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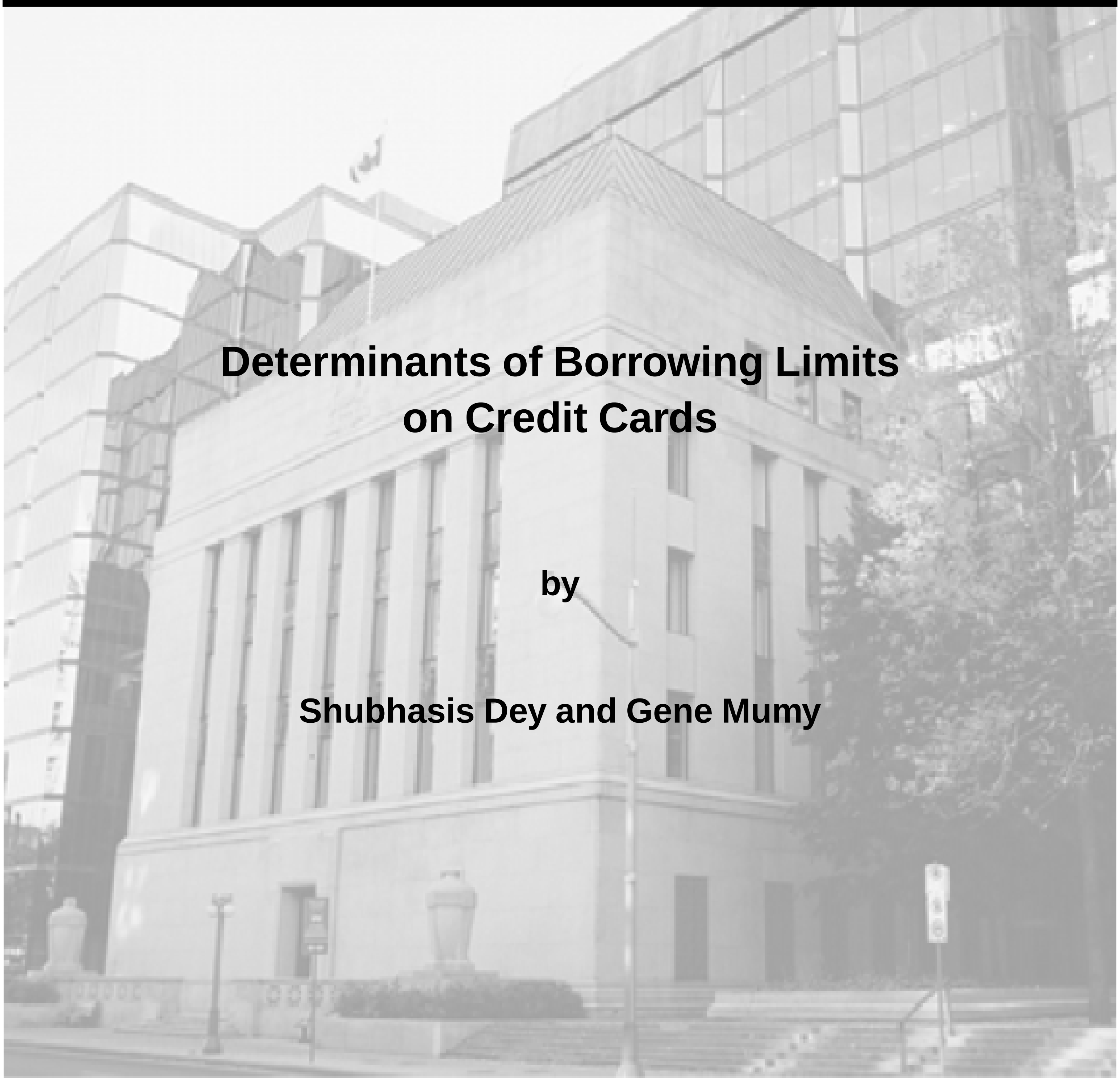

Determinants of Borrowing Limits on Credit Cards

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Abstract

The difference between actual borrowings and borrowing limits alone generates information asymmetry in the credit card market. This information asymmetry can make the market incomplete and create ex post misallocations. Households that are denied credit could well turn out to be ex post less risky than some credit card holders who borrow large portions of their borrowing limits. Using data from the U.S. *Survey of Consumer Finances*, the authors find a positive relationship between borrower quality and borrowing limits, controlling for banks' selection of credit card holders and the endogeneity of interest rates. Their estimation reveals how interest rates have a negative influence on the optimal borrowing limits offered by banks.

JEL classification: D4, D82, C3
Bank classification: Market structure and pricing; Econometric and statistical methods

Résumé

La différence entre le montant des limites de crédit accordées et le montant effectif des emprunts suffit à créer sur le marché des cartes de crédit une asymétrie d'information. Cette asymétrie peut rendre ce marché incomplet et produire, a posteriori, une mauvaise allocation des ressources. Il se pourrait fort bien, en effet, que les ménages qui se voient refuser un prêt représentent un risque moins élevé que certains détenteurs de cartes qui font largement appel à leur ligne de crédit. À partir des données de l'enquête menée par la Réserve fédérale américaine sur les finances des consommateurs, les auteurs concluent à une corrélation positive entre la solvabilité de l'emprunteur et la limite de crédit, une fois pris en compte l'endogénéité des taux débiteurs et les effets de la sélection par les banques des détenteurs de cartes. D'après leurs estimations, le niveau optimal des limites de crédit consenties varie en fonction inverse des taux d'intérêt.

