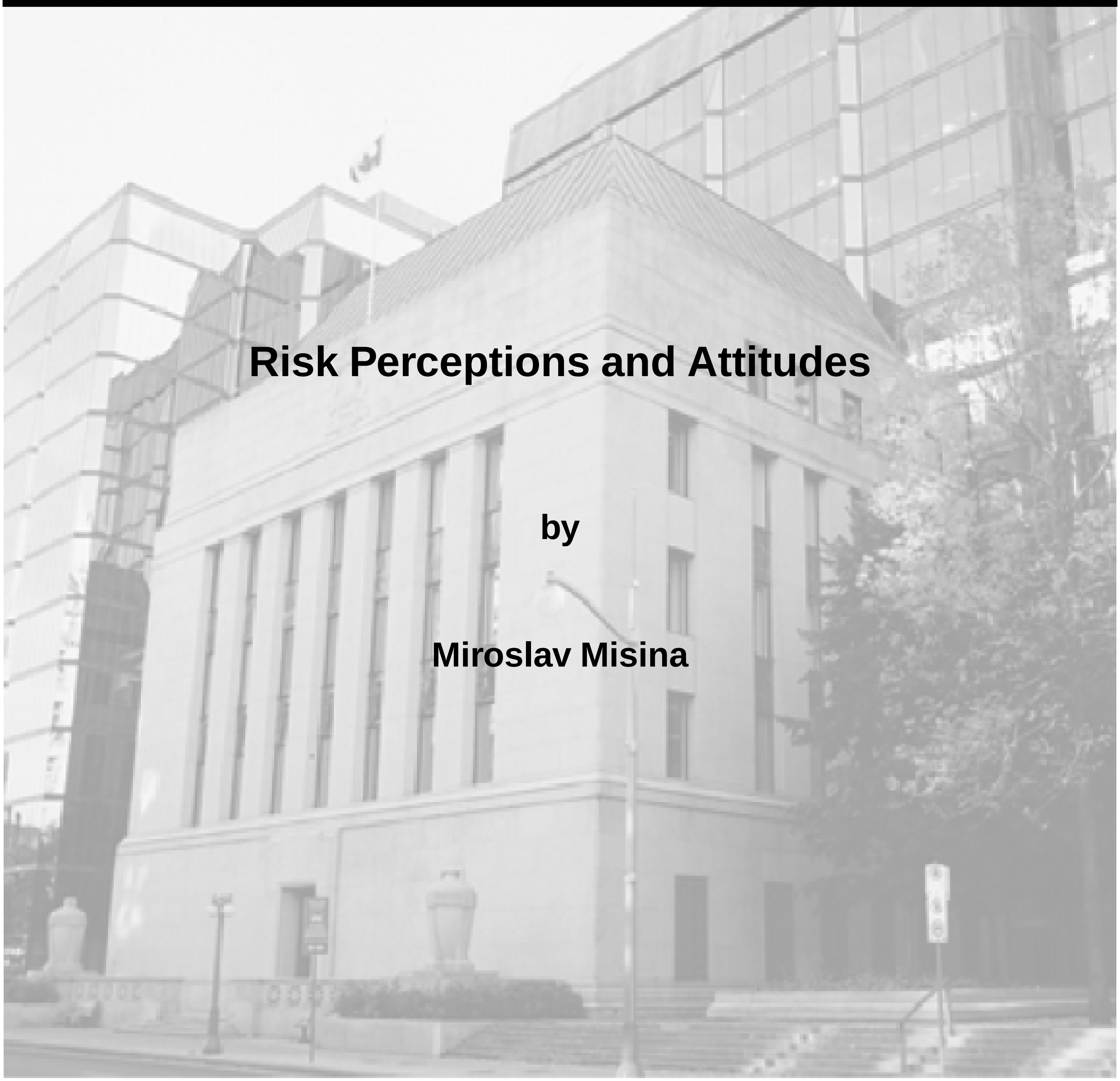

Risk Perceptions and Attitudes

by

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Abstract

Changes in risk perception have been used in various contexts to explain shorter-term developments in financial markets, as part of a mechanism that amplifies fluctuations in financial markets, as well as in accounts of “irrational exuberance.” This approach holds that changes in risk perception affect actions undertaken in risky situations, and create a discrepancy between the risk attitude *implied* by those actions and the a priori description of risk attitude as summarized by the Arrow-Pratt coefficients of risk aversion. The author characterizes this discrepancy by introducing the notion of risk perception within the expected utility theory, and proposes the concept of implied risk aversion as a summary measure of risk attitudes implied by agents’ actions. Properties of implied risk aversion are related to an individual’s future outlook. Key ideas are illustrated using an asset-pricing model.

JEL classification: D81, D84, G12
Bank classification: Economic models; Financial markets

Résumé

Les changements de la perception du risque ont servi à expliquer dans divers contextes l’évolution à court terme des marchés financiers, les phénomènes d’amplification des fluctuations sur ces marchés ainsi que les périodes d’« exubérance irrationnelle ». Selon les tenants de cette approche, les changements de perception influencent les actions entreprises en situation risquée, ce qui crée une divergence entre l’attitude face au risque dénotée par ces actions et celle mesurée a priori par les coefficients d’aversion pour le risque d’Arrow-Pratt. Pour tenir compte de cette divergence, l’auteur intègre la notion de perception du risque à la théorie de l’utilité espérée et propose de mesurer l’attitude qui transparaît implicitement dans les actions des agents au moyen d’un nouvel indicateur : l’aversion implicite pour le risque. Les propriétés de cet indicateur sont définies par les attentes de l’agent concernant l’avenir. Les idées maîtresses de l’étude sont illustrées à l’aide d’un modèle d’évaluation des actifs.

