Sectoral Default Rates under Stress: The Importance of Non-Linearities

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The purpose of aggregate-level stress testing is to identify the circumstances that could impair the functioning of the financial system and have economy-wide (systemic) implications. In models typically used for stress tests of aggregate credit risk, macroeconomic shocks are assumed to affect financial institutions via their impact on either individual or industry-level default probabilities.[1] Therefore, sound modelling of the relationship between macroeconomic variables and defaults is of considerable importance.

In this report, we examine how the functional form used in the specification of default regressions affects the nature of the responses of default probabilities under stress. In particular, we argue that the assumption of a linear relationship imposes severe restrictions on the responses of default probabilities to macroeconomic shocks. These restrictions are particularly undesirable in stress-testing exercises. To remedy this problem, we introduce non-linearities in a simple, but effective, way and illustrate their impact on responses with a series of examples.

We begin with a general discussion of the nature of the restrictions that linearity implies and their undesirability in the context of stress testing. This is followed by an empirical exercise in which we compare the performance of linear and non-linear models by varying the severity of a recession and the initial state of the economy. In the concluding section, we draw broader implications of our results for stress testing.

1. See, for example, Jiménez and Mencía (2007), Virolainen (2004), or Wilson (1997). Misina, Tessier, and Dey (2006), summarized in this *Review*, provides a general description of the structure of these models.

The Importance of Taking Non-Linearities into Account

Let π denote the default probability and x a set of explanatory variables. The relationship between π and x can be expressed as

$$\pi = f(x).$$

Specifying f as a linear function is a simple solution but has a number of undesirable consequences. To see this, consider the following example in which $\pi = ax$. The impact of changes in x is given by

$$\frac{d\pi}{dx} = a.$$

This simple expression makes it clear that the restrictions that linear models impose on responses are rather severe and have the following properties.

- *Symmetry:* the magnitude of the response is the same, regardless of whether the shock is positive or negative.
- *Proportionality:* the response is proportional to the change in the exogenous variable.
- *History independence:* the response is independent of initial conditions (x).

None of these restrictions is appealing in the context of stress-testing exercises, where asymmetry, non-proportionality, and history dependence would seem to be desirable properties. For example, one would expect a negative shock to have a different impact on companies, depending on whether the economy was in recession or in an expansionary phase.

Stress tests generally select scenarios that are severe but plausible, with the result that experimental shocks are usually quite large. With shocks of such magnitude, linear approximations to a possibly non-linear process might prove to be particularly poor.

To develop response profiles with features more suitable for stress testing, the assumption of linearity has to be relaxed. This can be done by introducing higher-order terms, while preserving additivity. The following non-linear specification,

$$\pi = ax + bx^2 + cx^3,$$

delivers the response function

$$\frac{d\pi}{dx} = a + 2bx + 3cx^2,$$

which generates asymmetric, non-proportional, and history-dependent responses. This type of response function implies that the impact of shocks would differ in good and bad economic states, both qualitatively and quantitatively.

Examples

The examples in this section build on the linear specification of default-probability regressions in Misina, Tessier, and Dey (2006). In that paper, regressions on sectoral default probability take the form

$$\ln\left(\frac{\pi_t}{1-\pi_t}\right) = \mu + \sum_{l=1}^{L} \beta_l X_{t-l} + e_t.$$

The explanatory variables are Canadian macroeconomic variables (real GDP and real interest rates) and their lags. One way to introduce non-linearities is to retain additivity but include higher-order terms:

$$\ln\left(\frac{\pi_t}{1-\pi_t}\right) = \mu + \sum_{l=1}^{L} \beta_l^{(1)} X_{t-l} + \sum_{l=1}^{L} \beta_l^{(2)} X^2_{t-l} + \sum_{l=1}^{L} \beta_l^{(3)} X^3_{t-l} + e_t$$

The key advantages of introducing non-linearities in this manner are simplicity and flexibility. The addition of other variables and higher-order terms does not present difficulties, since the relationship of the parameters remains linear.

The data used to estimate these regressions are the growth rate of real Canadian GDP, the real interest rate on medium-term business loans,[2] and sectoral default rates as proxies for sectoral default probabilities. The data cover the period 1987Q1 to 2005Q4. Details on constructing sectoral default rates are given in Box 1.

To examine the impact of introducing non-linearities, we focus on the behaviour of predicted sectoral default rates following the Canadian recession of the early 1990s, which peaked between 1990Q4 and 1991Q3. The forecasts are given for the period starting in 1991Q4.[3] Chart 1 contains the paths of historical and predicted default rates, where the latter are estimated using linear and non-linear models.[4] The benefit of non-linearities is particularly evident in this stressful period, when the default rate reached its historical peak. As is clear from the chart, the non-linear model captures the actual default rate over this period much better than the linear model. As the impact of the recession diminishes, the paths developed under these two specifications tend to converge.

