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A Modified P*-Model of Inflation Based on M1

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Abstract

This paper examines the performance of M1 in an indicator model of inflation over time horizons as long as 16 quarters into the future. The central conclusion of the paper is that, in addition to the output gap, the cumulative growth of M1 and the deviations of M1 from its long-run path provide “distant-early-warning” information about the future path of inflation.

Résumé

L’auteur cherche à évaluer le rôle joué par l’agrégat M1 dans un modèle indicateur servant à la prévision du taux d’inflation sur un horizon pouvant aller jusqu’à seize trimestres. Sa principale conclusion est que, à l’instar de l’écart de production, la croissance cumulative de M1 et les écarts de cet agrégat par rapport à son sentier de long terme renferment des éléments d’information sur l’évolution de l’inflation à long terme.

CONTENTS

1.0 Introduction and Summary

Research undertaken at the Bank of Canada, notably the work by Muller (1990), finds that, for one- to two-quarter horizons, broader monetary aggregates such as M2, M2+ and M2+ adjusted for substitution into long-term mutual funds are the best leading indicator of inflation and that M1 is the best monetary indicator of output. Recently, other work at the Bank has found that deviations of M1 from its long-run path (the M1 gap) provide information about the future path of inflation.[1]

The purpose of this paper is to further examine the role of M1 in an inflation-indicator model. The model used in the paper, which is based on the P^* model of inflation proposed by Hallman, Porter and Small (1989), suggests that the main factors that influence inflation are past values of inflation, past cumulative growth of M1, the M1 gap (which is derived from Hendry's (1995) long-run money-demand function) and an output gap generated as the deviation of real GDP from its linear-quadratic trend.

The main lessons to be drawn from the empirical exercises conducted in this paper are as follows:

- M1 provides leading information for inflation from six to sixteen quarters.
- The deviation of M1 from its long-run path leads inflation from one to sixteen quarters.
- The output gap is generally found to lead inflation at one- to twelve-quarter horizons.
- The term-structure spread, which is defined as the 10-year and over government bond yield minus the 90-day commercial paper rate, generally leads inflation at eight- to sixteen-quarter horizons.
- Lagged inflation provides leading information for inflation for a period up to four quarters.

1. See Section 5 of Atta-Mensah (1995).

The remainder of the paper is organized as follows. Section 2 presents a brief discussion of the model used in the paper. Section 3 presents the empirical results. Section 4 presents concluding remarks.

2.0 A Review of Inflation Influences

This section presents a brief background of an inflation-indicator model. This model, which is known as the P^* model of inflation, suggests that, besides the output gap, the deviation of money from its long-run path helps to explain the future path of inflation. Before presenting the model used for the empirical analyses, a brief discussion of the link between inflation and the current and expected future path of money supply is presented.

2.1 Inflation and Money

Inflation is generally perceived, in the long run, as a monetary phenomenon. However, it not very clear if money is linked to inflation in the short run. The evidence in this paper suggests that, in both the short run and long run, the link between money and inflation is so strong that it should not be overlooked. Before we examine the empirical evidence, a brief discussion of the link between inflation and current and expected future path of money will be presented.

As is well known, the demand for money is a function of income and the nominal interest rate. Since the nominal interest rate is linearly related to expected inflation, the presence of the nominal interest rate in the money-demand function provides a channel through which the money supply affects inflation. For instance, let the money-demand function be of the form:

$$m_t - p_t = \varphi_0 + \varphi_1 y_t + \varphi_2 i_t + \varepsilon_t, \tag{1}$$

where m_t, p_t, y_t and i_t are respectively money, the price level, real income and the nominal interest rate (opportunity cost of holding money). Also, $\varphi_1 > 0$ and $\varphi_2 < 0$. Let us assume that the economy is always at its potential, the real interest rate is constant and there are no money-demand shocks. Then, the money-demand function reduces to:

$$m_t - p_t = \varphi_0 + \varphi_1 y^* + \varphi_2 \bar{r} + \varphi_2 (p^e_{t+1} - p_t) \,, \tag{2}$$

where y^* is potential output, which is assumed to be a constant, $\bar{r}$ is the real interest rate and $(p^e_{t+1} - p_t)$ is expected inflation. Consider equation (2) as an equilibrium relation and rearrange to yield:

$$p_t = -\left(\frac{\varphi_0 + \varphi_1 y^* + \varphi_2 \bar{r}}{1 - \varphi_2}\right) + \frac{1}{1 - \varphi_2} m_t - \frac{\varphi_2}{1 - \varphi_2} p^e_{t+1} \,. \tag{3}$$

Equation (3) states that the current price level is the sum of a constant and a weighted average of the current money supply and the expected price level prevailing in the next period. It can be shown, through recursive substitution of the expected price level, that the current price level is a weighted average of the current and expected future money supplies:

$$p_t = -(\varphi_0 + \varphi_1 y^* + \varphi_2 \bar{r}) + \frac{1}{1 - \varphi_2} m_t + \frac{1}{1 - \varphi_2} \sum_{i=1}^{\infty} \left(\frac{-\varphi_2}{1 - \varphi_2}\right)^i m^e_{t+i} \,. \tag{4}$$

According to equation (4), a monetary policy action that causes the public to rationally expect that the future money supply, (m^e_{t+i}), will be raised will lead to higher inflation.[2]

2.2 The P* Model of Inflation

The P* model of inflation, which was proposed by Hallman, Porter and Small (1989; 1991) (herinafter HPS), is based on the quantity theory of money. In the model, the price level is determined by the money stock per

