Monetary Policy Rules in an Uncertain Environment

Gino Cateau and Stephen Murchison, Canadian Economic Analysis Department

- *Central banks have increasingly focused on a systematic approach to monetary policy. Simple monetary policy rules help to facilitate the communication of monetary policy to the public and enhance its predictability.*

- *Monetary policy rules have become an integral part of central bank models and are often fine-tuned to maximize economic welfare. However, uncertainty about the "true" model can seriously affect the performance of these rules and should therefore be accounted for when designing robust rules.*

- *Simple policy rules can often provide a good approximation to fully optimal policy under perfect information and are typically more robust to uncertainty.*

- *In ToTEM, an optimized simple rule that responds to a forecast of the price level is more robust to parameter uncertainty than a rule that responds to inflation.*

Monetary policy is most effective when the central bank's objectives, and the means of achieving those objectives, are well understood and regarded as credible by the public. This requires that the central bank communicate clearly what it seeks to achieve, such as inflation control over the medium term, and how its current and future actions can be expected to bring about the desired outcome(s). Since the collection and processing of information is costly for private agents, it is in the central bank's own best interest to respond to economic developments in a predictable fashion that is easy to communicate. Not only does this facilitate a better understanding of current policy actions, but it permits markets to better forecast the central bank's future actions.

Beginning with the seminal work of Taylor (1993), academic researchers and central banks have increasingly focused on the benefits of a systematic approach to the design of monetary policy. Monetary policy rules, or reaction functions, have become an integral part of central bank models and are often fine-tuned to maximize economic welfare. However, such fine tuning is inherently risky when the central bank has an imperfect understanding of how the economy functions.

This article discusses recent research on the influence of various forms of economic uncertainty on the performance of different classes of monetary policy rules: from simple rules to fully optimal monetary policy under commitment. Building on the research discussed in the Summer 2002 issue of the *Bank of Canada Review*, we explain why uncertainty matters for policy-rule design and provide quantitative examples from the recent literature, which has increasingly focused on structural models that feature rational expectations. We also present results for several policy rules in ToTEM, the Bank of Canada's

main projection and policy analysis model (Murchison and Rennison 2006), including rules that respond to the price level, rather than to inflation.

The article begins with a brief discussion of the theoretical arguments in favour of commitment to a policy rule and the role played by such rules in the design of real-world monetary policy. It then discuss the four major forms of uncertainty with which central banks must contend when formulating policy and how each type affects the performance of various rules. It concludes with a brief review of strategies for designing so-called robust rules: i.e., rules that perform well across a broad range of economic models.

What Is a Monetary Policy Rule?

For our purposes, a policy rule can be thought of as a mathematical equation that determines the appropriate level for the central bank's policy instrument as a function of one or more economic variables that describe the state of the economy.[1] Given that such rules are specified in terms of the policy instrument, they are often called instrument rules. An essential feature of such a rule is that while the policy interest rate varies through time in response to economic developments, its response to a given shock or state of the economy does not. Therefore, adherence to a rule is synonymous with predictability, and thus private agents in the economy understand how policy will respond now and in the future.

One might question why a central bank would adhere to a single rule, since doing so might constrain it in unfavourable ways. Even if the central banks' objectives do not vary through time, it may wish to maintain a high level of discretion in how it responds to the economy. The simple answer is that no central bank literally sets policy based on a single rule. For various reasons beyond the scope of this article, central banks do exercise a certain degree of judgment or discretion when setting policy. But this does not render the discussion of policy rules academic. What matters is that monetary policy is predictable from the viewpoint of private agents, whose decisions are influenced by current and future policy actions. From this perspective, the central bank's strict commitment

to a published rule can be seen as one extreme, whereas choosing policy at each point in time in a purely discretionary fashion can be seen as the opposite extreme.

Adherence to a rule is synonymous with predictability, and thus private agents in the economy understand how policy will respond now and in the future.

Recent empirical research generally supports the idea that monetary policy in many industrialized countries does contain a large systematic component. For instance, much of the interest in the so-called Taylor rule (Taylor 1993) is based on the observation that it predicts the actual behaviour of the federal funds rate in the United States over the period 1987–92 reasonably accurately. Thus, while no central bank literally follows a rule, their actual behaviour may be well approximated by such a rule. This is likely due, at least in part, to the fact that modern central bank projection models feature policy rules and that these models are used to provide policy advice.

So why do central banks behave in a manner broadly consistent with adherence to a rule? One key benefit is predictability. Monetary policy is most effective when households and firms understand both the objectives of monetary policy and how the central bank goes about achieving those objectives. By explicitly or implicitly committing to a certain pattern of behaviour, a central bank can influence private sector expectations of the future path of the policy rate, which, in turn, can help the central bank achieve its objectives. For instance, suppose a central bank has earned a reputation for responding aggressively to inflation whenever it strays from the target. Then, when an unanticipated shock causes inflation to deviate from the target, the deviation will be perceived as short lived. As a result, agents' expectations of future inflation will not respond to the shock, which, in turn, will dampen the current inflation response. In this way, a credible commitment to respond aggressively to shocks that affect inflation, combined with private sector expectations that factor in that commitment, can attenuate the required policy response.