Classification JEL : D81, D84, G12
Classification de la Banque : Modèles économiques; Marchés financiers

1. Introduction

Changes in the future outlook and in risk attitudes may be among practitioners' more popular explanations of asset-price movements, but it would certainly be an overstatement to say that these have been enthusiastically embraced in academic circles. There are signs of change, although these two explanations have fared somewhat differently. Whereas changes in the future outlook, modelled as changes in expectations, seem to be increasingly accepted as an explanation of a wide range of phenomena, the status of changes in risk attitudes seems to be controversial.[1] Practitioners' regular appeals to changes in risk attitudes are, perhaps equally regularly, dismissed by academics.[2] The grounds for dismissal are methodological: changes in risk attitudes amount to relaxing the assumption of constant preferences, which is thought to safeguard rigour in research.[3] Furthermore, changes in expectations—through learning, for example—are thought to be consistent with individual rationality,[4] while changes in risk attitudes are not.

There seems to be little reason for either this asymmetric treatment or explanatory dichotomy. The explanatory dichotomy has been breached by the use of changes in in-

1 Recent examples of the use of changes in expectations as explanatory devices include Cecchetti, Lam, and Mark (2000), Danthine et al. (2003), Kurz (1997, chapter 11), and Melino and Yang (2003). Campbell, Lo, and MacKinlay (1997, chapter 8.4) provide a discussion and additional references. For links to the business cycle literature, see Beaudry and Portier (2004). Misina (2003) discusses some pitfalls associated with the use of these models.

2 Changing risk aversion is accepted in the context of habit persistence; see Campbell, Lo, and MacKinlay (1997, chapter 8.4). However, changes in risk attitude obtained via habit persistence are unlikely to help explain shorter-term fluctuations, since risk attitudes are related to consumption, a variable that is quite stable over short periods of time.

3 The key issue is clearly summarized by Arrow (1982): "A fundamental element of rationality . . . is, in logicians' language, that of *extensionality*. The chosen element depends on the opportunity set from which the choice is made, independently of how that set is described. . . . The cognitive psychologists deny that the choice is in fact extensional; the framing of the question affects the answer."

4 Here, individual rationality means axiomatic consistency of individual choices, rather than the question of how individuals form expectations.

dividual risk *perceptions* in a variety of contexts to explain shorter-term developments in financial markets, as part of a mechanism that amplifies fluctuations in financial markets, and in explanations of "irrational exuberance."[5] This approach holds that changes in risk perception affect actions undertaken in risky situations, and create a discrepancy between the risk attitude *implied* by those actions and the a priori description of risk attitude summarized by the Arrow-Pratt coefficients of risk aversion. The problem is to characterize this discrepancy between implied and a priori risk attitudes, and arrive at a notion of risk attitude implied by agents' actions.

The first step in this process is to specify what determines risk perceptions. The literature on behavioural foundations of choice under uncertainty identifies a number of factors that influence risk perceptions.[6] Among them, individual future outlook has been established as an important determinant.[7] In this way, explanations based on changes in risk perceptions relate an individual's future outlook to their views regarding risks.[8]

This paper seeks to introduce the notion of risk perception within the context of the expected utility theory, while addressing the methodological concerns expressed above.

[5] For example, "the decline in longer-term interest rates and diminished perceptions of credit risk in recent months have provided a substantial lift to the market value of nearly all major categories of household assets" (Greenspan 2003). For examples related to financial cycles and irrational exuberance, see Borio (2003) and Shiller (2000, 46), respectively.

[6] For example, see Slovic et al. (2002) and references cited therein.

[7] See Hirshleifer (2001, 1550–51). For evidence on the relationship between future outlook and probability assessments, see Wright and Bower (1992). These authors also provide a discussion and further references regarding the impact of mood on risk assessments. More generally, the issue in this paper is the status of the extensionality axiom. Considerable evidence, starting with the early identification of the framing effects, has been accumulated in support of the claim that the extensionality axiom may be violated. See Tversky and Koehler (1994) for further discussion.

[8] I do not investigate the sources of revisions of subjective assessments, although the literature on cognitive psychology identifies a number of patterns: optimistic bias has been well documented (Weinstein and Klein 2002; Armor and Taylor 2002; and Hirshleifer 2001, 1550–51).

The starting point of the work is the formulation

$$U = \sum_s \pi_s^i u_s(x),$$

where π_s^i denotes agent i's probability of state s, and x represents relevant outcomes. In this context it is, in principle, straightforward to obtain the necessary links within the constant risk-aversion class by specifying that the risk-aversion parameter, ρ, depends on the individual's expectations, $\boldsymbol{\pi}^i = (\pi_1^i, ..., \pi_S^i)$; i.e., $\rho = \rho(\boldsymbol{\pi}^i)$. Assuming that there are σ "expectations states" (i.e., $\boldsymbol{\pi}^i \in \{\boldsymbol{\pi}_1^i, ..., \boldsymbol{\pi}_\sigma^i\}$) in this formulation, state-dependence is modelled through variation of the risk-aversion parameter across these states.