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2. Note that the weights on the expected future money supplies decline geometrically at rate $\varphi_2/(1 - \varphi_2)$. The rate of decline depends on φ_2, the sensitivity of demand for money to interest rate. If φ_2 is small, then $\varphi_2/(1 - \varphi_2)$ is small, and the weights decline quickly. In this case, the current money supply is the primary determinant of the price level. On the other hand, if φ_2 is large, then $\varphi_2/(1 - \varphi_2)$ is close to -1, and the weights decline slowly. In this case, the future money supply plays an important role in determining the current price level.

unit of potential output and the long-run equilibrium level of the velocity of money. Formally, the long-run equilibrium price level, P^*, is defined as:

$$P^* = \frac{M_t V^*}{Y^*}, \tag{5}$$

where M_t is the current level of an appropriately measured monetary aggregate (HPS use M2), V^* is the long-run equilibrium value of velocity, and Y^* is the current level of potential real output.[3]

Equation (5), which suggests that P^* is proportional to M_t per unit of potential output, implies that in the long run inflation is a monetary phenomenon. Also, given V^* and Y^*, P^* is the equilibrium price level that current holdings of M will support.

From the quantity theory of money, the current price level that current holdings of M will support is defined as:

$$P_t = \frac{M_t V_t}{Y_t}. \tag{6}$$

From equations (5) and (6), HPS obtain the price gap as:

$$p^* - p_t = (v^* - v_t) + (y_t - y^*), \tag{7}$$

where lower-case variables are the natural logarithms of their upper-case counterparts.

The proponents of the P^* model of inflation suggest that the discrepancy between the long-run equilibrium price level, P^*, and the actual price level is the major cause of inflation. They argue that excess money that is not yet reflected in the current price level may depress current velocity below its long-run level and/or raise output above its potential. This will generate an inflationary economy, since the current price level will be below

3. As noted by Hallman, Porter and Small (1991), the idea is not new. Humphrey (1989) documents that the quantity theorists from David Hume to Milton Friedman would have recognized the P^* model of inflation.

its equilibrium value. As lags in money demand and interest rates adjust to clear the excess money supply, current velocity will revert to its long-run equilibrium, v^*. Similarly, lags in the formation of inflationary expectations and adjustments in nominal wages will also force current output to converge toward potential output. At the end of these adjustments the price level will converge to its long-run equilibrium.

Although in the long-run, p^* determines p, HPS propose that the short-run dynamics for inflation are:

$$\Delta\pi_t = \alpha(p_{t-1} - p^*_{t-1}) + \sum_{i=1}^{4} \beta_i \Delta\pi_{t-i}, \tag{8}$$

where π is the rate of inflation and p, the natural logarithm of the price level. A less restrictive form of equation (8), which replaces the price gap with its components, is compared by HPS with equation (8). The unrestricted model is of the form:

$$\Delta\pi_t = \gamma_1(y_{t-1} - y^*_{t-1}) + \gamma_2(v^*_{t-1} - v_{t-1}) + \sum_{i=1}^{4} \beta_i \Delta\pi_{t-i}. \tag{9}$$

Equation (9) can be seen as a model that nests an accelerationist Phillips curve with that of a monetarist model of inflation. In the standard expectations-augmented Phillips curve, where inflation expectations are set adaptively, γ_2 would be set to zero. On the other hand, a monetarist specification would have only the velocity gap, and therefore γ_1 would be set to zero. HPS constrain the output gap and the velocity gap to enter the dynamic equation for inflation with equal weight, i.e., $\gamma_1 = \gamma_2 = \alpha$.

Equation (9) presents us with two competing views of how the rate of inflation adjusts from a disequilibrium position. First is the Phillips curve view ($\gamma_2 = 0$), where the rate of inflation adjusts to the output gap (goods market disequilibrium). In the second case, which is a monetarist view ($\gamma_1 = 0$), the inflation rate adjusts to monetary disequilibrium.

In the view of HPS, the current level of real output plays no direct role in the dynamics of inflation once its influence on p^* is taken into account through the quantity of money. This can be seen by substituting the expression for p^* into equation (8):[4]

$$\Delta\pi_t = \alpha(p_{t-1} - m^*_{t-1} - v^*_{t-1} + y^*_{t-1}) + \sum_{i=1}^{4} \beta_i \Delta\pi_{t-i} . \qquad (10)$$

It is obvious from equation (10) that current output has no direct impact on the dynamics of inflation. [5]

HPS apply U.S. data to estimate equation (9). They find both the velocity and the output gaps to be statistically significant; the study also does not reject the restriction that $\gamma_1 = \gamma_2$.

2.3 A Modified P* Model of Inflation

The authors of the P^* model assume that the velocity of money is stable and the income-elasticity of money demand is unity. However, research at the Bank of Canada has found that, unlike in the United States, the velocity of money in Canada is not stationary. Thus, the original version of the P^* model is not applicable to Canada. A modified version of the P^* model has therefore been developed at the Bank. The modified version begins by postulating a long-run money demand function — in other words, the cointegrating vector for money and other economic variables. Let long-run money demand be of the form:

4. The expression for p^* is obtained by taking the log of equation (1).

5. The P^* model has been the subject of considerable study in the United States. For example, HPS find that the P^* model performs better than a pure output-gap model, a velocity-gap model, an unconstrained price-gap model (where γ_1 and γ_2 are unrestricted) or an autoregressive model of inflation. However, Christiano (1989) has questioned the plausibility of assumptions underlying the model, and has argued that there is no meaningful difference between the forecasting ability of the P^* model and several other indicator models. Kuttner (1990) has also argued that the P^* model is deficient in that it ignores the output gap. See also Pecchenino and Rasche (1990), Rasche (1991), Tatom (1990), Ebrill and Fries (1991), Becsi and Duca (1994) and Hallman, Porter and Small (1991) for more on the empirical performance of the P^* model on U.S. data.