To get a better sense of the limitations of the linear model, we perform two sets of experiments: (i) a change in the severity of the recession; and (ii) a change in the initial conditions. The experiments are performed by exogenously changing Canadian GDP over the period 1990Q4 to 1991Q3, and deriving the implications for the GDP and interest rate in the subsequent period using a two-variable vector-autoregression model.[5]

2. The real medium-term rate is equal to the nominal rate minus inflation expectations, where the latter was calculated as a geometric mean of the five-years-ahead realized inflation rate.
3. Our specification includes four lags, which fully take into account the period 1990Q4 to 1991Q3.
4. In this report, we show the results for the manufacturing sector only. The results for other sectors (accommodation, construction, retail) are qualitatively similar.
5. We applied the method proposed in Jordà (2005), which uses a set of sequential regressions of the endogenous variable shifted several periods ahead.

Box 1

Constructing a Proxy for Sectoral Default Rates

Default probabilities are a key input in any model of credit risk. To arrive at reliable estimates of the relationship between the macroeconomic variables and defaults, a long series of data on historical defaults is required. Although some data are available for large publicly traded companies, a long series with broad coverage is not available for Canada. This box describes the construction of such a data set and the issues involved in this process.

Misina, Tessier, and Dey (2006) used bankruptcy rates (the ratio of bankruptcies in a sector to the total number of establishments in that sector) as a proxy for sectoral default probabilities. Data were obtained from the Office of the Superintendent of Bankruptcy (numerator) and Statistics Canada (denominator).

There are two issues with this choice. First, bankruptcy is not a good proxy for the events that affect banks and their economic capital. Bankruptcy is the last stage of a company's distress. Prior to that, a company would typically go through two stages (missed interest payments, distressed exchange),[1] both of which result in losses to the lender. To capture all these credit events, rating agencies use a broad category of default that includes anything from missed payments to bankruptcy. Use of the number of bankruptcies will lead to an underestimation of the number of credit events that affect the credit risk of banks.

Second, the total number of establishments in a sector does not accurately reflect banks' lending practices. Only the establishments that borrow from the banks are relevant. Use of the total number of establishments will, again, underestimate the number of credit events that have an impact on the credit risk of banks.[2]

To deal with these issues, we start with the data on bankruptcy rates and construct proxies that better reflect credit events that affect banks.

The adjustment was based on the following considerations:

- Reported data on default events from Moody's for the period 1989 to 2005 indicate that bankruptcies account for roughly one-third of default events.[3]
- Statistics Canada's (2004) "Survey of Financing of Small and Medium Enterprises" (SMEs) indicates that small and medium-sized enterprises account for 99.7 per cent of business establishments in Canada.[4]
- Statistics Canada's (2005) "Survey of Suppliers of Business Financing" offers an exceptionally detailed picture of banks' lending activities to small and medium-sized enterprises in Canada, which includes information on debt financing by authorization size of client businesses (Section B2), as well as debt losses by authorization size of client businesses (Section B6), for the years 2000–05. This information can be used to construct historical default rates for that period.[5]

1. This refers to a situation in which the issuer offers bondholders a new security or a package of securities that amount to a diminished financial obligation, with the purpose of helping the borrower avoid default.

2. In addition, the number of establishments overestimates the number of companies in a sector. Given that bankruptcies are reported at a company level, use of the number of establishments in the denominator will lead to a further underestimation of the bankruptcy rate.
3. "Default and recovery rates of Canadian corporate bond issuers, 1989–2005" (April 2006). Moody's provides the data on default rates as well, but the rates are computed relative to the number of companies they cover. That number is quite small, especially for the period prior to the mid-1990s, resulting in large fluctuations in default rates driven by a very small number of default events.
4. http://strategis.ic.gc.ca/epic/site/sbrp-rppe.nsf/en/rd00999e.html, Table 2.
5. Data prior to 2000 do not exist, since the first survey was conducted in that year. (http://sme-fdi.ic.gc.ca/epic/site/sme_fdi-prf_pme.nsf/vwapj/SurveyofSuppliersTables_Eng.pdf/$FILE/SurveyofSuppliersTables_Eng.pdf)

Box 1

Constructing a Proxy for Sectoral Default Rates (cont'd)

The adjustment process, then, consists of two steps:

- First, we use the information from Moody's to convert bankruptcies into defaults.[6] The adjustment for each year is done separately by scaling up the bankruptcy rate for that year by the ratio of defaults to bankruptcies for that year, to take into account the difference in dynamics between bankruptcies and defaults.[7]
- We then compare the adjusted series with the observed default rates in 2000–05, and make additional adjustments, as necessary. These adjustments involve scaling the whole series up or down to match the survey data as closely as possible.