This formulation, while incorporating links between expectations and risk attitudes, fails to meet the methodological objections raised above: preferences are formally not constant, but state-dependent. To deal with this objection, the primitive assumption in the analysis is that the subutility function is state-independent, and belongs to the constant risk-aversion class:

$$u_j(\cdot) = u_k(\cdot), \ \forall j, k.$$

This assumption will, within the class of constant risk-aversion utility functions, guarantee that individual risk attitudes are represented by an exogenously specified risk-aversion parameter. Changes in agent i's expectations are represented by changes in their probability distribution over future states, $(\pi_1^i, ..., \pi_S^i)$. The difficulty is that, under the assumption of constant subutility, establishing links between the future outlook and individual risk attitudes seems precluded.

To resolve this problem, I introduce the notion of *implied risk aversion*, and its corresponding coefficient. This coefficient captures the risk attitude implied by individual actions. Arrow-Pratt coefficients represent the a priori description of risk attitudes. The discrepancy between the implied risk aversion and constant Arrow-Pratt risk-aversion coefficients will be due to changing risk perceptions.

Moreover, it is possible to characterize the behaviour of implied risk aversion in qualitative terms as a function of future outlook. It will be shown that upward revisions of probabilities of good states are associated with lower implied risk aversion, and upward revisions of probabilities of bad states with increased implied risk aversion. If upward revisions of probabilities of good states can be interpreted as indicators of optimism, and downward revisions as indicators of pessimism, then this framework provides a description of the interaction between individual disposition towards the future (optimism, pessimism), individual actions, and risk attitudes implied by those actions.[9] For example, optimism about the future will induce individuals to undertake actions that would not have been undertaken for a given value of the Arrow-Pratt coefficient. The attitude towards risk implied by their actions will be captured by the implied risk-aversion coefficient. In this way, one can capture the anecdotal evidence of investors who claim that their risk attitudes have not changed but behave as if they have.[10]

The paper is organized as follows. In section 2, the key terms are defined and the main result is provided relating revisions of probabilities to risk attitudes. In section 3, the results obtained are interpreted in terms of the relationship between investors' disposition towards the future and risk perceptions. Section 4 illustrates the concepts

[9] Optimistic bias is defined as an upward bias in the assignment of probabilities of good states. The bias is established relative to a benchmark. This issue is discussed further in section 3. See Armor and Taylor (2002).

[10] Stories like this seem particularly frequent in periods of prolonged market upturns, as in the second half of the 1990s. Shiller (2000) provides detailed evidence.

by means of an example, using a standard asset-pricing model. Section 5 offers some conclusions.

2. Probability Beliefs and Risk Aversion

Consider the following setting:

- two states of nature: 1 is good; 2 is bad
- objective probabilities[11] defined over these states: π_1, π_2
- individual i with utility function

$$U^i(c) = \pi_1^i u(x_1) + \pi_2^i u(x_2),$$

where π_s^i is the subjective probability belief about state s.

Individuals can revise their beliefs about the likelihood of good and bad states in both directions: upward revisions of the good state, $\pi_1^i \geq \pi_1$, indicate that individuals believe that the good state is more likely than suggested by an objectively given measure. Upward revisions of the bad state, $\pi_2^i > \pi_2$, indicate that the bad state is considered more likely than is objectively warranted. The problem is to establish the link between these revisions and risk attitudes, and thus arrive at a formal expression of risk perception.

Individual risk aversion is specified by the subutility function, $u(\cdot)$. It is assumed that $u(\cdot)$ is state-independent, which, in this context, means that the exogenously specified coefficient of risk aversion does not vary across states. It will be demonstrated that revisions in individual beliefs have two effects: the effect on expected payoffs, and the

[11] I am not, at the moment, interested in the origin of these probabilities.

effect on risk attitudes via changes in risk perceptions. The latter effect is not captured by the risk-aversion coefficient, which is assumed to be constant.

A formal statement is given below. The key result is contained in Proposition 2.1. To get there, several preliminary steps are needed, which are summarized in Lemmas 2.1 and 2.2, and Definition 2.1.

Let S denote the (finite-dimensional) set of possible states of the world, and let $\boldsymbol{\pi} = [\pi(s)]$ denote a probability distribution defined on S. Each state has a payoff associated with it. Let $x = \boldsymbol{\pi} \cdot \mathbf{x}$ represent the expected payoff, where $\mathbf{x} = [x(s)]$. X is the space of expected payoffs.

Define S_γ and S_β so that $S = S_\gamma \cup S_\beta$, $S_\gamma \cap S_\beta = \emptyset$, with

$$s \in S_\gamma \text{ iff } x(s \in S_\gamma) > x(s \notin S_\gamma).$$

In words, S_γ is the set of payoff-dominant states with the associated probability $\pi(s \in S_\gamma) \equiv \sum_{s \in S_\gamma} \pi(s)$. The probability associated with S_β is $\pi(s \in S_\beta) \equiv \sum_{s \in S_\beta} \pi(s)$.

Lemma 2.1 *For any $x \in X$,*

$$\frac{\partial x}{\partial \pi(s \in S_\beta)} < 0,$$

$$\frac{\partial x}{\partial \pi(s \in S_\gamma)} > 0.$$

The proof is provided in the appendix.

Lemma 2.1 establishes the relationship between expected payoffs and revisions in individual assessments of good and bad states, respectively. Under the usual assumptions on the subutility function, a change in expected payoffs brought about by a change in these assessments will result in a change in the level of utility of the expected payoff.