Classification JEL : D4, D82, C3
Classification de la Banque : Structure de marché et fixation des prix; Méthodes économétriques et statistiques

1. Introduction

It is well accepted that borrowing limits on collateralized loans are primarily determined by the amounts of collateral pledged by the borrowers. However, for non-collateralized loans, such as those on credit cards, the information about borrowers' repayment abilities plays a crucial role in determining their credit card borrowing limits or credit limits. Asymmetric information between borrowers and lenders and the lack of collateral to mitigate that informational asymmetry are mainly responsible for the existence of credit rationing in some credit markets. Imperfect information about borrower risk induces banks to refuse credit to some borrowers even if the latter would accept higher interest rates for their loans. Credit bureau reports provide some critical information about borrower riskiness, which banks use to alleviate some of the informational asymmetry and to improve the quality of their loan-supply decision. Publicly available information about borrowers' creditworthiness helps banks sort their client pool into broad risk classes. Banks do not, however, have perfect knowledge about individual borrower risk. In the case of lines of credit, such as credit cards, banks particularly do not know how much a borrower will actually borrow on the line, which is a key determinant of the borrower's repayment probability. Therefore, credit rationing persists in the unsecured credit card market. Borrowers with no or "bad" credit reports are more likely to be refused access to credit cards by banks. Those credit card holders who have "better" creditworthiness are perceived to have higher repayment abilities and therefore are likely to be provided with higher credit card borrowing limits. Profit-maximizing banks choose to provide exactly the amount of credit to their borrowers that maximizes their expected profits. Therefore, a careful analysis of the elements of borrowers' creditworthiness and the optimal line of credit contracts will help us understand the determinants of credit card borrowing limits. We find that the difference between actual borrowings and offered credit limits is enough to generate information asymmetry in the credit card market. Moreover, individuals who are rationed out of the credit card market could very well turn out to have been "convenience users,"[1] and therefore ex post were less risky than some credit card holders who borrow large fractions of their credit limits. Thus, not only can information asymmetry in the credit

[1] "Convenience users" are individuals who use credit cards for transactions purposes only.

card market make the market incomplete (through credit rationing), but it can also result in ex post misallocations.

A typical credit card contract is two-dimensional. Banks offer a rate of interest along with a pre-set borrowing limit to their potential borrowers. The two-dimensional nature of loan contracts makes credit card interest rates endogenous. Empirical identification of the determinants of credit card borrowing limits requires us to correct for this endogeneity. Moreover, not all individuals have credit cards. The set of credit card holders is a selected sample and therefore our estimation needs to account for the sample selection bias. Controlling for the banks' selection of credit card holders and the endogeneity of credit card interest rates, we find a positive relationship between borrower quality and borrowing limits on credit cards. Our estimation also reveals how the endogenous interest rates negatively influence the optimal credit card borrowing limits. In section 2, we describe the background and previous research on these issues. In section 3, we introduce the theoretical model. The data are described and the econometric model built in sections 4 and 5, respectively. Section 6 describes the empirical results and section 7 offers some conclusions.

2. Background

Beginning with Ausubel (1991), researchers have examined consumer lines of credit, especially with regards to credit cards. The bulk of the literature on credit cards concentrates on explaining why the average credit card interest rates remain sticky at a high level. Ausubel (1991) argues that the reason for the downward rigidity of credit card interest rates and supernormal profits is the failure of competition in the credit card market. He partly attributes this failure to myopic consumers who do not foresee indebtedness and interest payments on their outstanding balances. Brito and Hartley (1995), however, argue that consumers carry high-interest credit card debt not because of myopia, but because low-interest bank loans involve transactions costs. Mester's (1994) view is that low-risk borrowers who have access to low-interest collateralized loans leave the credit card market. This makes the average client pool of the credit card market riskier, thereby preventing interest rates from going down. Park (1997) points to the option-value nature of credit cards to explain their price stickiness. He argues that the interest rate that produces zero profit for credit card issuers is higher than the interest rates on most other loans, because rational credit card holders borrow more money when they become riskier. An empirical paper by Calem and Mester (1995) finds evidence that consumers are reluctant to search for lower rates because of high search costs in this market. Cargill and Wendel (1996) suggest that, due to the high presence of convenience users, even modest search costs could keep the majority of consumers from seeking out lower interest rates. Kerr (2002) focuses on interest rate dispersion within the credit card market. He studies a two-fold information asymmetry: one between the banks (i.e., the lenders) and the borrowers, and the other within the banks themselves. Some banks (the external banks) have access to only the publicly available credit histories, while others (the home banks) have additional access to borrowers' private financial accounts. Kerr argues that, in equilibrium, the average rate of interest charged by the so-called external banks would be higher than that charged by the home banks, because the average borrower associated with the external banks would be riskier.

Most of the existing literature on the credit card market focuses on analyzing the various aspects of its pricing. Despite the fact that credit card loan contracts are essentially two-dimensional, researchers have largely ignored the credit-limit dimension of the contract. Gross and Souleles (2002) utilize a unique new data set on credit card accounts to analyze how people respond to changes in credit supply. They find that increases in credit limits generate an immediate and significant rise in debt, consistent with the buffer-stock models of precautionary saving, as cited in Deaton (1991), Carroll (1992), and Ludvigson (1999). Dunn and Kim (2002) argue that banks, in order to strategize against Ponzi-schemers in the credit card market, tend to provide lower credit limits to high-risk borrowers, despite giving them a larger number of cards. Though they find some empirical support for their hypothesis on credit limits, Dunn and Kim choose to focus their formal empirical analysis on an estimation of credit card default rates. Castronova and Hagstrom (2004), using simple two-stage least squares estimation, find that the action in the credit market is mostly in the limits and not in the balances.

In this paper, we build a general theoretical model that captures the key elements of credit card loan contracts, and we test the relationship between borrower quality and credit card borrowing limits, correcting for the banks' selection of credit card holders and the influence of endogenous interest rates.

3. The Theoretical Model

Consider a model where banks are competitively offering non-collateralized lines of credit, such as credit cards. A line of credit is a borrowing instrument whereby the borrower is offered a borrowing limit (or credit limit) and an interest rate. The borrower can borrow up to the credit limit. Interest charges accrue only if some positive amount is borrowed on the line. A line of credit contract incorporates the traditional fixed-loan contract as a special case when the entire credit limit is borrowed at the very outset. Banks are assumed to procure funds at a rate r_F. Based on publicly available credit reports, banks are able to partition their clients into broad risk classes. Let us assume that these classes, represented by i, are such that $i \in [\underline{i}, \bar{i}]$. The variable i can be considered the credit score that credit bureaus construct for all potential borrowers. Let us also assume for simplicity that there is only one borrower in every risk class, i.[2] A typical credit card contract offered to class i consists of a vector (L_i, r_i), where L_i is the credit limit and r_i is the interest rate. Using the framework put forward by Dey (2004), we argue that borrowers primarily use lines of credit to smooth consumption across various states and time periods. This framework essentially makes the desired borrowings on lines of credit become random variables – functions of the interest rates and the underlying wealth shocks. Let q_i represent the underlying wealth shock, such that we have the optimal borrowing as $B_i = B(r_i; q_i); q_i \sim G(q_i)$, where $q_i \in (-\infty, \infty)$. We can write $B_i \sim F(B_i)$, $F'(B_i) = f(B_i)$, where $B_i \in (-\infty, \infty)$. Moreover, q_i's are assumed to be independent of each other (and so are B_i's). Using the optimal borrowing function, we can derive an inverse demand curve for borrower i as $r_i = r(B_i; q_i)$. The repayment probability for a borrower increases with the risk-class measure, i, and decreases with the