$$m_t - p_t = \varphi_0 + \varphi_1 y_t + \varphi_2 i_t + \varepsilon_t, \quad (11)$$

where m_t, p_t, y_t and i_t are respectively money, the price level, real income and the nominal interest rate (opportunity cost of holding money).[6] With the exception of the interest rate, all the variables are expressed in their natural-logarithmic values. The long-run equilibrium price level (p^*) that will correspond to the current level of money, the long-run interest rate, and potential output can be expressed as:

$$p^* = m_t - \varphi_0 - \varphi_1 y_t^* - \varphi_2 i_t^*. \quad (12)$$

From equations (11) and (12) the price gap is obtained as:

$$p^* - p_t = \varphi_1 (y_t - y_t^*) + \varphi_2 (i_t - i_t^*) + \varepsilon_t \;. \quad (13)$$

In other words, the price gap is defined as the sum of the output gap (weighted by the income elasticity, φ_1), the interest-rate gap (weighted by the semi-interest elasticity, φ_2) and the money gap (ε_t).

Following HPS, the short-run dynamic-equation for inflation is of the form:

$$\pi_t = \Theta_0 + \Theta_1 ygap_{t-1} + \Theta_2 mgap_{t-1} + \Theta_3 rgap_{t-1} + \sum_{i=1}^{k} \Phi_i \pi_{t-i} + \zeta_t, \quad (14)$$

where *ygap*, *mgap* and *rgap* are the output gap, the money gap and theinterest-rate gap respectively.[7]

Traditional monetary inflation-indicator models suggest that current inflation is determined by its lagged values and the lagged values of a monetary aggregate. In order to examine the link between M1 and inflation,

6. The residual, ε_t, the difference between actual money balances and long-run money demand, is referred to as the "money gap."

7. Note that in our empirical work, we proxy the interest-rate gap by the term-structure spread, defined as the 10-year-plus government bond yield minus the 90-day commercial paper rate.

equation (14) was nested with a traditional indicator model. Thus, lagged cumulative growth rate of M1 was included as an explanatory variable in equation (14) in the empirical work of the paper.

3.0 Empirical Results

This section presents the results of the empirical examination of equation (14). The exercise involves the following steps. First, Hendry's (1995) long-run money demand function for M1 is used to construct the M1 gaps. Second, the modified version of the P^* models of inflation are estimated for CPI and the GDP deflator. Third, the performance of the models in forecasting inflation is assessed.

3.1 Data

The M1 gaps were created from Hendry's (1995) long-run demand-for-money function for M1. Hendry's money-demand functions used to create the money gaps are:

$$m1 = -0.503 + 0.930cpi + 0.597y - 0.038R90 - 0.141dum80 \qquad (15)$$

and

$$m1 = -0.986 + 1.020def + 0.428y - 0.023R90 - 0.115dum80, \qquad (16)$$

where *M1* is gross M1, *cpi* is the consumer price index, *def* is the GDP deflator, *R90* is the 90-day commercial paper rate, *y* is a measure of economic activity and *dum80* is a dummy variable that accounts for financial innovations beginning in 1980. With the exception of *R90* and *dum80*, all the variables are expressed in natural logarithms.[8]

The money gap created from equation (15) was used for the CPI model; the money gap created from equation (16) was used for the GDP

8. Note that equations (15) and (16) are the same as systems 10a and 11a in Hendry (1995). The shift dummy is zero before 1980Q1 and one after 1982Q4, and it increases linearly between those dates. This is designed to approximate the slow introduction and dissemination of financial innovations.

deflator models. The output gap (*YGAP*) was defined as the residual from a regression of output against a linear and quadratic time trend. The interest-rate gap is proxied by the term-structure spread, which is defined as the 10-year-plus government bond yield minus the 90-day commercial paper rate (*S10M90*). Also, the annualized *k*-quarter growth rate of prices (CPI and the GDP deflator) is defined as:

$$GkP_t = \frac{400}{k} \times \log\left(\frac{P_t}{P_{t-k}}\right), \tag{17}$$

where k is the quarter (1, 4, 6, 8, 12 and 16) and P is the price level (CPI and the GDP deflator). Similarly the annualized k-quarter growth rate of M1 is defined by equation (17) by replacing P with M1. The sample period for the estimation of the models in the paper is 1956Q1 to 1994Q4.

As explained earlier, the motivation for including M1 in equation (14) is to examine the link between the cumulative growth rates of M1 and their corresponding rates of inflation. Figure 1 plots the eight-quarter growth rates of CPI versus the eight-quarter growth rate of M1 (lagged eight quarters). The graph reveals a correlation between M1 and inflation.

Figure 2 also plots the six-quarter growth rate of CPI versus the M1 gap (lagged six quarters).[9] The graph shows a link between inflation and the money gap.