An upward revision of the probability of the good state will lead to an increase in the expected payoff and to an increase in the level of utility of the expected payoff. To compare the initial and the final states, it is necessary to bring the individual back to the original level of utility. This can be achieved by notionally changing the risk-aversion parameter. The change in the risk-aversion parameter that is necessary to bring the individual to the original level of utility, after a revision in probabilities, is called the *equivalent variation*, denoted EV_ρ.

Definition 2.1 *Let $u = \bar{u}$ denote the level of utility associated with the expected payoff prior to a revision of probabilities and let x' denote the expected payoff after the revision of probabilities. Then,*

$$EV_\rho \equiv \left.\frac{\partial \rho}{\partial \pi \left(s \in S_s\right)}\right|_{u=\bar{u},x=x'},\ s = \beta, \gamma.$$

Using this definition, the implied risk aversion, ρ^ι, can be defined as the value of the risk-aversion parameter that, after a change in the expected payoff, results in the original level of utility:

$$\rho^\iota \equiv \rho + EV_\rho.$$

The concepts of equivalent variation and implied risk aversion establish the link between revisions in the probability assessments and risk attitudes. The problem is to characterize this relationship in qualitative terms. To accomplish this, one more result, given in the following lemma, is needed.

Lemma 2.2 *Let $u\left(\cdot\right)$ denote a differentiable utility function of the constant risk-aversion type. Then, for any utility function in this class,*

$$\frac{\partial u}{\partial \rho} > 0.$$

The proof is provided in the appendix.

This lemma characterizes the relationship between changes in risk aversion and the level of utility. The result holds for the class of constant risk-aversion utility functions (both constant absolute risk aversion and constant relative risk aversion).

These results enable me to characterize the relationship between revisions in probability assessments and risk attitudes. This is the content of the following proposition.

Proposition 2.1 *Let $x \in X$, and let $\bar{u} = u(x;\rho)$. Then,*

$$\frac{\partial x}{\partial \pi(s \in S_\beta)} < 0 \Rightarrow \left.\frac{\partial \rho}{\partial \pi(s \in S_\beta)}\right|_{u=\bar{u},x=x'} > 0,$$

$$\frac{\partial x}{\partial \pi(s \in S_\gamma)} > 0 \Rightarrow \left.\frac{\partial \rho}{\partial \pi(s \in S_\gamma)}\right|_{u=\bar{u},x=x'} < 0.$$

The proof is provided in the appendix.

This proposition characterizes the relationship between changes in payoffs due to revisions in probability assessments and equivalent variation. The relationship between the two is inverse: an increase in the probability of a bad state will result in an increase in EV_ρ. Similarly, an increase in the probability of a good state will result in a decrease in EV_ρ.

The effect of revisions in probabilities on implied risk aversion follows directly from this result, in conjunction with the definition of implied risk aversion.

Corollary 2.1

$$\frac{\partial x}{\partial \pi(s \in S_\gamma)} > 0 \Rightarrow \frac{\partial \rho^\iota}{\partial \pi(s \in S_\gamma)} < 0.$$

$$\frac{\partial x}{\partial \pi(s \in S_\beta)} < 0 \Rightarrow \frac{\partial \rho^\iota}{\partial \pi(s \in S_\beta)} > 0.$$

The proof follows directly from the main proposition and the definition of implied risk aversion.

The corollary establishes that

- upward revisions of the probability of a good state lead to lower implied risk aversion, and
- upward revisions of the probability of a bad state lead to higher implied risk aversion.

The concept of implied risk aversion allows for the possibility that risk attitudes implied by individual actions when their future outlook changes diverge from the exogenously specified risk attitudes, summarized by the Arrow-Pratt measure. This discrepancy is due to changes in risk perceptions, captured by the EV_ρ term. This is accomplished without formally relaxing the assumption of constant preferences, understood as the state-independent subutility function, $u(\cdot)$.[12]

3. Disposition towards the Future and Risk Perception

The results of section 2 can be given a more precise interpretation by examining more closely the revisions of individual assessments discussed there. The objective is not to relate revisions to a particular underlying cause, but to suggest that the revisions investigated above are consistent with the notions of optimism and pessimism. Optimism is a state in which an individual assigns a greater probability to the good state than the one implied by some objective measure. The key feature of optimism is that an individual overestimates the probability of a good state. Pessimism manifests itself in overestimat-

[12] The assumption of extensionality of choices is implicitly relaxed here. Alternative descriptions of the same event are assumed to influence individual probability assessments and thus lead to different judgments. See Tversky and Koehler (1994, 548).

ing the probability of a bad state. States of optimism and pessimism are jointly referred to as a "disposition towards the future."

To make these notions operational, one needs to define the term "normal state." This can be done in two ways:

- by specifying a benchmark, or
- by comparison with a previous state.

It is important to distinguish between these two methods. By defining disposition towards the future relative to a benchmark, it is possible to focus on trends in disposition changes. Definitions relating the current state to a previous state capture small variations in disposition, but may obstruct the identification of trends. In the following discussion, the definitions will be established relative to a benchmark.[13]

Individual i is said to be *optimistic* if

$$\pi^i \left(s \in S_\gamma \right) > \pi \left(s \in S_\gamma \right),$$

where $\pi \left(s \in S_\gamma \right)$ is the probability of a good state associated with some benchmark. In models that rely on the assumption of rational expectations, the benchmark is represented by the equilibrium process. In practical applications, the benchmark can be taken as the relative frequency of visiting a particular state obtained from past data.[14]

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13 Whereas the construction of the benchmark is outside the scope of this paper, a natural candidate is the stationary measure based on the restrictions on individual beliefs. See Kurz (1997, Introduction) for an exposition of the basic ideas.