---

[1] This is a somewhat narrow definition. In the economics literature, a rule can either describe how the policy instrument reacts to the state of the economy, or it can prescribe a particular economic outcome, such as the achievement of the central bank's inflation target—hence the label "targeting rules," (Svensson 1999). In the latter case, the behaviour of the policy instrument can be inferred only in the context of a full model that links the policy instrument to the targeting variables included in the rule.

Types of Rules

Since the general consensus among central bankers is that the long-run objective of monetary policy should be price stability, a natural starting point would be to design a rule that ensures long-run price stability. For example, the Bank of Canada aims to maintain the growth rate of the consumer price index (CPI) at the 2 per cent midpoint of a 1 to 3 per cent control range. According to the conventional view of the monetary policy transmission mechanism, inflation tends to decline when interest rates are high, other things being equal, and increase when interest rates are low. Therefore, an appropriate rule would stipulate that the Bank raise the target overnight interest rate[2] when current CPI inflation exceeds 2 per cent and lower it when inflation is below 2 per cent.

Restricting one's focus to the long-run objective of price stability represents an overly narrow view of the role of monetary policy. It is generally acknowledged that monetary policy can focus on, although not necessarily fully achieve, multiple *short-run* objectives. For instance, a central bank may care about stabilizing both inflation around the target and real GDP around potential GDP. To the extent that certain shocks push inflation and the output gap in opposite directions, a short-run trade-off exists, which will be reflected by the inclusion of both inflation and the output gap in the policy rule.

Perhaps the best-known policy rule is the Taylor rule (Taylor 1993), which was estimated using U.S. data and is given by:

$$R_t = 4.0 + 1.5(\pi_t - 2) + 0.5\tilde{y}_t, \qquad (1)$$

where R_t is the U.S. federal funds rate, π_t is the rate of price inflation, and $\tilde{y}_t$ is the output gap, all in period t. According to the Taylor rule, when inflation equals 2 per cent and output equals potential output, the federal funds rate should be set equal to 4 per cent—400 basis points (bps). Moreover, that rate should be adjusted by 150 bps up or down for every 1-percentage-point difference between actual inflation and the desired level of 2 per cent, and 50 bps for every 1-percent difference between output and potential output. The Taylor rule's greatest virtue may be its simplicity,

since the policy rate in a given period can be described in terms of just two economic variables.[3]

The Taylor rule is a special case of a broader class of so-called simple rules. There are important extensions to this basic set-up that include (a) lagged interest rates as an additional argument in the rule, and (b) replacing current inflation by a forecast of future inflation. A lag of the interest rate was initially added because it resulted in a better fit of the data (Clarida, Galí, and Gertler 2000), and it suggests that, in response to a change in economic conditions, central banks adjust the policy rate gradually over several months, rather than all at once, as suggested by the Taylor rule. Woodford (1999) has argued that interest rate smoothing or *inertia* is actually consistent with optimal central bank behaviour when economic agents form their expectations in a forward-looking manner. As the relative weight on the lagged interest rate increases, the future value of the policy rate becomes easier to predict, since it is determined to a greater extent by the current rate.

Responding to a forecast of future, rather than current, inflation is also consistent with optimal behaviour if monetary policy exerts its maximum effect on inflation with a lag and if the central bank is good at forecasting inflation. The policy rule currently used in ToTEM includes a role for both the lagged policy interest rate and a forecast of future inflation, and is described by the equation:

$$R_t = \rho R_{t-1} + (1 - \rho)\big[R^* + \varphi_\pi(E_t\pi_{t+k} - \pi^T) + \varphi_y(\tilde{y}_t)\big], \qquad (2)$$

where R_t is the target overnight interest rate in period t, R^* is the long-run, neutral rate of interest, $E_t\pi_{t+k}$ is the period t expectation of inflation in period $t + k$, and $\tilde{y}_t$ is the output gap. ρ, φ_π, and φ_y are fixed parameters that determine the degree of interest rate smoothing and the sensitivity of the policy rate to deviations of inflation from target and to the output gap, respectively.[4] Note that k determines the degree to which policy is forward looking and is referred to as the "feedback horizon."

The rules discussed so far summarize the behaviour of monetary policy in terms of just a few economic variables, such as expected inflation and the output

---

[3] Potential output was proxied by a simple linear trend of log GDP in Taylor's specification, which is straightforward to calculate.