[2] We therefore assume that banks offer the same contract to all individuals within a particular risk class (with the same credit score), despite the potential heterogeneity in their repayment abilities.

amount owed, D_i, where $D_i = R_iB_i$ and $R_i = (1 + r_i) = R(B_i;\, q_i)$.[3] We represent the class i repayment probability as $r_i = r(D_i, i)$, such that $\frac{\partial r(.)}{\partial D_i} < 0$, $\frac{\partial r(.)}{\partial i} > 0$, and $r_i \in [0,1]$. The only uncertainty that banks have about borrowers' repayment probabilities results from their inability to know the actual borrowings to be undertaken on the lines they extend. In the following section, we consider a typical bank's profit-maximization problem where it is offering an unsecured line of credit, such as a credit card.

3.1 A bank's profit-maximization problem

The expected profit from offering an unsecured line of credit contract (L_i, r_i) to class i is represented by p^i. For class i, a bank's profit-maximization problem is given by:

$$\underset{L_i}{Max}\, p^i = \int_{-\infty}^{L_i} [r(R(B_i;q_i)B_i, i)R(B_i;q_i) - R_F]B_i f(B_i)dB_i +$$

$$\int_{L_i}^{\infty} [r(R(L_i;q_i)L_i, i)R(L_i;q_i) - R_F]L_i f(B_i)dB_i$$

$$= \int_{-\infty}^{L_i} [r(R(B_i;q_i)B_i, i)R(B_i;q_i) - R_F]B_i f(B_i)dB_i +$$

$$[1 - F(L_i)][r(R(L_i;q_i)L_i, i)R(L_i;q_i) - R_F]L_i.$$

Let us assume that $p^i{}_{L_iL_i} < 0$.

Partially differentiating p^i with respect to L_i and setting it to zero, we get,

$$p^i_{L_i}\Big|_{L_i^*} = [r(.)|_{L_i^*} R(L_i^*;q_i) - R_F]L_i^* f(L_i^*) +$$

$$[1 - F(L_i^*)][L_i^*\{R(L_i^*;q_i)\frac{\partial r(.)}{\partial D_i}\Bigg|_{L_i^*} (R(L_i^*;q_i) + R'(L_i^*;q_i)L_i^*) + r(.)|_{L_i^*} R'(L_i^*;q_i)\} +$$

$$r(.)|_{L_i^*} R(L_i^*;q_i) - R_F] - [r(.)|_{L_i^*} R(L_i^*;q_i) - R_F]L_i^* f(L_i^*) = 0,$$

[3] Banks know that actual borrowings undertaken on credit cards change borrowers' repayment probabilities. Since banks have no way of knowing how much a borrower in a particular risk class will borrow, line of credit contracts cannot be conditioned on actual borrowings.

or,

$$p^{i}_{L_i}\Big|_{L_i^*} =$$

$$[1-F(L_i^*)][L_i^*\{R(L_i^*;q_i)\frac{\partial r(.)}{\partial D_i}\Big|_{L_i^*}(R(L_i^*;q_i)+R'(L_i^*;q_i)L_i^*)+r(.)\big|_{L_i^*}R'(L_i^*;q_i)\}+$$

$$r(.)\big|_{L_i^*}R(L_i^*;q_i)-R_F]=0. \qquad (1)$$

***Proposition*:**

(i) Banks choose L_i^* and $r_i^* = r(L_i^*;q_i),$ such that $p^{i}_{L_i}(L_i^*, r_i^*) = 0 = p^{i}(L_i^*, r_i^*).$

For all risk classes yielding $p^{i}(L_i^*, r_i^*) < 0,$ the banks choose $L_i^* \leq 0.$

(ii) For all banks, maximizing the total expected profit over all risk classes is equivalent to integrating over all risk classes the maximized expected profit of every risk class.[4]

The optimal credit card contract offered to borrower *i*, given by the pair (L_i^*, r_i^*), is chosen such that, if L_i^* is actually borrowed at price $r_i^* = r(L_i^*;q_i),$ our bank's profit maximization and zero-profit conditions are simultaneously satisfied for the risk class that borrower *i* represents. We show that the difference between actual borrowings and offered credit limits is enough to generate information asymmetry in the credit card market. Moreover, individuals who are rationed out of the credit card market could very well turn out to have been ex post less risky than some credit card holders who borrow large fractions of their credit limits. Thus, not only can information asymmetry in the credit card market make the market incomplete (through credit rationing), but it can also result in ex post misallocations.

Our bank's optimal credit card contract for risk class *i* can be represented by the following triangular structure:

(i) The equilibrium credit limit equation,

$$L_i^* = L(r_i^*, r_F, q_i, i), \qquad (2)$$

(ii) The equilibrium price equation,

[4] This follows from the fact that wealth shocks, q_i's, (and therefore actual borrowings, B_i's,) are independent of each other and banks are forced to make zero expected profits in every risk class.

$$r_i^* = R^*(r_F, q_i, i). \qquad (3)$$

Therefore, our theory of lines of credit contracts yields to a bivariate equation system that can be used for empirical analysis. Using household-level data, we can potentially test the signs of the following partial derivatives: $\frac{\partial L_i^*}{\partial r_i^*}, \frac{\partial L_i^*}{\partial r_F}, \frac{\partial L_i^*}{\partial i}, \frac{\partial r_i^*}{\partial r_F}$, and $\frac{\partial r_i^*}{\partial i}$.[5]

Given the second-order condition ($p^i_{L_iL_i} < 0$), and assuming that $p^i_{r_i} > 0$, $p^i_{L_i i} > 0$, and $p^i_{L_i r_i} < 0$, our derivations shown in the appendix can generate the following testable predictions out of our theoretical model of line of credit contracts: $\frac{\partial r_i^*}{\partial i} < 0$, and $\frac{\partial r_i^*}{\partial r_F} > 0$. Though we have unambiguous signs for the reduced-form partial derivatives, the signs for the partial derivatives of the structural-form optimal borrowing limit function, such as $\frac{\partial L_i^*}{\partial i}$, $\frac{\partial L_i^*}{\partial r_F}$, and $\frac{\partial L_i^*}{\partial r_i^*}$, are ambiguous.[6] We hope to use our econometric model and the empirically verified results to shed some light on these ambiguous signs.