3.2 Estimation of P^* Model of Inflation

In this section we estimate the P^* indicator models of inflation using the term spread (*S10M90*), the money gap and the output gap. The form of the regression is:

$$\begin{aligned} GkP_t = {} & \alpha_0 + \alpha_1 GkP_{t-k} + \alpha_2 GkM1_{t-k} + \alpha_3 M1GAP_{t-k} + \alpha_4 YGAP_{t-k} \\ & + \alpha_5 S10M90_{t-k} + \varepsilon_t \end{aligned} \tag{18}$$

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9. Note that similar results were obtained for the GDP deflator. Also note that the money gaps were multiplied by a factor of 20 before the graphs were plotted.

where the variables are as defined earlier. The results are reported in Table 1 in appendix.[10]

The data used in the paper are quarterly, but the forecasting horizon *k* varies from one to sixteen quarters ahead. The overlapping data generate a moving-average error of order *k-1*, which produces inconsistent standard errors (not the coefficient estimates). To obtain correct inferences on the coefficients, the Hansen and Hodrick (1980) adjustment method is applied to correct for a moving-average process of order *k-1* in the residuals.

The results in Table 1 show that the coefficient of past inflation is statistically significant at one- and four-quarter horizons for the CPI model and at one-, four- and six-quarter horizons for the GDP deflator. The results also indicate that, in addition to the GDP deflator model at the four-quarter horizon, the coefficient of lagged growth rate of M1 is positive and statistically significant at the six- to sixteen-quarter horizon for both models. The coefficient of the money gap is found to be statistically significant at the one- to sixteen-quarter horizon for both models. The coefficient of the output gap is observed to be statistically significant at one to sixteen quarters for the CPI model. In the case of the GDP deflator model, the output gap is significant at the four- and six-quarter horizons. The spread term is observed to be significant at eight- to sixteen-quarter horizons for both models. Figures 3 and 4 plot the actual and in-sample fitted values of the six-quarter growth rates of prices, which suggest that these models fit the data reasonably well.

Table 2 reports the results of the diagnostics exercise conducted on the residuals from the estimates of the models at various quarters. The Breusch-Godfrey and Box-Pierce tests were conducted to check for serial correlation in the residual. The tests reject the null hypothesis of no autocorrelation. The results in the table also show that the null hypothesis of no autoregressive-conditional heteroscedasticity (ARCH) of the residuals

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10. Note that there is no generated regressor problem in equation (18) since the estimates used to calculate the money and output gaps measures are extremely consistent.

can also be rejected. Finally, based on the Jacque and Bera test, the results show that the null hypothesis of normally distributed residuals cannot be rejected for the following models: G8CPI, G16CPI and all the deflator models except G4DEF and G8DEF. An intuitive explanation for the presence of autocorrelation in the residuals is the moving-average (MA) error process generated by the use of overlapping data in the regression. The ARCH residuals, however, suggest the presence of some systematic structure not captured by the current models.

3.3 Forecast of Inflation

This part of the paper examines the out-of-sample forecasting performance of the inflation-indicator models estimated in Section 3.2. The predictive performance of the models was assessed by running recursive regressions over the sample period. This method involves the estimation of the model from 1956Q1 to 1982Q1 and a *k*-quarter-ahead forecast, where *k*= 1, 4, 6, 8, 12 and 16. The models are re-estimated by extending the sample size by a quarter at time and *k*-quarter-ahead forecasts are made at each time. The process continues until the data points are exhausted. The root-mean-square error (RMSE), mean-absolute error (MAE) and Theil's (1966, 28) U-statistic for each model are then calculated.[11] The results are reported in Table 3 in appendix.

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11. The U-statistic, which is dimensionless, is defined as:

$$U = \left(\left(\frac{1}{n^0}\sum_i (y_i - \hat{y}_i)^2\right) \Big/ \left(\frac{1}{n^0}\sum_i y_i^2\right)\right)^{0.5}$$

where n^0 is the number of forecasting periods being forecasted, y_i and $\hat{y}_i$ are actual and predicted values of the goal variables respectively. Note that the U-statistic can be defined in terms of the root-mean-square error (RMSE) as:

$$U = RMSE \Big/ \left(\frac{1}{n^0}\sum_i y_i^2\right)^{0.5}$$

It is easily seen that if U = 0 then the forecasts are all perfect ($y_i = \hat{y}_i$ for all i). Thus, since U can only be positive, the larger U is, the worse the forecasting performance of the model. Also if U > 1 then the forecasting performance of the model is inferior to that of a random-walk model.

Based on the RMSE, the U-statistic and the MAE, the results in Table 3 suggest that the out-of-sample-forecast performance of the CPI model is reasonably good at the one- to sixteen-quarter horizons. However, the models are poor predictors of the GDP deflator measure of inflation.

Figures 5 and 6 plot the actual and out-of-sample predicted values of the six-quarter growth rates of the two prices. The graphs show that the CPI model predicts inflation better than the GDP deflator model. However, the graphs also show that the models over-predict inflation in the early and late 1980s and the early part of the 1990s.

3.4 A Comparison to a Vector Error-Correction Model

Recently Armour et al. (1996) used a vector error-correction-model (VECM) to forecast inflation eight quarters ahead. This study, which is based on Hendry (1995), finds that the VECM provides significant leading information for inflation and forecasts very well. The next question is: How do the models studied in this paper compare with the VECM?