[4] In the current version of ToTEM used for projections, the optimized parameter values are $\rho = 0.95$, $\varphi_\pi = 20$, $\varphi_y = 0.35$, $k = 2$, and $R^* = 4.75$ per cent.

[2] The target for the overnight interest rate is the conventional policy instrument in Canada.

gap. Explaining the movements in the policy rate from one period to the next is, therefore, straightforward. But this simplicity typically comes at the price of reduced performance in terms of economic stabilization. To see why, consider first that the forecast of inflation depicted in equation (2) will depend on every variable in the economic model, and in a fully articulated model, such as ToTEM, the number of economic variables can be considerable. Implicitly, the strength of the central bank's response to each of these variables is governed by a single parameter: φ_π in equation (2).

But suppose that instead of forcing monetary policy to respond to forecast inflation, we allocate a separate response parameter for each variable that influences future inflation, including the exogenous shocks that hit the economy. Such a set-up describes the essential features of fully optimal monetary policy under commitment. Such a rule will better stabilize the economy if the central bank's model is correct and if the data used in the model are well measured. But as we discuss in the next section, such a rule may perform quite poorly if one or both of these assumptions turns out to be false.

Types of Uncertainty Faced by Central Banks

In this section we discuss the four main types of economic uncertainty facing policy-makers and how each affects the performance of different policy rules.

Shock uncertainty

In practice, a monetary policy rule represents one equation in a central bank's model of the economy. At a minimum, the model will also include equations governing the behaviour of the variables that enter the policy rule, such as inflation and the output gap. Taken together, these equations form a self-contained system that can be simulated through time to generate a path for the policy interest rate that is consistent with the outlook for inflation, and vice versa.

Economic models, however sophisticated, are by construction simple caricatures of the true economy (Coletti and Murchison 2002). They are intended to capture those linkages between households, firms, governments, and the central bank believed to be the most important, on average. Nevertheless, the deliberate omission of many idiosyncratic factors means that models will make prediction errors, which are referred to as shocks, and the associated uncer-

tainty is referred to as *shock uncertainty*. To understand the impact of shock uncertainty on the performance of a policy rule, it is helpful to understand how policy rules are parameterized.

> *Economic models, however sophisticated, are by construction simple caricatures of the true economy.*

For central bank models, such as ToTEM, that are used to provide policy advice, the parameters of the policy rule are normally chosen to minimize an assumed loss function,[5] which in ToTEM includes the variance of CPI inflation relative to the 2 per cent inflation-control target, the variance of the economy-wide output gap, and the variance of the change in the target overnight interest rate.[6] The variances of these endogenous variables will depend on the structure and calibration of the economic model, the policy rule, and the variances and covariances of the shocks included in the model, which are normally estimated using historical data. Choosing optimal parameters for the rule involves using the covariance matrix of shocks, in conjunction with the model, to compute variances for the endogenous variables that appear in the loss function. The task then is to choose parameter values in the policy rule that minimize the expected loss.

In general, the optimal parameter values in the rule will depend importantly on which shocks were most important over history, as well as on the covariances among shocks.[7] This is because simple rules must trade off performance for simplicity. As a very simple example, consider an economy with just two shocks: a demand shock that pushes output and inflation in the same direction, and a supply shock that moves them in opposite directions. Also assume that while the central bank seeks to stabilize output and inflation, the policy interest rate responds only to inflation. In this set-up, the optimal response to a demand shock will be larger than the optimal response to a supply shock, since the policy response to a supply shock pushes output away from potential output. Therefore, the optimal response to inflation in the

5   Details of the loss function and of the optimized rule currently used in ToTEM are described in Cayen, Corbett, and Perrier (2006) and Murchison and Rennison(2006).
6   The respective weights in the loss function are 1, 1, and 0.5.
7   Cayen, Corbett, and Perrier (2006) provide examples using ToTEM.

policy rule will depend on the relative importance of demand versus supply shocks in the economy.

This simple example illustrates that the performance of optimal simple rules will depend on the nature of the shocks that hit the economy. If the relative importance of various shocks changes through time, the performance of a simple rule will no longer be optimal. In contrast, since a fully optimal rule responds optimally to each shock, the parameter values of the rule do not depend on the relative importance of the various shocks.[8] Relative to other sources of uncertainty discussed in this article, shock uncertainty is unique in that it renders simple rules less robust than optimal rules.

Data and measurement uncertainty

Much of the data used in economic models, with the notable exceptions of the CPI and the labour force survey in Canada, is subject to periodic revision. As a general rule, recently released data are subject to larger revisions than data that have already been revised several times. When formulating policy, central banks must therefore be aware that the data on which they rely to gauge the current state of the economy contain a potentially important noise component.