4. Data

The data used in this study are from the 1998 U.S. *Survey of Consumer Finances* (SCF). SCF is a nationwide survey conducted by the National Opinion Research Center and the U.S. Federal Reserve Board. The 1998 SCF provides a large and rich data set on household assets, liabilities, demographic characteristics, and a number of variables that capture household attitudes. In 1998, 4,305 households were surveyed and 3,233 of them had at least one bank-type credit card, which amounts to 75.1 per cent of the total number of households in the sample.

Table 1 defines the variables used in our econometric analyses. Table 2 compares the mean characteristics of consumers with credit cards against those without.

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[5] See the appendix for a discussion of the partial derivatives.

[6] The ambiguity regarding the structural parameters arises from the fact that it is impossible to theoretically identify a supply function when the price is endogenous. Identification of the structural parameters requires further restrictions, which will be addressed in more detail in section 5.

Table 1: Definition of Variables

Variables	Type	Explanation
RHI	Continuous	HELOC* interest rate (Maximum interest rate charged among the different HELOCs taken out by the household)
DELINQUENCY	Binary	1 – Got behind in payments by two months or more 0 – Otherwise
BANKRUPTCY	Binary	1 – Declared bankruptcy 0 – Otherwise
RCI	Continuous	Credit card interest rate
LOGCLIMIT	Continuous	Logarithm of credit card borrowing limit
LOGINCOME	Continuous	Logarithm of income
ALPHAI	Continuous	Fraction of HELOC and/or mortgage debt repaid
HOUSEHOLDSIZE	Continuous	Household size
AGE	Continuous	Age of the household
EMPLOYMENT1	Binary	1 – Not working 0 – Otherwise
EMPLOYMENT2	Binary	1 – Retired 0 – Otherwise
EMPLOYMENT3	Binary	1 – Working and not self-employed 0 – Otherwise
EMPLOYMENT4	Binary	1 – Working and self-employed 0 – Otherwise
LIQCONSTRAINT	Binary	1 – Did not obtain as much credit as applied for despite reapplying or did not reapply after the first refusal 0 – Otherwise
HELOC	Binary	1 – Has taken out a HELOC 0 – Otherwise
SHOPINVEST	Categorical	0 – Almost no shopping for the very best terms 1 – Moderate shopping 2 – Great deal of shopping

* HELOC = Home equity line of credit

Table 2: Credit Card Holders and Non-Holders

Variables	Credit card Holders	Credit card Non-holders
	Mean	Mean
HOUSEHOLDSIZE	2.7	2.6
DELINQUENCY	0.03	0.1
BANKRUPTCY	0.06	0.1
LOGINCOME	11.4	9.4
EMPLOYMENT1	0.06	0.3
EMPLOYMENT2	0.2	0.2
EMPLOYMENT4	0.3	0.1
SHOPINVEST	1.2	0.9
ALPHAI	0.01	0.01
AGE	50.8	47.0
LIQCONSTRAINT	0.1	0.1
HELOC	0.1	0.02
RHI	0.6	0.1
RCI	14.5	-
LOGCLIMIT	9.5	-

Note: All monetary variables are in thousand dollars.

5. The Econometric Model

Household i now represents the risk class (or borrower) i. Let L_i^* denote the profit-maximizing borrowing limit that all banks collectively extend to household i. According to our theoretical model describing the equilibrium in the credit card market, we have $L_i^* = L(r_i^*, r_F, q_i, i)$.

The variable r_F has no variation across households. Therefore, the effect of r_F on L_i^* cannot be empirically tested. Moreover, let the vector X_{1i} denote the information on household i that banks use to define the household's risk measure, i. The vector X_{1i} consists of variables included in publicly available credit reports and the variables that banks gather while processing credit card applications. Table 3 provides a complete list of variables included in an individual credit report. Hence, the vector X_{1i} consists of

Table 3: Credit Report Details

Personal information	• Name • Current and previous address • Social security number • Telephone number • Date of birth • Current and previous employers
Credit History	Type of accounts: 1. Retail credit cards 2. Bank loans 3. Finance company loans 4. Mortgages 5. Bank credit cards Information available: 1. Account number 2. Creditor's name 3. Amount borrowed 4. Amount owed 5. Credit limit 6. Dates when accounts were opened, updated, or closed 7. Timeliness of payments 8. Late payments
Public records	▪ Tax liens ▪ Bankruptcies ▪ Court judgments
Inquiries	List of all parties who have requested a copy of your credit report

Source: TransUnion

personal information, such as employment status, age, and the size of the household *i*. It also contains information on credit history variables, such as access to alternative lines of credit (e.g., HELOCs), the fraction of mortgage and/or HELOC debt repaid, timeliness of payments, bankruptcy records, and credit inquiries. The vector X_{1i} also includes information on household income, which is gathered during the application process for a credit card. We postulate a linear structural-form equation for L_i^* as

$$L_i^* = \gamma r_i^* + b_1' X_{1i} + v_{1i}. \tag{4}$$

In equation (4), the banks' opportunity cost of funds, r_F, contributes to the constant term, and the underlying wealth shock that influences a household's desired borrowing level (q_i) goes into the error term, v_{1i}.

Since the optimal credit card interest rate is given by $r_i^* = R^*(r_F, q_i, i)$, the linear reduced-form equation for r_i^* is given by

$$r_i^* = b_2' X_{2i} + v_{2i}. \tag{5}$$

Also in equation (5), the variable r_F contributes to the constant term and the underlying wealth shock (q_i) goes into the error term, v_{2i}.

The vector X_{2i} consists of all the variables present in vector X_{1i} and some identifying variables (which are influential yet absent in public credit reports) that capture aspects of a household's search behaviour, such as a household's propensity to shop around for the best rates before making major savings and investment decisions, and the interest rates on alternative lines of credit, such as HELOCs.