A comparison of the results in this paper and those of Armour et al. indicates that the VECM forecasts the eight-quarter growth rate of CPI better than the models studied in this paper. However, the results in this paper are useful to an empirical understanding of the link between inflation, on the one hand, and M1, the M1 gap, the output gap and the term spread, on the other. Also, this paper assesses the impact of the above variables on inflation at a longer horizon (one to sixteen quarters), while the study in Armour et al. was an examination of inflation at eight quarters.[12]

3.5 The Money-Gap: Disequilibrium Money

Many economists might argue that the money gap, which is an important variable in the P^* model of inflation, would disappear rapidly in equilibrium: the underlying forces of financial markets will react so as to

12. Following Chong and Hendry (1986), encompassing tests were conducted on the forecasts from the VECM and from the P* model. The test results show that the VECM forecasts encompass the P* model.

ensure that the supply and demand for money are equal in perpetual equilibrium. The question then is: Why is the money gap an important component of the P^* model of inflation, which in turn is derived from an equilibrium model of the quantity theory of money? The answer to this question lies in the disequilibrium view of money.[13]

Laidler (1990) argues that economic agents plan to hold money for a particular interval rather than for a particular point in time. Laidler suggests that the quantity of money demanded by agents must therefore be interpreted as an average or target value of an inventory or buffer stock of money. One implication of this buffer-stock view of the demand for money is that agents do not instantaneously adjust their money holdings in response to shocks to the economy. Agents hold money in order to minimize the cost of information and uncertainty in the economy. The slow adjustment of prices and wages may result in agents finding themselves with excess money.

Laidler begins by explaining why uncertainty and information costs motivate a demand-for-money function:

> To hold a buffer stock of generally acceptable purchasing power prevents, at least in part, surprises in markets where the agent is a seller from impinging upon his buying activities, and vice versa. Indeed the matter goes deeper: an agent's chances of being surprised in his market activity are not independent of the time and effort he puts into seeking and processing information relevant to those activities. If money holdings enable him to endure the consequences of surprises at lower cost than would otherwise be the case, then money becomes a substitute for information. Thus the very existence of a monetary system which enables him so to protect himself will cause the agent to be more prone to surprises. Hence fluctuations in holdings of money about their target value are of the very essence of economic activity co-ordinated by monetary exchange.[14]

An intuitive explanation of Laidler's quote can be illustrated with the behaviour of a firm in an uncertain economic environment. The cost to the firm of adjusting to changed prices, wages, production and inventories in the wake of a macroeconomic shock may outweigh the opportunity cost

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13. The discussion on the of the disequilibrium-money view in this paper is drawn from Laidler (1990, Chapter 2), Goodhart (1989, 75-81) and Cuthbertson and Taylor (1987). Caramazza, Hostland and Poloz (1990) also provide a discussion of this subject.

14. Laidler (1990, 26).

(interest foregone) of holding money balances. As well, shocks to the firm may result in changes to the money balances the firm is willing to hold in the short run until the shocks are perceived to be permanent or temporary, motivating the persistence of a money gap. A similar argument can be made for households.

Although the buffer-stock view suggests that aggregate demand and supply for money may differ, the view is very much consistent with the concept of "full equilibrium." An economy is said to be in full equilibrium if each economic agent, based on ex ante expectations, is able achieve all planned economic activities, such as buying and selling goods, services and assets ex post. By defining full equilibrium in this manner, it is possible for agents to be away from their target holdings of money and still be able to achieve their planned economic activities.

The buffer-stock view of money demand also suggests that money acts like a "shock absorber" so that economic agents can temporarily postpone costly adjustments to their portfolio. Households and firms incur costs in making adjustments to their respective portfolios. The costs include brokerage fees, the costs of determining the various yields and prices of financial assets, and the cost of knowledge and research of the fundamentals of the economy. As a result of these costs, economic agents will make portfolio adjustments at certain intervals and no changes in between. Thus agents may not react immediately to every macroeconomic shock, like an unexpected rise in the money supply, that occurs between the decision dates for portfolio adjustment. Hence, an unexpected inflow of money may cause agents to hold excess money until the next portfolio adjustment date.

Following the work of Miller and Orr (1966), Milbourne (1987) uses an inventory model to demonstrate how agents decide on their holdings of money and other interest-bearing assets. Milbourne argues that because of costs of portfolio adjustments, agents will allow their money holdings to fluctuate within a range. However, when the money held by an agent reaches either an upper or lower threshold, a portfolio adjustment is instantaneously triggered, and the money holding is returned to a target level. Thus, under this framework, economic agents will only respond (instantaneously) to unexpected money supply shocks if the shocks cause the money holdings of agents to lie outside the allowable range for money holdings to fluctuate. Otherwise, agents will always wait until the next portfolio-adjustment period to react to the shocks.

One question asked by critics of the buffer-stock view of money is: How does the disequilibrium in money market disappear? To answer this question, consider a situation where the monetary authorities embark on an expansionary monetary policy that leads agents to hold excess money. The excess money created can be eliminated by a combination of a rise in prices and output and a fall in interest rates. However, the removal of the disequilibrium may not be instantaneous since the excess money will be removed slowly over time, given that prices and wages adjust sluggishly.[15]

It must be mentioned that a fall in interest rates may not be sufficient to remove the excess money in the economy. Laidler (1990) argues that the desire of agents to rid themselves of excess money may lead to an increase in the demand for bonds in the economy, which in turn will lead to a fall in interest rates. The fall in interest rates would make agents willing to hold more money but, according to Laidler, not all the excess money is eliminated in the economy. Laidler stresses the role of price changes in eliminating the excess money. Thus, the fall in interest rates is not sufficient to equilibrate the supply and demand for money.

Note that the buffer-stock view implies that discrepancies between the actual and desired money holdings of individuals may not cancel out when one aggregates over all agents in the economy. While one individual might be able to get rid of his or her excess money by passing it to another

15. The classical and rational-expectation views could suggest that prices will rise instantaneously to clear the money market. The Keynesian economist would argue that interest rates will fall instantaneously to remove the excess money in the system. Both schools of thought believe that excess money or a money gap cannot persist in the long run.

individual instantaneously, on aggregate, the whole economy cannot eliminate the excess money supply if prices and wages are sluggish to adjust.[16]

The above presentation briefly explains the demand for money and argues that, in the short run, aggregate money demand could differ from money supply. The above explanation supports our empirical demonstration of the relevance of the money gap in a P^* model of inflation.