Charts A and B contain the adjusted series, and Chart C compares the adjusted rates with the historical default rates for 2000–05. The match over the past five years is quite close, both in year-to-year and average comparisons. Nonetheless, it should be kept in mind that the variable adjustment is based on a small sample of bankruptcies and defaults documented by Moody's.

Chart A Constructed Proxies for Sectoral Default Rates

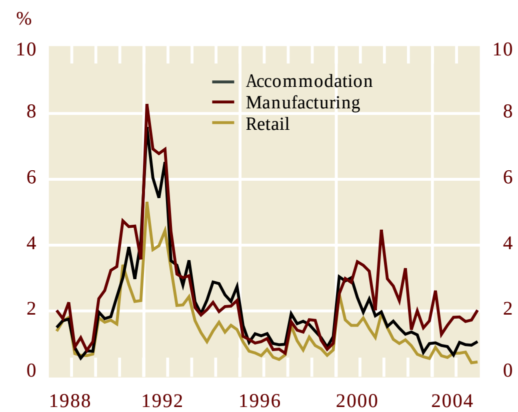


Chart B Constructed Proxies for Sectoral Default Rates

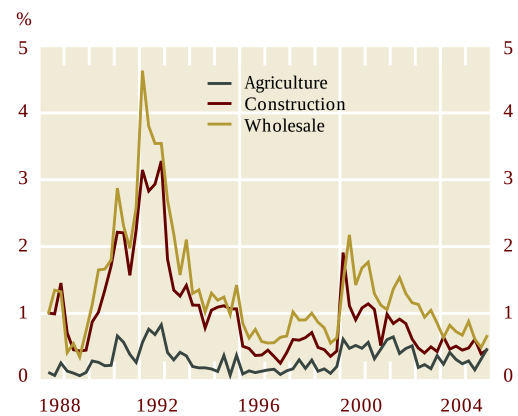
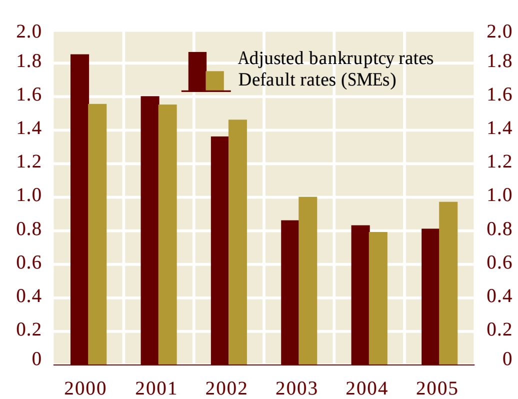


Chart C Comparison of Average Default Rates



6. Given that the Moody's data cover mostly large publicly traded companies, the relationship between bankruptcies and defaults in Moody's data set may not be representative of that relationship more generally. One can argue, however, that the second step of the adjustment process corrects for any biases that might be present here.
7. The difference in dynamics is due to the fact that credit events, such as missed interest payments, are much more sensitive to changes in business conditions than bankruptcies, which represent the last stage of distress and typically occur with a lag.

Chart 1 Historical and Predicted Default Rates: Manufacturing Sector

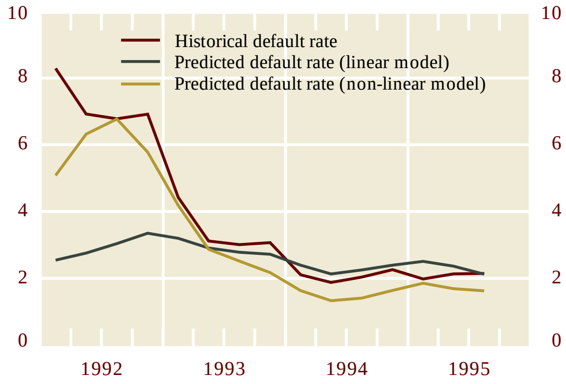


Chart 2 Impact of a Change in the Severity of Recession on Default Rate: Linear Model