The combination (L_i^*, r_i^*) is observed if the banks decide to offer a credit card to household *i*; i.e., if $L_i^* > 0$. Let us therefore consider the following econometric model:

$$\left.\begin{aligned} L_i &= L_i^* = g r_i^* + b_1' X_{1i} + v_{1i} \\ r_i &= r_i^* = b_2' X_{2i} + v_{2i} \end{aligned}\right\} \quad \text{if } L_i^* > 0 \text{, and}$$

$$\left.\begin{aligned} r_i &= 0 \\ L_i &= 0 \end{aligned}\right\} \quad \text{otherwise.}$$

In the equations above, L_i and r_i represent the observed credit card borrowing limit and interest rate, respectively; X_{1i} and X_{2i} are vectors of exogenous variables; v_{1i} and v_{2i} follow bivariate normal with means zero, variances s_1^2 and s_2^2, respectively, and covariance s_{12}. If X_{2i} contains at least one variable that is not included in X_{1i}, then all the parameters of the model are identified. To identify the effect of the endogenous interest rate on the credit card borrowing limit that is offered to household *i*, we use the estimated credit card interest rate, $\hat{r}_i$, as an instrument.[7]

Maximum-likelihood estimation is used to estimate the proposed econometric model. To form the likelihood function, we have to relate the dependent variables to

[7] See Lee, Maddala, and Trost (1980), and Wales and Woodland (1980) for technical details.

their empirical counterparts, and describe the process by which the observable counterparts are generated in terms of the underlying stochastic components. Household i is observed to have a credit card if

$$L_i^* > 0.$$

Substituting the equation for the credit card interest rate into the borrowing limit equation, the decision on whether to offer a credit card can be written as

$$b_1' X_{1i} + \gamma b_2' X_{2i} > - (v_{1i} + \gamma v_{2i}),$$

or, $\quad I_i > v_i$

where $v_i \sim N(0,\ s_1^2 + \gamma^2 s_2^2 + 2\gamma s_{12}) \equiv N(0,\ s_v^2)$, and

$$I_i = b_1' X_{1i} + \gamma b_2' X_{2i}.$$

The likelihood of observing household i without a credit card is

$$\text{Prob}\,(I_i < v_i) = \int_{I_i/s_v}^{\infty} \frac{1}{\sqrt{2p}} e^{-\frac{v^2}{2}} dv = 1 - \Phi(\frac{I_i}{s_v}) = \Phi(-\frac{I_i}{s_v}),$$

where Φ is the standard normal cumulative density function.

Hence, the likelihood of observing the data consisting (say) of N households with M households without credit cards is

$$L = \prod_{i=1}^{M} \Phi(-\frac{I_i}{s_v}) \prod_{i=M+1}^{N} b(v_{1i}, v_{2i}) = \prod_{i=1}^{M} \Phi(-\frac{I_i}{s_v}) \prod_{i=M+1}^{N} n(v_{2i}) n(v_{1i} | v_{2i}),$$

where $b(.)$ is the bivariate normal density function, $n(.)$ is the normal density function, and

$$v_{1i} = L_i - b_1' X_{1i} - \gamma r_i$$

$$v_{2i} = r_i - b_2' X_{2i}.$$

Now, $\quad v_{2i} \sim N(0,\ s_2^2)$

$$v_{1i} | v_{2i} \sim N\left(\frac{r s_1 v_{2i}}{s_2}, s_1^2 (1 - r^2) \right),$$

where $r = \dfrac{s_{12}}{s_1 s_2}$.

Let $s_c^2 = s_1^2 (1 - r^2)$, and

$$T_i = \frac{v_{1i} - \dfrac{r s_1 v_{2i}}{s_2}}{s_c}.$$

The corresponding log-likelihood function can be written as

$$\log L = \sum_{i=1}^{M} \log[\Phi(-\frac{I_i}{s_v})] + \sum_{i=M+1}^{N} \frac{f\left(\dfrac{v_{2i}}{s_2}\right)}{s_2} + \sum_{i=M+1}^{N} \frac{f(T_i)}{s_c},$$

where $f(.)$ is the standard normal density function.

A multi-step procedure is used to estimate the parameters of the model. First, the parameters of the econometric model are estimated using the two-stage probit method described by Lee, Maddala, and Trost (1980). This two-step procedure yields consistent estimates of all the parameters of the model. To obtain asymptotically efficient parameter estimates, the consistent estimates are used as initial values for the final maximization of the log-likelihood function.

Let $d_i = -\frac{I_i}{s_v}$ and $l(d_i) = \frac{f(d_i)}{1 - \Phi(d_i)}$. Let us further define a dummy variable, D_i, such that,

D_i = 1 if household i has a credit card (i.e., $L_i^* > 0$)

= 0 otherwise.

For credit card holders,

$$L_i = b_1' X_{1i} + g r_i + s_v l(d_i) + v_{3i},$$

$$r_i = b_2' X_{2i} + m l(d_i) + v_{4i},$$

where $m = \frac{g s_2^2 + s_{12}}{s_v}$.

Consistent estimates of d_i can be obtained by running a probit of the decision to offer a credit card. Then, we can estimate the following equation using ordinary least squares (OLS) and obtain $\hat{r}_i$:

$$r_i = b_2' X_{2i} + m l(\hat{d}_i) + \overline{v_{4i}}.$$

We then estimate the following equation using OLS:

$$L_i = b_1' X_{1i} + g\hat{r}_i + s_v\, l(\hat{d}_i) + \overline{v_{3i}}.$$

This two-step procedure (after correcting for the standard errors) gives us consistent estimates of all the parameters of the model, which are used as starting values for the maximum-likelihood procedure.

6. Results and Discussion

Table 4 reports the results of a probit estimation that explains a bank's decision to offer a credit card to a potential borrower.[8] The household's income, age, and self-employed status significantly improve their likelihood of getting a credit card. The fact that self-employed households are more likely to receive credit cards seems counter-

Table 4: A Bank's Decision to Offer a Credit Card

Variables	Coefficient	Standard error (S.E.)
CONSTANT	-2.878***	0.183
HOUSEHOLDSIZE	-0.039**	0.017
DELINQUENCY	-0.309***	0.101
BANKRUPTCY	-0.229***	0.085
LOGINCOME	0.304***	0.016
EMPLOYMENT1	-0.614***	0.074
EMPLOYMENT2	-0.073	0.09
EMPLOYMENT4	0.469***	0.076
SHOPINVEST	0.298***	0.033
ALPHAI	0.386	0.79
AGE	0.004*	0.002
LIQCONSTRAINT	-0.31***	0.087
HELOC	0.645***	0.187
RHI	0.012	0.025

*** Significant at 1 per cent; ** significant at 5 per cent; * significant at 10 per cent.

intuitive. Self-employed households face higher variations of income and are likely to have higher default risks; therefore, banks should extend lower amounts of credit to them. The size of the household, unemployment, delinquency, and a declaration of bankruptcy

[8] Table 1 defines the variables used in all the tables of this section.

diminish the chance of obtaining a credit card. If the household is liquidity constrained, then there is a higher probability that they will be denied a credit card. The more the household looks around for the best rates, or if they have taken out a HELOC, the more likely they are to have a credit card. In general, the results indicate that the higher the household's creditworthiness, the greater their likelihood of obtaining a credit card.