14 This allows for the possibility of extreme events. One could think of a bad event, the frequency of which in past data is zero. An individual would be said to display pessimism if they assign a positive probability to that event.

Individual i is said to be *pessimistic* if

$$\pi^i\left(s \in S_\beta\right) > \pi\left(s \in S_\beta\right).$$

In words, individuals are said to display optimism (pessimism) if they overweigh the probability of good (bad) states.

These definitions, in conjunction with the results of section 2, enable me to establish precise links between a disposition towards the future and risk perceptions. Changes in disposition towards the future affect agents' actions by affecting the way they perceive risks. Optimism implies lower risk perception, whereas pessimism implies that a given situation will be perceived as riskier than before.

The implications for individual behaviour are immediate. In situations where individuals are optimistic, they undertake actions that they would not usually undertake. They tend to downplay the risks associated with particular types of assets, and do not demand the risk premium that they usually would. This opens the possibility of bidding up the prices of assets. Similarly, when individuals are pessimistic, they demand higher risk premiums. They perceive most assets as riskier than usual and may decide to withhold their investment funds.

In the above framework, changes in risk attitudes due to changes in disposition towards the future are captured by the *implied* risk-aversion coefficient. Changes in risk perceptions will have an impact on individual actions, even when the risk-aversion coefficient in the utility function is unchanged. It follows that risk perceptions may have an important role to play in explaining individual behaviour in dynamic settings.

The above analysis does not imply that changes in risk perceptions will necessarily lead to "irrational exuberance" or similar events. I do not analyze the factors leading to exuberance.[15] My analysis demonstrates that these phenomena *can* be captured within the standard framework, and that the role of risk perceptions may have been underemphasized.

4. Risk Perceptions and Asset Prices

This section illustrates the relationship between changes in disposition towards the future, risk perceptions, and price behaviour.

4.1 Model

A standard consumption-based asset-pricing model is used, with a representative agent in an exchange economy that has a single consumption good. I focus on analyzing the behaviour of a risky asset. The utility function is a constant relative risk-aversion type, given by

$$u(c) = \frac{c^{1-\gamma}}{1-\gamma}.$$

The only source of uncertainty is the time-varying nature of the risky asset's payoffs. It is assumed that there are two possible states of the world ($s_t = \{1, 0\}$, good and bad), and that d^h and d^l are dividend payments of this asset associated with each state, respectively. Dividends at each date are selected according to the following rule:

$$d_t = \begin{cases} d^h & \text{if } s_t = 1 \\ d^l & \text{if } s_t = 0 \end{cases}.$$

The future state is drawn so that $\Pr(s_t = 1) = \Pr(s_t = 0) = 0.5$. One can interpret these as the unconditional probabilities of the two states. The associated transition ma-

[15] Shiller (2000) analyzes factors that create environments in which people are susceptible to mass exuberance.

trix is

$$T = \begin{bmatrix} 0.5 & 0.5 \\ 0.5 & 0.5 \end{bmatrix}. \tag{1}$$

To isolate the effect of changes in the future outlook on asset prices, it is assumed that consumption grows at a known constant but positive rate, g_c. With these assumptions, the first-order conditions result in the following expression for the stock price in each state:

$$p_i^s = \beta \sum_j \pi_{ij} \left(g_c\right)^{-\gamma} \left(p_j^s + d_j\right). \tag{2}$$

4.1.1 Agents' disposition towards the future

Agents form beliefs about future dividend payments and this affects their current demand for equity. For simplicity, it is assumed that agents know the unconditional state probabilities. They have at their disposal past data and are trying to infer something about the future. At each point in time, they decide whether to revise their future outlook upwards or downwards, or to leave it unchanged. The decision taken can be related to some well-documented attitudes in the literature on psychology, such as overreaction and conservatism.[16] Revisions in any direction will have an impact on asset prices.

4.1.2 Computation of expected returns

The expected returns at any point in time are computed using the expression:

$$R\left(s_{t+1} = i\right) = \frac{p\left(s_{t+1} = i\right) + d\left(s_{t+1} = i\right)}{p\left(s_t = i\right)},$$

[16] Revisions of the future outlook could be due to new information, or to a different interpretation of existing information. While the underlying motives are taken as exogenous here, the mechanism is general enough to accommodate either of the causes.

for expected returns if the future state is unchanged, and

$$R\left(s_{t+1}=j\right)=\frac{p\left(s_{t+1}=j\right)+d\left(s_{t+1}=j\right)}{p\left(s_{t}=i\right)},$$

if the future state changes. The expected returns at time t are

$$R_{t+1}^{ex}=\pi\left(s_{t+1}=i|s_{t}=i\right)R\left(s_{t+1}=i\right)+\pi\left(s_{t+1}=j|s_{t}=i\right)R\left(s_{t+1}=j\right).$$

Equivalent formulas apply when $s_t = j$.

4.1.3 Computation of implied risk aversion

The benchmark value of the risk-aversion parameter is taken to be an exogenously given value, which is assumed to be related to the risk attitude when states are generated according to (1). The associated level of utility is the benchmark level of utility, $\bar{u}$. Revisions of the future outlook result in changes in expected payoffs. For the new value of expected payoffs, the implied risk aversion is the value of the risk-aversion parameter that would yield the original level of utility.