In addition to errors associated with data collected by statistical agencies, central banks must often construct data for variables that are not directly measurable. An important example is the trend level of labour productivity. While measures of actual labour productivity are available from Statistics Canada, the underlying trend or permanent component must be estimated, and this is typically done using a statistical filter.[9] Since these filters are often two-sided (i.e., the estimate of the trend in a given period is based on both past and future observations of the data being filtered), their accuracy declines as they approach the end of the sample, since there are fewer future observations on which to condition the estimate.

In designing an optimal monetary policy rule, a central bank would typically respond more cautiously to a variable measured with error. To see why, we refer back to the example in which the estimated level of trend labour productivity is a noisy measure of the true level. Since potential output is constructed using trend labour productivity, the output gap will inherit

much of this noise. Now, consider a central bank that uses a policy rule of the form given by equation (1), which can now be written in terms of the true output gap and the noise component, ε_t^y, as

$$R_t = R^* + \varphi_\pi(\pi_t - \pi^T) + \varphi_y(\tilde{y}_t + \varepsilon_t^y). \quad (3)$$

Equation (3) reveals the nature of the information problem. By choosing to respond positively to the output gap (the variable measured with error), the policy-maker inadvertently reacts to the noise. This introduces undesirable movements in the interest rate, which feed back to the economy and generate unnecessary fluctuations in output and inflation. Cateau, Desgagnés, and Murchison (forthcoming) illustrate this point using an inflation-targeting rule in ToTEM. The results are presented in **Table 1**.

Table 1: Effects of data uncertainty

	σ_π	$\sigma_{\tilde{y}_t}$	$\sigma_{\Delta R}$	Loss
No data uncertainty				
$R_t = 1.09R_{t-1} + 0.54\pi_t + 0.13\tilde{y}_t$	1.06	1.09	0.55	1
Data uncertainty ignored				
$R_t = 1.09R_{t-1} + 0.54\pi_t + 0.13\tilde{y}_t$	1.31	1.10	0.56	+12%
Data uncertainty accounted for				
$R_t = 1.14R_{t-1} + 0.61\pi_t + 0.08\tilde{y}_t$	1.04	1.25	0.51	+6%

The top panel of Table 1 shows an optimized inflation-targeting rule under the assumption that the output gap in ToTEM is perfectly measured; the middle panel evaluates the performance of that rule when the output gap is, in fact, not perfectly measured.[10] Ignoring the measurement errors in the output gap leads to additional volatility in inflation, the output gap, and the change in the interest rate, culminating in a 12 per cent deterioration in the rule's performance.

Of course, a policy-maker who recognizes that the information at his disposal is not accurate need not naively follow a rule that is efficient only in the absence of data uncertainty. Indeed, as is clear from equation (3), by choosing to respond less aggressively

---

8   For this reason, optimal policy under commitment is said to be *certainty equivalent*.
9   Butler (1996) provides a detailed discussion of the estimations of trend labour productivity and trend labour input that are used in the Bank of Canada's conventional measure of potential output.

10   Cateau, Desgagnés, and Murchison (forthcoming) allow for data uncertainty by computing the discrepancies between the real-time and revised values of the Bank of Canada's conventional estimate of potential output and modelling the resulting measurement errors as an AR(2) process.

to the central bank's measure of the output gap, the influence of the noise can be reduced. The bottom panel of Table 1 presents an optimized rule that accounts for the presence of measurement errors in the output gap. Owing to the difficulty of accurately measuring the output gap, the resulting rule gives it a lower weight[11] but places higher weights on inflation and policy inertia. This leads to a more volatile output gap but allows better control of inflation and of changes in the interest rate. Ultimately, the new rule reduces the influence of output gap mismeasurement relative to the baseline rule by half.

Parameter uncertainty

While economic theory can guide modellers on the nature of certain economic relationships, it rarely provides much guidance on the exact strength of the relationship. For instance, theory says that Canadian exports to the United States will strengthen, other things being equal, following a depreciation of the real Canada/U.S. exchange rate, since Canadian goods become more competitive. But the size of the export response is unknown. It must therefore be estimated using historical data and will be subject to sampling uncertainty, even if the underlying theory is correct. In this sense, policy-makers should regard the parameters of their model as random variables with some underlying distribution, rather than as known, fixed quantities.

Viewed from this perspective, it is natural to ask what differentiates parameter uncertainty from shock uncertainty, since shocks are also modelled as random variables. The crucial difference lies in the fact that a model's parameters enter multiplicatively, meaning that they interact with the model's endogenous variables, whereas shocks are additive. Thus, while the optimal parameter values of a simple policy rule depend on the *relative* variances of the model's shocks, the absolute variances are unimportant.[12] If we think about the model's parameters as random variables, however, absolute variances do matter.