Table 5: Two-Stage Probit for Interest Rates

Variables	Two-stage probit	
	Coefficient	S.E.
CONSTANT	12.655***	1.347
HOUSEHOLDSIZE	-0.038	0.057
DELINQUENCY	1.567***	0.481
BANKRUPTCY	0.591*	0.357
LOGINCOME	0.141	0.094
EMPLOYMENT1	-0.042	0.412
EMPLOYMENT2	0.211	0.288
EMPLOYMENT4	-0.08	0.208
LIQCONSTRAINT	1.361***	0.398
ALPHAI	0.2	0.32
AGE	0.007	0.007
SHOPINVEST	-0.346**	0.135
HELOC	0.445	0.387
RHI	-0.09*	0.05
LAMBDA[a]	0.749	0.655
	$\bar{R}^2 = 0.02$ F-value = 5.11*** $s_2 = 4.5$ $N = 3233$	

*** Significant at 1 per cent; ** significant at 5 per cent; * significant at 10 per cent.

[a] LAMBDA = $\frac{\hat{f}_i}{1 - \hat{\Phi}_i}$.

Table 5 reports the results for the two-stage probit regression for interest rates among credit card holders. Delinquency or a declaration of bankruptcy induces banks to charge higher credit card interest rates. If the household is liquidity constrained, then it is also likely to result in a higher credit card interest rate offered by banks. If the household shops around, then they can obtain a lower interest rate on their credit cards. The rate on an alternative line of credit, such as a HELOC, also reduces the offered credit card interest rate. Finally, the estimated value of μ is 0.749. Since this estimated value is not significantly different from zero, we conclude that there is no empirical evidence of sample selection in our estimates of the equation for the credit card interest rate. We conclude that the better the creditworthiness of the household, the lower the credit card interest rate charged by banks, which means that we have empirical support for the following: $\frac{\partial r_i^*}{\partial i} < 0$. Moreover, controlling for the credit risk of the household, the more pronounced is their search behaviour, the lower is their credit card interest rate.

Table 6 reports the two-stage probit estimates of the equation for the credit card borrowing limit for the credit card holders. The higher the income of the household, the higher the credit limit offered by banks. If the household is self-employed or has already taken out a HELOC, then again they receive a higher credit card borrowing limit from banks. The endogenous variable, the credit card interest rate, has a negative effect on banks' line of credit supply. Charging a higher credit card interest rate will raise the default probability of a borrower of any given risk type. A typical bank's optimal credit limit should, therefore, fall to compensate for this rise in default risk. Finally, the estimated value of s_v is 0.796. Since the estimated value is also significant, we conclude that there is empirical evidence of sample selection in the estimates of the equation for the credit card borrowing limit. Moreover, the estimated effect of sample selection is positive. Therefore, the higher the creditworthiness of the borrower, the higher is their likelihood of being offered a credit card and the higher is the borrowing limit extended by banks. Hence, our empirical results support the following predictions: $\frac{\partial L_i^*}{\partial r_i^*} < 0$ and $\frac{\partial L_i^*}{\partial i} > 0$.

$\frac{\partial r_i^*}{\partial i}<0$.

Table 6: Two-Stage Probit for Credit Limits

Variables	Two-stage probit	
	Coefficient	S.E.
CONSTANT	10.485***	2.15
HOUSEHOLDSIZE	-0.004	0.038
DELINQUENCY	-0.167	0.4
BANKRUPTCY	-0.319	0.23
LOGINCOME	0.359***	0.07
EMPLOYMENT1	-0.326	0.244
EMPLOYMENT2	0.063	0.177
EMPLOYMENT4	0.353***	0.122
ALPHAI	0.108	0.195
AGE	0.014***	0.005
LIQCONSTRAINT	0.101	0.334
HELOC	0.432**	0.168
RCI	-0.428**	0.194
LAMBDA	0.796*	0.466
	-LogL = -7717.693 s_1 = 2.679 s_{12} = 9.263 N = 3233	

*** Significant at 1 per cent; ** significant at 5 per cent; * significant at 10 per cent.

[a] LAMBDA = $\frac{\hat{f}_i}{1-\hat{\Phi}_i}$.

Table 7 reports the full-information maximum likelihood (FIML) estimates of the equation for the credit card interest rate. Delinquency or a declaration of bankruptcy induces banks to charge higher credit card interest rates. If the household is unemployed or liquidity constrained, then it is again likely to result in banks offering a higher credit card interest rate. The income of the household or their self-employed status depresses the offered interest rate in credit card contracts. If the household shops around for the best rates, then they can obtain a lower interest rate on credit cards. Hence, an active search for a lower rate results in a lower rate for a household of any risk type. Moreover, our maximum-likelihood estimates conform to the two-stage probit prediction of $\frac{\partial r_i^*}{\partial i}<0$.

Table 7: FIML Estimates for Interest Rates

Variables	Maximum Likelihood	
	Coefficient	S.E.
CONSTANT	20.959***	1.142
HOUSEHOLDSIZE	0.036	0.067
DELINQUENCY	2.072***	0.482
BANKRUPTCY	0.975***	0.341
LOGINCOME	-0.411***	0.082
EMPLOYMENT1	1.756***	0.426
EMPLOYMENT2	0.342	0.338
EMPLOYMENT4	-0.425*	0.231
LIQCONSTRAINT	2.079***	0.404
ALPHAI	0.09	1.344
AGE	0.003	0.008
SHOPINVEST	-0.944***	0.142
HELOC	-0.441	0.46
RHI	-0.075	0.059

*** Significant at 1 per cent; ** significant at 5 per cent; * significant at 10 per cent.

Table 8 reports the FIML estimates of the equation for the credit card borrowing limit. The higher the income or the age of the household, the higher the credit limit offered by banks. If the household is self-employed or already has taken out a HELOC, then again they receive a higher credit card borrowing limit from banks. Unemployed households, however, obtain lower credit card borrowing limits from banks. The endogenous variable, the credit card interest rate, again has a negative effect on the bank's loan supply. Hence our maximum-likelihood estimates are consistent with the two-stage probit results supporting $\frac{\partial L_i^*}{\partial r_i^*} < 0$ and $\frac{\partial L_i^*}{\partial i} > 0$.