4.1.4 Parameter values

The purpose is not to match particular properties of the data, but to investigate the impact of changes in the future outlook on asset prices. The process generating the states is assumed to be (1). Other parameter values are as follows:

$$\rho=3,\ \beta=0.95,\ \left(d_{1},d_{2}\right)=\left(2,0\right),\ g_{c}=1.01.$$

4.2 Results

Consider a case in which the agent has recently observed a realization of the following states: $\{1, 1, 1\}$. The question is whether this will lead the agent to revise their future outlook. There are three possibilities, corresponding to different investment styles.

Case 1: No revisions of the agent's future outlook

The agent does not attach any significance to the fact that the last three periods were good states. The agent believes that the process generating states is a coin toss with equal probabilities, and that this process does not preclude the possibility of a sequence of three good states. Indeed, the probability of getting three good states in a row under the postulated process is 12.5 per cent. As a consequence of this reasoning, the agent does not revise their future outlook. This reasoning would correspond to a conservative strategy, which discounts the most recent realizations. The price in period 4 remains the same. The implied risk aversion is unchanged.

Case 2: Upward revisions

The agent believes that a sequence of three good states is a sign of a possible regime change. The agent also believes that the probabilities assigned by the postulated process are too low, and revises the probability of a future state upwards. The agent is more confident than warranted by past data that the good state will be realized tomorrow, and displays optimism. The implied risk aversion decreases, even though the Arrow-Pratt measure of risk aversion remains the same. The investment outlook is perceived as less risky, and the agent invests, putting upward pressure on the price.

Figure 1: Upward Revisions

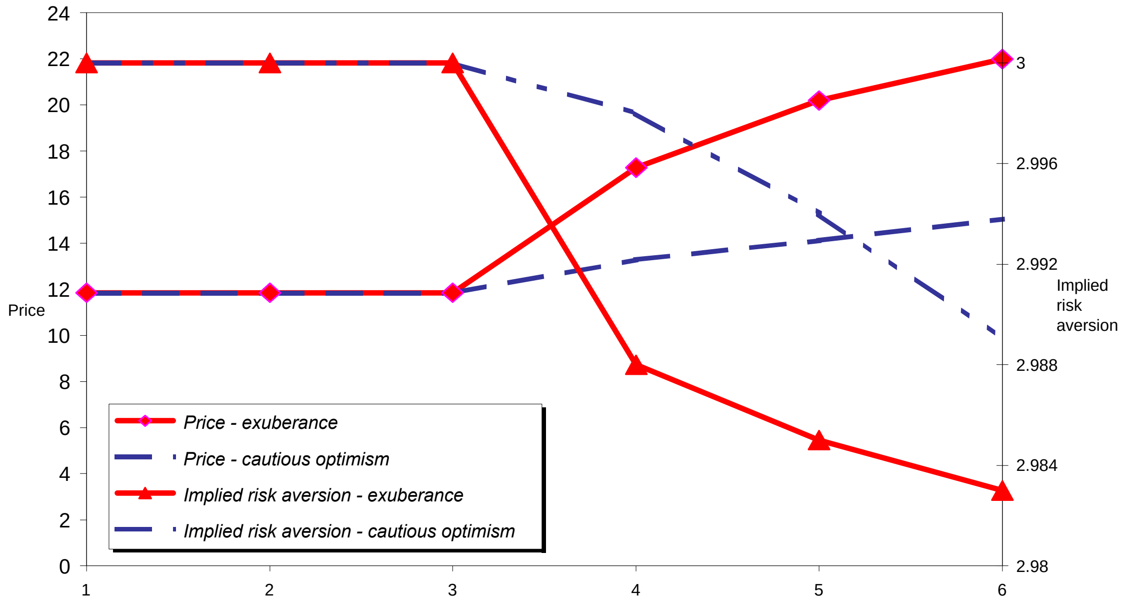

Figure 1 illustrates two cases, termed cautious optimism and exuberance. In the case of cautious optimism, the agent revises their future outlook upwards, but in a gradual fashion. The probability of a future good state is revised from $\pi_{11} = 0.5$ in period 3 to $\pi_{11} = 0.61$ in period 4. The unconditional probability assigned to the good state increases from 0.5 in period 3 to 0.63 in period 6, as implied risk aversion decreases. This leads to a total increase in price of approximately 27 per cent.

In the case of exuberance, the initial revision is from $\pi_{11} = 0.5$ in period 3 to $\pi_{11} = 0.81$ in period 4. The unconditional probability of the good state is revised from 0.5 in period 3 to 0.92 in period 6. Moreover, the agent considers that the continuation of the good state is virtually certain and sets $\pi_{11} = 0.96$ in period 6. This leads to a dramatic increase in prices of approximately 86 per cent. Implied risk aversion declines more than in the case of cautious optimism.

Case 3: Downward revisions

The agent believes that it is highly unlikely that the good times will continue. They consider the three consecutive good states to be "too good to be true" and consequently revise their future outlook downwards. The agent displays pessimism relative to the postulated process. As a consequence, the investments are perceived as more risky than before and the agent will decide to sell, or demand a higher expected return, to be compensated for the higher perceived risk. The agent's implied risk aversion increases.