Consider the famous example given by Brainard (1967), in which inflation is linearly related to the policy instrument, and there is an exogenous demand shock, u_t:

$$\pi_t = -\theta R_t + u_t,$$

and the central bank's objective is to minimize the variance of inflation. The optimal policy rule with no parameter uncertainty sets the interest rate in each period to $(1/\theta)u_t$, and inflation is perfectly stabilized at zero each period. However, if the parameter relating the instrument to the target is not known with certainty, the central bank's model will be characterized by:

$$\pi_t = -(\theta - \varepsilon)R_t + u_t = -\theta R_t + u_t + R_t\varepsilon,$$

where ε is a random variable. There are now, in effect, two shocks in the model, and the multiplier on the second one is the nominal interest rate. If the central bank implements the same policy as discussed above, the variance of inflation will be unnecessarily high. The optimal policy rule that accounts for parameter uncertainty in this example is $[\theta/(\theta^2 + \sigma_\varepsilon^2)]u_t$, where σ_ε^2 is the variance of ε. As the degree of parameter uncertainty increases, the optimal response coefficient in the rule declines. This finding is called the "Brainard conservatism principle" (Blinder 1998).

In addition to introducing uncertainty regarding the linkages between observed variables, such as inflation and the policy interest rate, parameter uncertainty also creates uncertainty about the correct level of unobserved, model-defined variables. For instance, in ToTEM, the real marginal cost of production in the consumption-goods sector is the key driver of core CPI inflation (Murchison and Rennison 2006). Since Statistics Canada does not provide a measure of real marginal cost, it is calculated within ToTEM, and its properties reflect both the structure and the parameterization of the model. As a result, parameter uncertainty introduces additional uncertainty about the future evolution of inflation through its influence on marginal cost.

---

[11] This result is in accordance with the literature. Smets (1999) shows that when measurement error in the output gap becomes very large, the efficient Taylor rule parameter on the output gap falls towards zero. Orphanides (2003) shows that once the measurement errors between real-time and ex-post data are properly taken into account, optimized policy reactions are more cautious than otherwise.

[12] Slightly more technically, multiplying the covariance matrix of shocks by a scalar will not affect the optimal parameter values of a simple rule, since doing so will not affect the relative variances of the endogenous variables that enter the central bank's loss function.

Table 2: Robustness of optimized inflation- and price-level-forecast rules

Benchmark rule	IF	PLF	FO
No parameter uncertainty			
Performance: $\dfrac{loss(rule\ j)}{loss\ (IF)} - 1$	1	-4.3%	-11.4%
Parameter uncertainty			
Robustness: $\dfrac{E\ loss(rule\ j|Under\ uncertainty)}{loss(rule\ j|No\ uncertainty)} - 1$	+80%	+81%	+142%
Overall average performance: $\dfrac{E\ loss(rule\ j|Under\ uncertainty)}{E\ loss\ (IF|Under\ uncertainty)} - 1$	1	-3.4%	+21%

Finally, any time that a monetary policy rule responds to a forecast of inflation (or of any other variable), the performance of that rule will be influenced by parameter uncertainty, since the forecast will not be as precise. Parameter uncertainty can thus be thought of as introducing noise into the inflation forecast in a manner similar to measurement uncertainty (see equation 3), thereby rendering that variable less reliable as a guide for policy. In the end, whether it is better to respond to current inflation or to a forecast of future inflation, will depend on the benefit of being forward looking, in the absence of parameter uncertainty, relative to the cost of introducing additional noise in the policy rule.[13]

Cateau, Desgagnés, and Murchison (forthcoming) derive optimized inflation-forecast (IF) and price-level-forecast (PLF) rules for ToTEM and compare their performance with fully optimal policy under commitment (FO).[14] They then investigate the robustness of these rules to parameter uncertainty by analyzing how they would fare if the structural parameters that actually characterize the behaviour of private agents differed from those assumed by the policy-maker in

deriving the optimized rules **(Table 2)**. These types of comparisons are of particular interest in light of the Bank of Canada's interest in evaluating the potential welfare gains of switching from its current inflation-targeting regime, to a price-level-targeting regime.[15] Furthermore, most of the research to date that explores this issue ignores altogether the issue of uncertainty.

The top panel of Table 2 compares the performance of the optimized inflation-forecast rule, price-level-forecast rule, and optimal policy under full commitment, using ToTEM's baseline calibration. Without parameter uncertainty, fully optimal policy under commitment offers an 11.4 per cent improvement in performance over IF, while PLF would offer a 4.3 per cent improvement.