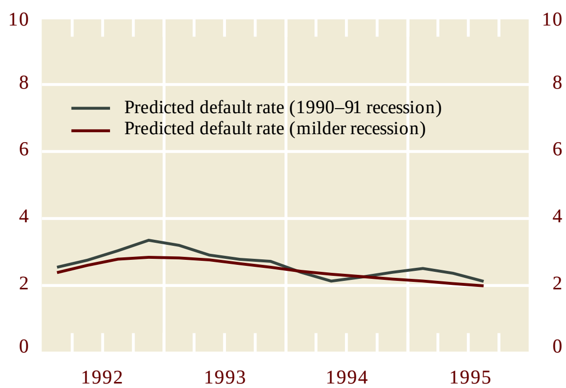
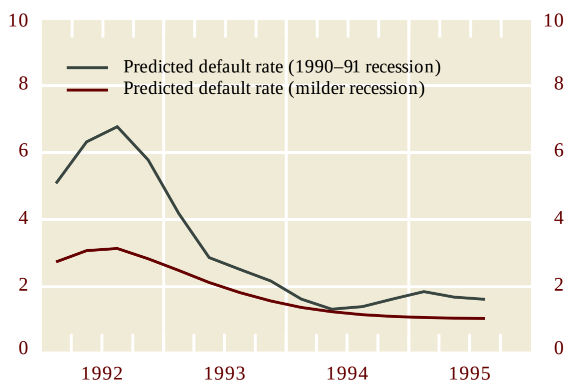


Chart 3 Impact of a Change in the Severity of Recession on Default Rate: Non-Linear Model



Change in the severity of recession

In this experiment, we assume that the recession is very mild (10 per cent of the 1990–91 recession). This is done by multiplying the observations of GDP in the 1990Q4–1991Q3 period by 0.1. All else being the same, this should result in a significant decrease in default rates predicted by the model.

Charts 2 and 3 contain the results for linear and non-linear models, respectively. In both charts, we compare the default rate paths predicted under the 1990–91 recession to the paths predicted under our much milder hypothetical recession. The non-linear model is clearly more responsive than the linear one, and the difference is more significant the larger the shock. The key reason is that the non-linear model is not bound by the assumption of proportionality, and therefore the shocks are magnified. This is not the case with the linear model.

Change in the initial conditions

In this experiment, we change the conditions prior to the recession by converting them from unfavourable (approximately zero per cent GDP growth) to favourable (3 per cent GDP growth). The latter is similar to the conditions in Canada over the past few years. One would expect that, starting from these more favourable conditions, a decline in GDP of the magnitude observed in 1991 would have a much smaller impact than was the case at that time, since favourable economic conditions put companies in a better position to absorb shocks.

Charts 4 and 5 contain the results for linear and non-linear models. In both cases, there is a decline in default rates relative to the original setting, but it is much more significant in the case of the non-linear model. Indeed, this model now predicts only a slight change in default rates, while the responses in the linear model are limited to an approximately parallel shift down.[6] This example highlights the invariance of the shape of the response in the linear specification to changes in initial conditions.

6. The shift would be exactly parallel if the changes in both explanatory variables were fixed exogenously. In our model, the interest rate is determined endogenously.

One implication of this result is that if the initial conditions are favourable, a much larger decline in GDP would be needed to induce a response in the default rates comparable to that observed in the 1991 recession.

Conclusions

The findings described here raise questions about the suitability of linear models for stress testing. The net result of the limited ability to generate plausible behaviour around extreme events, together with a limited responsiveness to initial conditions, is that these models tend to underestimate the impact of shocks during bad times, and fail to take into account the fact that favourable initial conditions put the economy in a relatively better position to withstand shocks of a given magnitude. Our solution to this problem is to relax the assumption of linearity and replace it with a more plausible alternative.

Of course, the importance of non-linearities will depend on the nature of the sample and the incidence of stressful episodes. Even when there is only one stressful episode in the sample, the non-linear terms may capture it well, but the robustness of the specification might be an issue. To fully assess the extent of the problem, if any, a sample with more than one stressful episode is needed.

References

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Chart 4 Impact of a Change in Initial Conditions on Default Rate: Linear Model

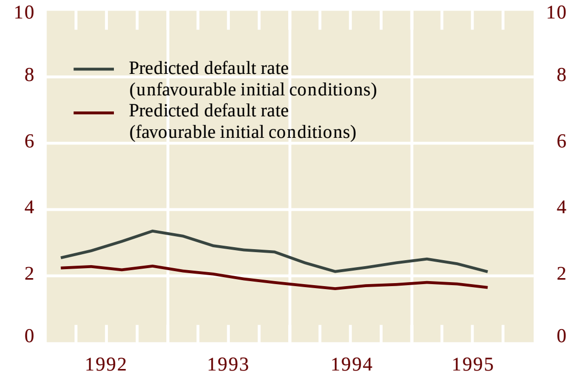

Chart 5 Impact of a Change in Initial Conditions on Default Rate: Non-Linear Model