Figure 2: Downward Revisions

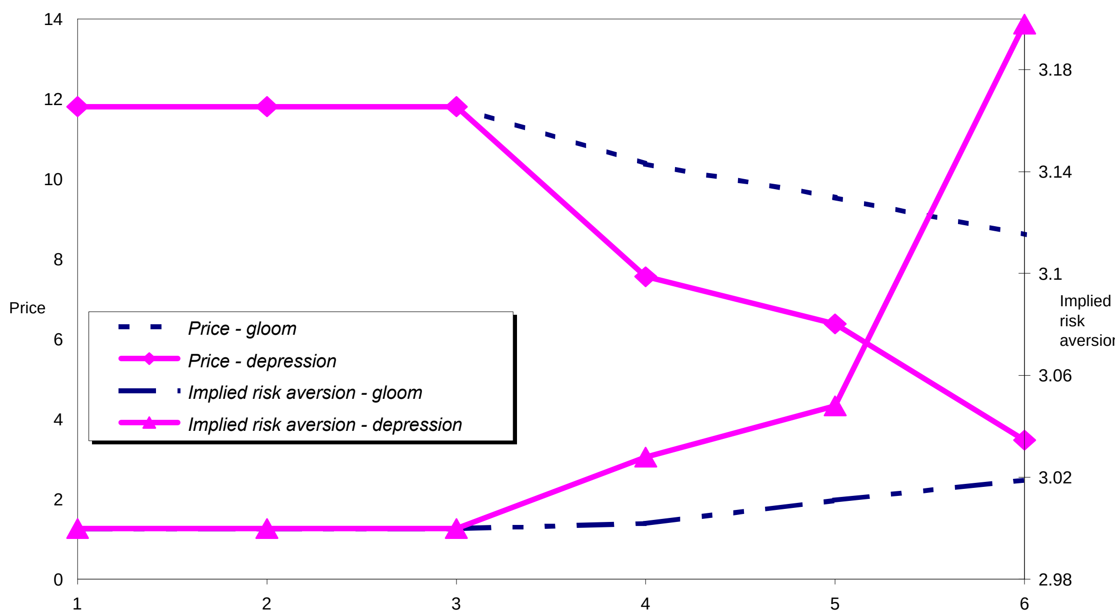

Figure 2 illustrates two cases, termed gloom and depression. In each case, the agent believes that the present state is unlikely to continue, and this is reflected in downward revisions of the probability of the good state. In the case of gloom, the unconditional probability of a bad state increases from 0.5 in period 3 to 0.63 in period 6, whereas the probability of staying in a bad state once there increases from 0.5 to 0.71. This leads to a decline in price of approximately 37 per cent as the implied risk aversion increases.

In the case of depression, the results are more dramatic: the unconditional probability of the bad state is set to 0.84 in period 6, and the probability of remaining in the bad state is 0.91. This results in a drop in price of approximately 70 per cent as the implied risk aversion increases.

4.3 Comments

The foregoing examples illustrate the links between changes in future outlook and asset-price movements. A great variety of patterns can be produced in this way. Persistent optimism can create upward movements in asset prices, whereas persistent pessimism will create downward movements. This effect is well known as bull and bear markets, but it is important to emphasize that the above results are obtained through changes in risk perceptions. The same series of events can be perceived differently at different points in time, and this gives rise to changing attitudes towards risk. This effect is not captured by either the constant risk-aversion class of utility functions or the class of utility functions that relies on habit persistence.

5. Conclusions

In this work, I have provided a formal description of changes in risk perceptions, which bridges the gap between explanations based on changes in expectations and those that are based on changes in risk attitudes. The concept of equivalent variation was introduced as a representation of risk perception, and the notion of implied risk aversion was introduced to capture the discrepancy between an exogenosuly specified Arrow-Pratt measure of risk attitudes and the risk attitudes implied by agents' actions when their risk perceptions change.

Although there are similarities between this approach and the approach that relies on a state-dependent risk-aversion parameter, I believe that the approach proposed in this paper has some advantages over the former: state-dependency does not in itself offer any explanation of reasons for a change in investors' risk attitudes. Under the proposed approach, there is a direct link between revisions of subjective probability assessments and risk perceptions. While this link can be interpreted in terms of disposition towards the future, and in terms of risk perceptions, this interpretation is not exhaustive. Indeed, one could interpret revisions in probability assessments as a way of capturing violations of the extensionality axiom, regardless of the underlying cause. The interpretation proposed in this paper relies on the observation that optimistic individuals tend to downplay risks, whereas pessimism often leads to extreme caution and overweighing of risks. While there is some empirical support for this type of interpretation, the results do not depend on it.

Furthermore, the approach that relies on state-dependent risk aversion does not allow for the discrepancy between a priori specified risk attitudes and the risk attitudes implied by agents' actions. As such, that approach does not allow for a distinction between risk perceptions and risk attitudes, which is the key aspect of the framework proposed in this paper.

In my analysis, no assumptions were made regarding the reasons for revisions of individual probability assessments. Several possibilities come to mind, all of which can be accommodated within the proposed framework:

– the absence of an objective standard for determining the impact of current news on future prospects,

- framing effects, or some other underlying cognitive biases, and
- learning limitations. Agents do not believe that what they learn from past data is the correct statistical data-generating process. Among the possible reasons for this are structural uncertainty, or cognitive biases.