The authors go on to investigate how parameter uncertainty affects these rankings by evaluating the performance of each benchmark rule in 5000 alternative parameter configurations drawn randomly from the Bayesian posterior distribution of the estimated parameters. The bottom panel of Table 2 contains two important messages. First as recently emphasized by Orphanides and Williams (2008), while fully optimal policy under commitment is the best policy if the parameters are known, it is often the least robust policy under uncertainty. Indeed, relative to the case of no uncertainty, its performance deteriorates 60 percentage points more than the other rules. Second, while IF is slightly more robust than PLF, on average, PLF still performs better than IF under parameter uncertainty. Therefore, while the reduction in loss associated with moving from inflation targeting to

---

13 The extent of the benefit of setting policy in a forward-looking manner depends on the speed of the monetary policy transmission mechanism. All else being equal, the faster policy actions get transmitted to output and inflation, the less need there is to be forward looking.

14 The optimized inflation-forecast rule for ToTEM is a rule that responds to current inflation, the lagged interest rate, and the output gap. In contrast, the optimized price-level-forecast rule responds to the price-level forecast four quarters ahead, the lagged interest rate, and the output gap. The price-level-forecast rule is an example of a rule that implements price-level targeting, since this rule will eventually return the price level to the desired level following a shock. Optimal policy under full commitment is the policy that is optimally tuned to the model. It is, by design, a very complicated rule that depends on every variable that affects the state of the economy. Optimal policy does not, in general, fully reverse price-level movements following a shock in ToTEM and, therefore, is not fully consistent with a price-level-targeting regime.

15 See Bank of Canada (2006).

price-level targeting in ToTEM is modest, this reduction is robust to parameter uncertainty.[16]

Model uncertainty

So far, we have discussed uncertainty about the underlying shocks that drive business cycles, uncertainty about the data used in a particular model, and uncertainty about the parameter values used in the model. But what about the economic model itself? A model may be misspecified for various reasons: it may be built around an economic paradigm that is further from economic reality than assumed (Engert and Selody 1998); it may ignore economic relationships that are, in fact, relevant; or it may be built under simplifying assumptions that make the model tractable (e.g., linearity) but less realistic. Since a model is ultimately only one view of how the economy works, a policy rule that is tuned to work well in a particular model may perform poorly across alternative but plausible views.

Côté et al. (2002) analyze the performance of various simple rules in 12 models of the Canadian economy. They find that simple outcome-based rules (rules where the policy instrument responds to current and lagged variables) are not particularly robust. In particular, they find that rules with high degrees of inertia often induce substantial volatility in output and inflation and are even unstable in many models.

Since a model is ultimately only one view of how the economy works, a policy rule that is tuned to work well in a particular model may perform poorly across alternative but plausible views.

More recently, Tetlow (2010) evaluates the performance of 8 alternative simple rules in 46 vintages of the Federal Reserve Board FRB/US model used by the Board's staff for forecasting and policy analysis from July 1996 to October 2007. He concludes that model

uncertainty is a substantial problem: model properties differ importantly according to vintage and so do the policy rules optimized by vintage. Further, while some rules offer satisfactory performance, many that are promoted as being robust to some specific type of uncertainty perform poorly when confronted with real-time model uncertainty.

Once we acknowledge that any particular model is potentially misspecified, the results above indicate that model uncertainty can seriously affect the performance of policy rules in stabilizing the economy and, hence, should be taken into account when designing effective policy rules. In the next section, we review recent strategies for designing rules that are robust to specific forms of uncertainty, including model uncertainty.

Robust Policy Rules

When designing policy rules, it is important to seek a robust rule—one that yields a satisfactory performance in an uncertain environment. There are two approaches to designing a robust rule. The first involves deriving optimized coefficients that formally account for specific uncertainties. That is, given a specific rule, we determine how strongly the policy instrument should respond to each variable in the rule, taking into account the features about which we are uncertain. The second approach involves determining a functional form for the rule (i.e., what variables the policy instrument responds to) that is less susceptible to yielding a poor performance, given specific uncertainties. These approaches are complementary and are often combined when pursuing a robust simple rule. In this section, we review how they have been or could be applied to design rules robust to each of the uncertainties discussed.

Robustness to data uncertainty

There are two main approaches for designing a rule robust to data uncertainty. The first, alluded to earlier, involves formally taking into account that data are observed with noise and will subsequently be revised. A common strategy for dealing with this problem follows Orphanides (2003) in modelling the measurement errors between real-time and ex-post data and incorporating these equations in the model prior to optimizing the rule. To the extent that future measurement errors may behave like historical errors, this strategy helps the policy-maker design a rule that accounts for likely mismeasurement of the data.

---

[16] This is an important result, since ToTEM's baseline calibration assumes a very small role for dynamic indexation to lagged price and wage inflation. Thus, the distributions for these parameters are positively skewed, and nearly all of the risk is on the upside. In addition, research (see Ambler 2009 for a review) suggests that higher levels of dynamic indexation (or rule-of-thumb behaviour) will cause a deterioration in the relative performance of price-level-forecast rules.