4.0 Concluding Remarks

This paper has examined, in addition to the output gap and the term spread, the roles of the cumulative growth of M1 and the M1 gap for two measures of inflation. The examination was conducted within the framework of the P^* model of inflation. The analysis also paid attention to distant-early-warning (DEW) models of inflation. The central conclusion from all the estimated models is that, like the output gap, the cumulative growth of M1 and the M1 gap play significant roles in inflation-indicator models. Thus, DEW models for inflation suggest that the cumulative growth of M1 and M1 gaps are useful for predicting the long-run path of inflation.

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16. David Laidler likes explaining this point with a “hot potato” analogy. Like a hot potato, each participant in the economy will immediately get rid of any excess money. The excess money which is passed from agent to another will be eliminated as prices change.

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Table 1: P*-Model of Inflation

$$GkPRICE_t = \alpha_0 + \alpha_1 GkPRICE_{t-k} + \alpha_2 GkM1_{t-k} + \alpha_3 M1GAP_{t-k} + \alpha_4 YGAP_{t-k} + \alpha_5 S10M90_{t-k} + \varepsilon_t$$

Sample: 1956Q1 + k to 1994Q4 (t-statistics noted in parentheses)

PRICE	Annualized price growth, k quarters ahead											
	k = 1						**k = 4**					
	α_1	α_2	α_3	α_4	α_5	$\bar{R}^2$	α_1	α_2	α_3	α_4	α_5	$\bar{R}^2$
CPI	0.363	0.027	11.835	13.273	0.019	0.748	0.218	0.041	13.798	17.824	0.201	0.775
	(5.631)	(1.853)	(5.592)	(3.754)	(0.143)		(2.139)	(1.001)	(3.819)	(4.925)	(1.106)	
GDP Def	0.260	0.017	23.570	7.670	0.116	0.605	0.472	0.070	13.475	11.514	0.306	0.742
	(2.845)	(0.914)	(4.414)	(1.114)	(0.701)		(4.626)	(2.010)	(2.478)	(2.449)	(1.393)	
	k = 6						**k = 8**					
PRICE	α_1	α_2	α_3	α_4	α_5	$\bar{R}^2$	α_1	α_2	α_3	α_4	α_5	$\bar{R}^2$
CPI	0.150	0.135	13.396	17.241	0.206	0.768	-0.008	0.183	16.000	17.298	0.474	0.752
	(1.289)	(2.479)	(3.330)	(3.850)	(1.017)		(-0.049)	(2.819)	(3.068)	(3.555)	(1.730)	
GDP Def	0.300	0.151	16.243	11.163	0.362	0.716	0.127	0.198	19.118	11.376	0.536	0.671
	(2.392)	(2.558)	(2.475)	(1.883)	(1.397)		(0.899)	(2.188)	(2.518)	(1.687)	(1.894)	
	k = 12						**k = 16**					
PRICE	α_1	α_2	α_3	α_4	α_5	$\bar{R}^2$	α_1	α_2	α_3	α_4	α_5	$\bar{R}^2$
CPI	-0.149	0.264	17.486	16.695	0.890	0.727	-0.057	0.313	12.549	17.812	0.924	0.670
	(-0.757)	(3.297)	(3.011)	(2.675)	(2.346)		(-0.243)	(5.396)	(1.907)	(2.159)	(2.196)	
GDP Def	-0.079	0.288	21.788	9.705	0.872	0.632	0.032	0.340	14.172	8.791	0.883	0.566
	(-0.531)	(2.820)	(3.638)	(1.373)	(3.015)		(0.156)	(5.152)	(2.238)	(0.850)	(2.761)	

Table 2: Residual Diagnostics

Model	LM(1)	LM(4)	Q(1)	Q(4)	ARCH(1)	ARCH(4)	Jarque-Bera
G1CPI	123.945 (0.000)	124.465 (0.000)	0.037 (0.848)	1.756 (0.781)	0.038 (0.844)	0.435 (0.979)	199.793 (0.000)
G4CPI	110.017 (0.000)	111.302 (0.000)	81.716 (0.000)	133.638 (0.000)	68.476 (0.000)	68.938 (0.000)	45.717 (0.000)
G6CPI	104.442 (0.000)	104.131 (0.000)	97.548 (0.000)	197.300 (0.000)	71.287 (0.000)	71.185 (0.000)	14.291 (0.001)
G8CPI	112.402 (0.000)	111.879 (0.000)	92.405 (0.000)	204.202 (0.000)	48.124 (0.000)	52.144 (0.000)	3.253 (0.196)
G12CPI	109.849 (0.000)	107.563 (0.000)	91.719 (0.000)	230.485 (0.000)	56.915 (0.000)	58.482 (0.000)	8.081 (0.018)
G16CPI	107.738 (0.000)	105.626 (0.000)	94.042 (0.000)	248.503 (0.000)	57.871 (0.000)	57.988 (0.000)	3.376 (0.184)
G1DEF	129.512 (0.000)	124.605 (0.000)	2.188 (0.139)	12.247 (0.016)	6.635 (0.010)	4.893 (0.298)	1.943 (0.378)
G4DEF	115.283 (0.000)	120.415 (0.000)	83.233 (0.000)	144.505 (0.000)	65.394 (0.000)	68.233 (0.000)	23.732 (0.000)
G6DEF	111.845 (0.000)	110.925 (0.000)	103.381 (0.000)	245.308 (0.000)	68.189 (0.000)	76.645 (0.000)	2.458 (0.293)
G8DEF	120.261 (0.000)	119.831 (0.000)	101.069 (0.000)	249.681 (0.000)	48.280 (0.000)	48.317 (0.000)	1.949 (0.000)
G12DEF	113.004 (0.000)	111.027 (0.000)	95.632 (0.000)	253.197 (0.000)	57.809 (0.000)	58.013 (0.000)	2.181 (0.336)
G16DEF	112.998 (0.000)	110.966 (0.000)	102.367 (0.000)	280.979 (0.000)	72.937 (0.000)	71.238 (0.000)	1.837 (0.399)