Regardless of the underlying causes, the example provided in section 4 suggests that the impact on asset prices depends on the extent of these revisions. The example does not specify the mechanics of revisions, but one could use Bayesian updating at least as a benchmark and then investigate the implications for asset prices.[17] Alternatively, rather than specifying the revision mechanism, one could impose a priori restrictions on the extent of admissible revisions, given the past history of the data, as in Kurz (1997). Whereas different learning mechanisms may have different quantitative implications for the asset-price movements, the basic relationship between disposition towards the future, risk perceptions, and asset-price movements analyzed in this work remains intact.

[17] This learning procedure might be of limited use, however, in environments where complete learning is precluded.

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Appendix

A.1 Proof of Lemma 2.1:

Let $\boldsymbol{\pi} \equiv [\pi(s \in S_\beta), \pi(s \in S_\gamma)]$, and $\mathbf{x} \equiv [\mathbf{x}(s \in S_\beta), \mathbf{x}(s \in S_\gamma)]$, so that

$$x = \begin{bmatrix} x(s=\beta) & x(s=\gamma) \end{bmatrix},$$

where

$$x(s \in S_\beta) = \sum_{s \in S_\beta} \pi(s)\, x(s),$$

and

$$x(s \in S_\gamma) = \sum_{s \in S_\gamma} \pi(s)\, x(s).$$

Then,

$$\begin{aligned} \frac{\partial x}{\partial \pi(s \in S_\beta)} &= x(s \in S_\beta) - x(s \in S_\gamma) \\ &< 0. \end{aligned}$$

The proof of the second part follows from the above.

A.2 Proof of Lemma 2.2:

Case 1: Constant relative risk aversion

From the definition of the Arrow-Pratt measure of relative risk aversion,

$$R_R = -\frac{xu''}{u'},$$

it follows that, in the constant relative risk-aversion case,

$$-\frac{xu''}{u'} = \rho.$$

From here,

$$xu'' + \rho u' = 0.$$

This is a second-order Euler differential equation. The solution guess takes the form $u = x^r$. From here,

$$xr\left(r-1\right)x^{r-2} + \rho r x^{r-1} = 0,$$

$$x^{r-1}\left[r\left(r-1\right) + \rho r\right] = 0,$$

which implies that

$$r^2 - \left(1-\rho\right)r = 0,$$

so that

$$r_1 = 0,\ r_2 = \left(1-\rho\right).$$

The solution then takes the form

$$u = c_1 x^{1-\rho} + c_2.$$

Any constant relative risk-aversion function will take this form. From here,

$$\frac{\partial u}{\partial \rho} = c_1 x^{1-\rho} \ln x.$$

It follows that

$$\frac{\partial u}{\partial \rho} > 0 \text{ if } \ln x > 0 \ \Rightarrow x > 1.$$

If the constant $c_2 = -1$, $x > 1$ guarantees a well-behaved utility function. Other properties of utility functions are used to restrict parameter values for c_1.

Case 2: Constant absolute risk aversion

Starting from the definition of the Arrow-Pratt measure of absolute risk aversion $R_A = -\frac{u''}{u'}$, one gets, in the case of constant absolute risk-aversion utility function,

$$R_A = -\frac{u''}{u'} = \rho,$$

i.e.,

$$u'' + \rho u' = 0. \tag{3}$$

The solution to this differential equation is standard. The solution guess takes the form $u = e^{rx}$. Substituting the appropriate derivatives of the guess into (3) yields

$$e^{rx}\left(r^2 + \rho r\right) = 0,$$

which implies that

$$r_1 = 0, \ r_2 = -\rho.$$

The general solution thus takes the form

$$u = c_1 e^{-\rho x} + c_2.$$

From here,

$$\frac{\partial u}{\partial \rho} = c_1 \left(-\rho\right) e^{-\rho x}.$$

Then,

$$\frac{\partial u}{\partial \rho} > 0 \text{ if } c_1 < 0.$$

Condition $c_1 < 0$ is needed in order for the utility function to be well-behaved.

A.3 Proof of Proposition 2.1:

From $u \equiv u\left(x\left(\pi\right);\rho\left(\pi\right)\right)$, we have

$$\frac{du}{d\pi\left(s \in S_s\right)} = \frac{\partial u}{\partial x}\frac{\partial x}{\partial \pi\left(s \in S_s\right)} + \left.\frac{\partial u}{\partial \rho}\frac{\partial \rho}{\partial \pi\left(s \in S_s\right)}\right|_{u=\bar{u}}, \; s = \beta, \gamma.$$

$u = \bar{u}$ (by assumption) implies that $\frac{du}{d\pi} = 0$ and

$$\frac{\partial u}{\partial x}\frac{\partial x}{\partial \pi\left(s \in S_s\right)} = -\left.\frac{\partial u}{\partial \rho}\frac{\partial \rho}{\partial \pi\left(s \in S_s\right)}\right|_{u=\bar{u}}, \; s = \beta, \gamma.$$

Given that $\frac{\partial u}{\partial x} > 0$ (positive marginal utility) and $\frac{\partial u}{\partial \rho} > 0$ (Lemma 2.2), it follows that

$$\frac{\partial \rho}{\partial \pi\left(s \in S_s\right)} \; > 0 \text{ when } s = \beta,$$

since, by Lemma 2.1, $\frac{\partial x}{\partial \boldsymbol{\pi}\left(s \in S_\beta\right)} < 0.$ Similarly,

$$\frac{\partial \rho}{\partial \pi\left(s \in S_s\right)} \; < 0 \text{ when } s = \gamma,$$

since, by Lemma 2.1, $\frac{\partial x}{\partial \boldsymbol{\pi}\left(s \in S_\gamma\right)} > 0.$

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