Table 8: FIML Estimates for Credit Limits

Variables	Maximum Likelihood	
	Coefficient	S.E.
CONSTANT	16.348***	2.958
HOUSEHOLDSIZE	-0.068	0.071
DELINQUENCY	0.797	0.608
BANKRUPTCY	0.009	0.381
LOGINCOME	0.658***	0.091
EMPLOYMENT1	-1.108**	0.482
EMPLOYMENT2	0.161	0.36
EMPLOYMENT4	0.441*	0.246
ALPHAI	0.284	2.79
AGE	0.02**	0.009
LIQCONSTRAINT	0.776	0.535
HELOC	0.688*	0.375
RCI	-1.126***	0.164
s_1	5.317***	0.617
s_2	5.018***	0.158
r	0.675***	0.081
	-Log-L = -13941.71	

*** Significant at 1 per cent; ** significant at 5 per cent; * significant at 10 per cent.

7. Conclusions

Line of credit contracts (such as credit card contracts) are fundamentally different from traditional fixed-loan contracts. An understanding of the key elements of credit card contracts requires a theoretical separation of the choice of the amount of borrowing and the choice of the amount of credit limit, the two-dimensional nature of the contract and the market structure under which the borrowers and lenders operate. We have shown that the difference between actual borrowings and offered credit limits is enough to generate information asymmetry in the credit card market. Moreover, individuals who are rationed out of the credit card market could very well turn out to have been ex post less risky than some credit card holders who borrow large fractions of their credit limits. Therefore, not only can information asymmetry in the credit card market make the market incomplete (through credit rationing), but it can also result in ex post misallocations.

We have theoretically identified the crucial features of credit card contracts offered by banks, and examined the association between borrower quality and the offered menu of credit card borrowing limit and interest rate. We have also been able to capture the effect of endogenous interest rates on the offered borrowing limits of households.

Our results support the fact that banks use publicly available information on potential borrowers to assess their credit risk and to formulate the type of credit card contracts to offer. The credit card market shows clear evidence of credit rationing. Banks refuse lowest-quality borrowers access to credit cards. Among the credit card holders, those with "better" credit reports are perceived to have higher repayment probabilities and therefore are provided with higher credit card borrowing limits and lower interest rates. Controlling for the borrower's risk type, we find that a greater search for the best rates on loans significantly reduces the interest rate that they are charged on credit cards. We also have found empirical support for a negative relationship between the credit card borrowing limit (loan supply) and the credit card interest rate (the price of the loan). A higher interest rate will raise a borrower's default probability, regardless of their risk type; therefore, the optimal borrowing limit should fall to compensate for this rise in default risk.

Several empirical studies have shown that a household's actual amount of borrowing on credit cards (or their credit card utilization rate) does affect the credit card borrowing limit that banks offer them. For instance, the credit card borrowing limit offered to a "convenience user" of a given risk class is typically higher than that offered to a household in the same risk class and yet already borrowing up to the credit limit. As an extension to this paper, one could postulate a dynamic theoretical model where banks update their ex ante repayment probabilities and their credit card contracts after observing the actual amount borrowed on credit cards. The extended econometric model should have, among other things, the credit card interest rate and the observed credit card borrowing as endogenous variables.[9]

[9] Such a model may be able to explain why we found self-employed households receiving higher credit card borrowing limits and lower credit card interest rates. Self-employed households are primarily "convenience users" because they use credit cards not for borrowing purposes, but to minimize the liquidity risks of their day-to-day business operations.

References

Ausubel, L.M. 1991. “The Failure of Competition in the Credit Card Market.” *American Economic Review* 81(1): 50–81.

Brito, D.L. and P.L. Hartley. 1995. “Consumer Rationality and Credit Cards.” *Journal of Political Economy* 103(2): 400–33.

Calem, P.S. and L.J. Mester. 1995. “Consumer Behavior and Stickiness of Credit Card Interest Rates.” *American Economic Review* 85(5): 1327–36.

Cargill, T.F. and J. Wendel. 1996. “Bank Credit Cards: Consumer Irrationality versus Market Forces.” *Journal of Consumer Affairs* 30(2): 373–89.

Carroll, C. 1992. “The Buffer-Stock Theory of Saving: Some Macroeconomic Evidence.” *Brookings Papers on Economic Activity* 2: 61–156.

Castronova, E. and P. Hagstrom. 2004. “The Demand for Credit Cards: Evidence from the Survey of Consumer Finances.” *Economic Inquiry* 42(2): 304–18.

Deaton, A. 1991. “Saving and Liquidity Constraints.” *Econometrica* 59(5):1221–48.

Dey, S. 2004. “Line of Credit as Instruments of Consumption Smoothing across State and Time: The Choice between Credit Cards and Home Equity Lines of Credit.” The Ohio State University, Columbus, OH. Unpublished paper.

Dunn, L.F. and T.H. Kim. 2002. “An Empirical Investigation of Credit Card Default: Ponzi Schemes and Other Behaviors.” The Ohio State University, Columbus, OH. Unpublished paper.

Gross, D. and N.S. Souleles. 2002. “Do Liquidity Constraints and Interest Rates Matter for Consumer Behavior? Evidence from Credit Card Data.” *Quarterly Journal of Economics* 117(1): 149–85.

Kerr, S. 2002. “Interest Rate Dispersion Due to Information Asymmetry in the Credit Card Market: An Empirical Study.” The Ohio State University, Columbus, OH. Unpublished paper.

Lee, L.F., G.S. Maddala, and R.P. Trost. 1980. “Asymptotic Covariance Matrices of Two-Stage Probit and Two-Stage Tobit Methods for Simultaneous Equations Models with Selectivity.” *Econometrica* 48(2):491–503.

Ludvigson, S. 1999. “Consumption and Credit: A Model of Time-Varying Liquidity Constraints.” *Review of Economics and Statistics* 81(3): 434–47.