LM statistics denote the Breusch-Godfrey LM test for serial correlation, and are distributed as χ^2; Q statistics denote the Box-Pierce test for serial correlation of first and fourth order, and are also distributed as χ^2; ARCH statistics denote Engle‘s test for ARCH of orders one and four, which again are distributed as χ^2; Jarque-Bera statistics denote the Jarque-Bera test for normality, and are distributed as $\chi^2(2)$. P-values are given in parentheses. They give the probability of not rejecting the null hypothesis.

Table 3: Out-of-Sample Forecast of the Annualized Growth Rates of Prices at Various Quarters Ahead
Forecast Horizon: 1982Q1+k - 1994Q4

$$GkPRICE_t = \alpha_0 + \alpha_1 GkPRICE_{t-k} + \alpha_2 GkM1_{t-k} + \alpha_3 M1GAP_{t-k} + \alpha_4 YGAP_{t-k} + \alpha_5 S10M90_{t-k} + \varepsilon_t$$

Quarters	RMSE		Theil's U-statistic		MAE	
	Prices		Prices		Prices	
	CPI	GDP Def	CPI	GDP Def	CPI	GDP Def
1	1.7509	2.2591	0.4280	0.6681	1.3038	1.8063
4	1.4851	1.8645	0.3808	0.5892	1.2664	1.5340
6	1.5518	1.9764	0.4005	0.6323	1.2757	1.6739
8	1.7780	2.2869	0.4571	0.7266	1.4565	1.9353
12	1.8347	2.5033	0.4655	0.7814	1.5352	2.0826
16	2.6527	3.1748	0.6637	0.9684	2.2917	2.7921

FIGURE 1: Eight-quarter inflation rate and the eight-quarter growth rate of M1 (lagged eight quarters)

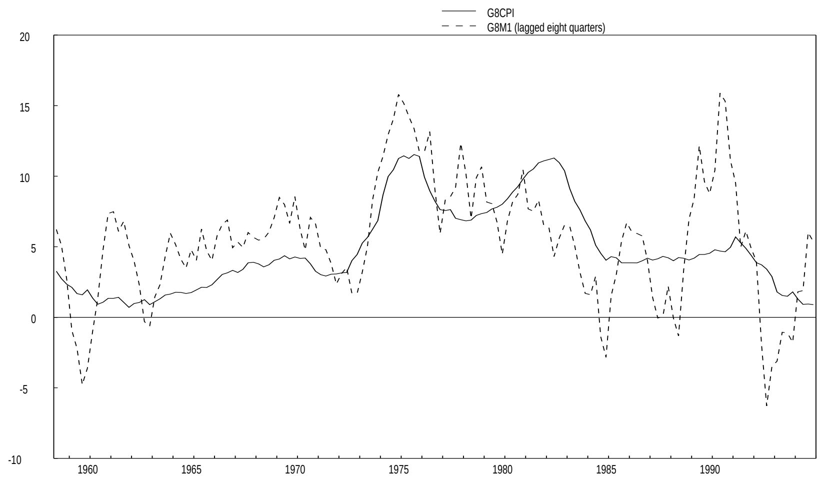


FIGURE 2: Six-quarter inflation rate and M1 gap (lagged six quarters)