Table 3: Robust inflation- and price-level-forecast rules

$$R_t = \rho R_{t-1} + \varphi_\pi E_t \pi_{t+k} + \varphi_P E_t P_{t+k} + \gamma \tilde{y}_t$$

Rule j	ρ	φ_π	φ_P	γ	k	σ_R	$\dfrac{loss(j)}{loss\,(IF)} - 1$	Robustness: $\dfrac{E\,loss(j)}{loss\,(j)} - 1$	Overall: $\dfrac{E\,loss(j)}{loss\,(IF)} - 1$
		Coefficients of rule				**Benchmark parameters**		**Parameter uncertainty**	
No uncertainty									
IF	1.09	0.54	0	0.13	0	1.48	1	+80%	+80%
PLF	0.99	0	0.07	0.17	4	1.84	-4.3%	+81%	+73%
Parameter uncertainty									
IF	1.01	0.46	0	0.14	0	1.56	+1%	+70%	+72%
PLF	1.01	0	0.08	0.21	3	2.04	-4.1%	+68%	+61%

An alternative approach is to design a rule that does not respond to variables measured with error. Taylor's original rule was criticized by Orphanides et al. (2000) and by Orphanides and Williams (2002) for including unobservable variables, such as the natural rate of interest and potential output (or natural rate of unemployment). Given the difficulty of measuring these variables in real time, Orphanides and Williams (2002) propose difference rules in which the short-term nominal interest rate is raised or lowered from its existing level in response to inflation and to changes in economic activity (change in unemployment or growth rate of output). These rules do not require knowledge of the natural rates of interest or unemployment (or potential output) for setting policy and are consequently immune to mismeasurement. Orphanides et al. (2000) and Orphanides and Williams (2002) show that, in the presence of data uncertainty, these difference rules outperform rules that respond to levels of economic activity. But how do such difference rules perform in environments characterized by other forms of uncertainty?

Tetlow (2010) evaluates the performance of the difference rule proposed by Orphanides and Williams (2002) in 46 vintages of the Federal Reserve Board FRB/US model. The experiment provides an ideal laboratory for evaluating the robustness of a rule since it incorporates real-time model and parameter uncertainty in a model used for policy-making. Tetlow observes that the difference rule does lead to robust performance in the sense that a difference rule optimized for a particular vintage maintains good stabilization properties across all other vintages.

Robustness to parameter uncertainty

The most popular approach for deriving a rule robust to parameter uncertainty is the Bayesian approach, which assumes that unknown parameters come from known distributions. That is, even though the precise values of parameters are not known, it is possible to determine the range of values that they can take, together with their associated probabilities. A robust rule is then derived by choosing the coefficients of the rule to minimize the expected loss, given the distribution of parameters. **Table 3** presents the results of Cateau, Desgagnés, and Murchison (forthcoming) who derive robust inflation-forecast and price-level-forecast rules for ToTEM under parameter uncertainty.[17]

The top panel of Table 3 displays the optimized inflation-forecast (IF) and price-level-forecast rule (PLF) with the estimated parameters of ToTEM as benchmark. The bottom panel displays the robust versions of the IF and PLF rule under parameter uncertainty. The results suggest three important messages:

1. PLF is more robust than IF under parameter uncertainty. The last column compares the overall performance of each rule under parameter uncertainty. The robust PLF rule dominates the robust IF rule by 11 percentage points.

---

17 Cateau, Desgagnés, and Murchison (forthcoming) allow for parameter uncertainty by allowing a set of key parameters to take 5000 possible values drawn randomly from the Bayesian posterior distribution of the estimated parameters. The robust inflation-forecast and price-level-forecast rules minimize expected loss; i.e., the weighted average of the losses across the draws.

2. Robustness to parameter uncertainty in ToTEM leads to more aggressive policy responses. For instance, the robust PLF rule requires more aggressive responses to the lagged interest rate, forecast price level, and output gap. This translates into more aggressive policy responses as shown by an increase in the unconditional standard deviation in the interest rate, σ_R, from 1.84 to 2.04 per cent. The robust IF rule, on the other hand, requires weaker responses to the lagged interest rate and current inflation but stronger responses to the output gap. The stronger response to the output gap dominates, making policy responses slightly more aggressive (the standard deviation of the interest rate increases from 1.48 to 1.56 per cent).[18]

3. While Bayesian robust rules improve policy performance under parameter uncertainty, they do not offer a significant improvement. The second to last column assesses the robustness of the various rules by comparing their average performance under parameter uncertainty with their performance under no uncertainty. Although the robust IF and PLF rules improve performance over the benchmark IF and PLF rules by 10 and 13 percentage points, respectively, they still lead to a high average loss under uncertainty (respectively 70 per cent and 68 per cent higher than the loss that the benchmark IF rule leads to under no uncertainty). Note, however, that this increase in average loss may also reflect that, on average, the alternative parameterizations of the model make inflation and the output gap more difficult to control, relative to the baseline calibration.