Mester, L.J. 1994. “Why Are Credit Card Rates Sticky?” *Economic Theory* 4(4): 505–30.

Park, S. 1997. “Option Value of Credit Lines as an Explanation of High Credit Card Rates.” Federal Reserve Bank of New York Research Paper No. 9702.

Wales, T.J. and A.D. Woodland. 1980. “Sample Selectivity and the Estimation of Labor Supply Functions.” *International Economic Review* 21(2): 437–68.

APPENDIX

The optimal line of credit contract offered to risk class i can be represented by the following triangular structure:

$$L_i^* = L(r_i^*, r_F, q_i, i), \tag{A1}$$

$$r_i^* = R^*(r_F, q_i, i). \tag{A2}$$

Substituting equation (A2) into equation (A1), we obtain the following reduced-form representation of the optimal credit card borrowing limit:

$$L_i^{R*} = L^R(r_F, q_i, i). \tag{A3}$$

Using equations (A2) and (A3), we can write the following:

$$p^i_{L_i}(L_i^{R*}, r_i^*; r_F, q_i, i) = 0, \tag{A4}$$

$$p^i(L_i^{R*}, r_i^*; r_F, q_i, i) = 0. \tag{A5}$$

Partially differentiating equation (A4) with respect to i, we have

$$p^i_{L_iL_i}(L_i^{R*}, r_i^*; r_F, q_i, i)\frac{\partial L_i^{R*}}{\partial i} + p^i_{L_ir_i}(L_i^{R*}, r_i^*; r_F, q_i, i)\frac{\partial r_i^*}{\partial i} +$$

$$p^i_{L_ii}(L_i^{R*}, r_i^*; r_F, q_i, i) = 0. \tag{A6}$$

Partially differentiating equation (A5) with respect to i, we have

$$p^i_{L_i}(L_i^{R*}, r_i^*; r_F, i, q_i)\frac{\partial L_i^{R*}}{\partial i} + p^i_{r_i}(L_i^{R*}, r_i^*; r_F, i, q_i)\frac{\partial r_i^*}{\partial i} +$$

$$p^i_i(L_i^{R*}, r_i^*; r_F, i, q_i) = 0.$$

Using the first-order condition of a bank's profit-maximization problem, we can write

$$p^i_{r_i}(L_i^{R*}, r_i^*; r_F, i, q_i)\frac{\partial r_i^*}{\partial i} + p^i_i(L_i^{R*}, r_i^*; r_F, i, q_i) = 0. \tag{A7}$$

Solving equations (A6) and (A7), and suppressing the arguments, we obtain

$$\frac{\partial L_i^{R*}}{\partial i} = -\frac{\begin{vmatrix} p^i_{L_ii} & p^i_{L_ir_i} \\ p^i_i & p^i_{r_i} \end{vmatrix}}{\begin{vmatrix} p^i_{L_iL_i} & p^i_{L_ir_i} \\ 0 & p^i_{r_i} \end{vmatrix}} = \frac{C}{D}, \tag{A8}$$

$$\frac{\partial r_i^*}{\partial i} = -\frac{\begin{vmatrix} p^i_{L_iL_i} & p^i_{L_i i} \\ 0 & p^i_i \end{vmatrix}}{\begin{vmatrix} p^i_{L_iL_i} & p^i_{L_ir_i} \\ 0 & p^i_{r_i} \end{vmatrix}} = \frac{E}{D}. \tag{A9}$$

Similarly, we can solve for

$$\frac{\partial L_i^{R*}}{\partial r_F} = -\frac{\begin{vmatrix} p^i_{L_ir_F} & p^i_{L_ir_i} \\ p^i_{r_F} & p^i_{r_i} \end{vmatrix}}{\begin{vmatrix} p^i_{L_iL_i} & p^i_{L_ir_i} \\ 0 & p^i_{r_i} \end{vmatrix}} = \frac{F}{D}, \tag{A10}$$

$$\frac{\partial r_i^*}{\partial r_F} = -\frac{\begin{vmatrix} p^i_{L_iL_i} & p^i_{L_ir_F} \\ 0 & p^i_{r_F} \end{vmatrix}}{\begin{vmatrix} p^i_{L_iL_i} & p^i_{L_ir_i} \\ 0 & p^i_{r_i} \end{vmatrix}} = \frac{G}{D}. \tag{A11}$$

Moreover, we know that

$$L_i^{R*} = L(R(r_F, q_i, i), r_F, q_i, i).$$

Hence, we have

$$\frac{\partial L_i^{R*}}{\partial i} = \frac{\partial L_i^*}{\partial r_i^*}\frac{\partial r_i^*}{\partial i} + \frac{\partial L_i^*}{\partial i}, \tag{A12}$$

$$\frac{\partial L_i^{R*}}{\partial r_F} = \frac{\partial L_i^*}{\partial r_i^*}\frac{\partial r_i^*}{\partial r_F} + \frac{\partial L_i^*}{\partial r_F}. \tag{A13}$$

Therefore, the theoretically predicted signs of the partial derivatives derived above, and those of $\frac{\partial L_i^*}{\partial r_i^*}$, $\frac{\partial L_i^*}{\partial i}$, and $\frac{\partial L^*}{\partial r_F}$, will depend on the signs of the determinants *C*, *E*, *F*, *G* and *D*.

We know that the repayment probability for borrower *i*, $r_i = r(D_i, i)$, satisfies $\frac{\partial r(.)}{\partial D_i} < 0$, $\frac{\partial r(.)}{\partial i} > 0$, and $r_i \in [0,1]$. Therefore, we have $p^i_i > 0$. The bank's profit function also makes $p^i_{r_F} < 0$ and $p^i_{L_ir_F} < 0$. Let us assume that $p^i_{r_i} > 0$, $p^i_{L_i i} > 0$, and $p^i_{L_ir_i} < 0$.

Given our assumptions and the second-order condition ($p^{i}_{L_i L_i} < 0$), we have $\frac{\partial r_i^*}{\partial i} < 0$, $\frac{\partial r_i^*}{\partial r_F} > 0$, $\frac{\partial L_i^{R*}}{\partial i} > 0$, and $\frac{\partial L_i^{R*}}{\partial r_F} < 0$. From equations (A12) and (A13), we conclude that the signs of $\frac{\partial L_i^*}{\partial i}$, $\frac{\partial L_i^*}{\partial r_F}$, and $\frac{\partial L_i^*}{\partial r_i^*}$ are ambiguous.

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