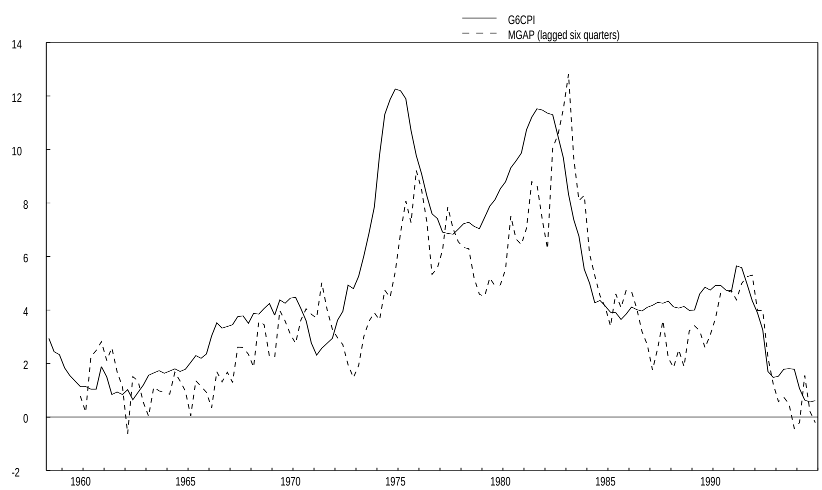


FIGURE 3: In-sample fit of the six-quarter growth of CPI

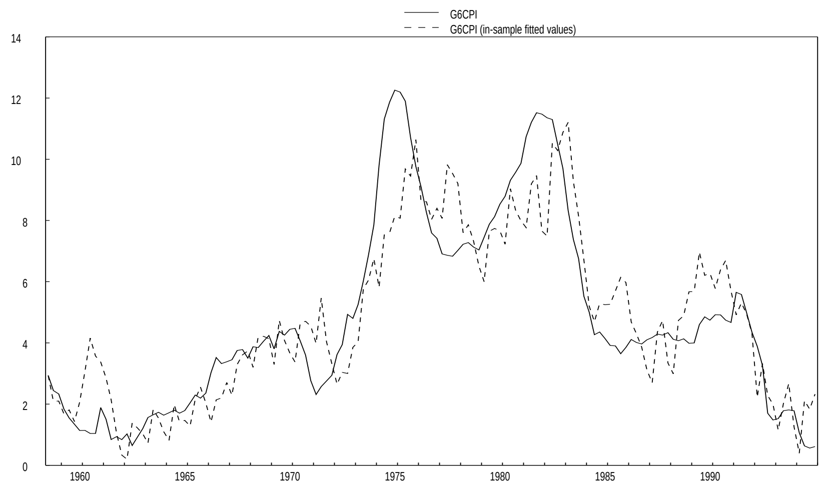


FIGURE 4: In-sample fit of the six-quarter growth of the GDP deflator

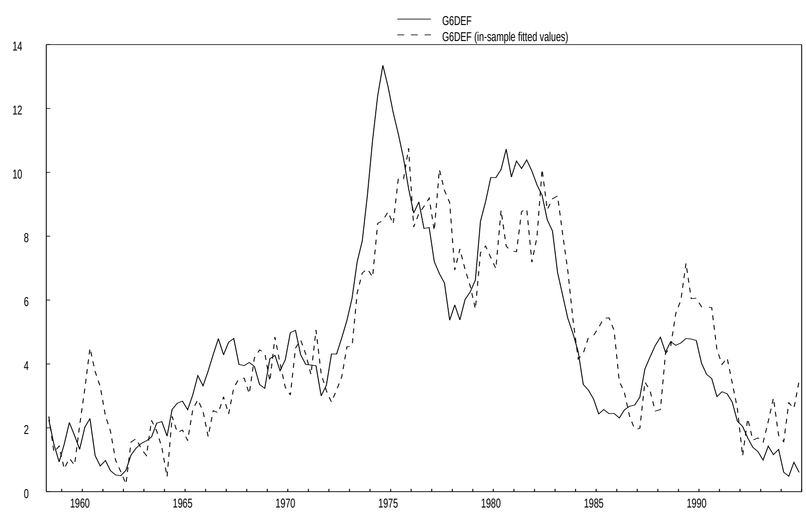


FIGURE 5: Out-of-sample forecast of the six-quarter growth of CPI

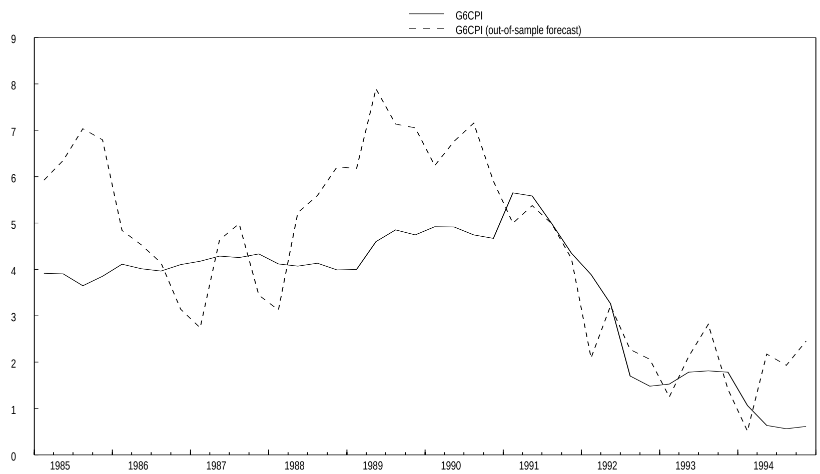
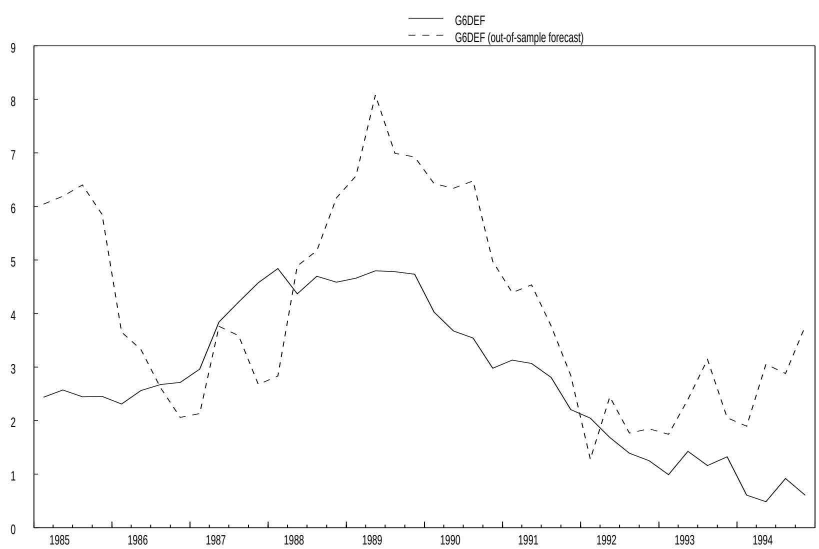


FIGURE 6: Out-of-sample forecast of the six-quarter growth of the GDP deflator



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