The third result illustrates a disadvantage of the Bayesian approach as a tool for deriving robust rules. By design, the Bayesian approach tunes the policy rule to work best across those parameter configurations that are the most probable: i.e., receive the most probability weight. This yields a policy rule that works well in parameter configurations that are most likely to be true but whose performance suffers in the more extreme, but less likely, parameter configurations.

An alternative approach that offers more robustness to extreme parameter configurations is the worst-case approach. For example, Giannoni (2002) proposes a worst-case approach that does not require knowledge of the distribution of the unknown parameters. Instead the policy-maker knows only the bounds for each parameter and seeks robust policy rules that minimize loss in the worst-case parameterization within those bounds. Giannoni (2002) finds that a policy-maker that seeks to mitigate the effect of parameter uncertainty in a standard New Keynesian model would choose Taylor rules that respond more aggressively to both inflation and the output gap.

Both approaches are useful in determining robust versions of a particular choice of rule. Levin et al. (2006) use a micro-founded model to investigate what types of simple rules are effective when the central bank faces parameter uncertainty. They find that the performance of optimal policy is closely matched by a simple operational rule that responds to the lagged interest rate and focuses solely on stabilizing nominal-wage inflation. Furthermore, this simple wage-stabilization rule is robust to uncertainty about the structural parameters and to various assumptions regarding the nature and incidence of the innovations. However, the performance of the rule is sensitive to the specification of wage contracts in the labour market. Indeed, when Taylor contracts rather than Calvo contracts are assumed, rules that respond to price inflation and real economic variables perform better than the wage-inflation rule. Hence, the robustness of wage-inflation rules hinges critically on structure and wage determination in labour markets.

Robustness to model uncertainty

There are two popular approaches to deriving robust rules under model uncertainty. The first allows the policy-maker to consider different candidate models (e.g., those reflecting different paradigms of the monetary policy transmission mechanism) and seeks policy choices that perform well on average (Brock, Durlauf, and West 2007) or on a worst-case basis. Cateau (2007) proposes a decision-making framework where a policy-maker can consider various non-nested models for choosing policy. His framework distinguishes between two types of risk: within-model risk (risk arising because of the stochastic nature of a particular model) and across-model risk (risk arising as a result of contemplating various models). He shows that the policy-maker's aversion to across-model risk determines the extent to which the policy-maker wants to trade off good average performance

---

18 Edge, Laubach, and Williams (2010) also find that parameter uncertainty leads to more aggressive policy in a micro-founded model. Uncertainty about the structural parameters in their model leads to uncertainty about the implicit "natural" rates of output and interest. They find that optimal Taylor rules under parameter uncertainty respond less to the output gap and more to price inflation than would be optimal without parameter uncertainty. But the more aggressive response to inflation dominates, making policy more aggressive.

for robustness: as the degree of aversion to across-model risk increases, the policy-maker wants to achieve more robustness at the expense of good average performance. Cateau shows that when the policy-maker wants to achieve more robustness, the policy-maker chooses less-aggressive Taylor rules that are in line with those estimated in the data.

Levin, Wieland, and Williams (2003) compare the performance of various outcome-based and forecast-based rules with the objective of identifying one rule that would perform well across five distinct models of the U.S. economy. For their model set, they find that a robust rule is a forecast-based rule that responds to a short-horizon forecast of inflation (less than one year), the current output gap, and also involves a high degree of inertia.

The second approach derives policy choices that are robust to misspecification of the policy-maker's baseline model. In this approach, the policy-maker takes into account that his baseline model is only an approximation of some unknown true model and, hence, can potentially be misspecified. In particular, the dynamics of the baseline model may omit important explanatory variables, as in Hansen and Sargent (2008), or parameters affecting the relationship between different variables may be unknown, as in Onatski and Stock (2002). The policy-maker deals with these misspecifications by choosing policy according to the worst-case model in a set of plausible models. Sargent (1999), Onatski and Stock (2002), and Tetlow and von zur Muehlen (2001) find that robust rules are, in fact, more aggressive than those obtained when potential misspecifications are ignored.

Conclusion

Monetary policy is most effective when the central bank's objectives, and the means of achieving those objectives, are well understood and regarded as credible by the public. This requires that the central bank communicate clearly what it seeks to achieve and, further, requires the central bank to respond to economic developments in a predictable and systematic fashion that is easy to communicate.

Since Taylor (1993), academic researchers and central banks have increasingly used simple rules as a guide to setting monetary policy. Simple rules have the advantage of being easier to communicate to the public than more complex policies and, by virtue of their simplicity, offer the promise of making monetary policy more easily understood and predictable. But what simple rule should a central bank use? The various uncertainties that central banks must contend with make the choice and design of a simple rule difficult.

The results surveyed here suggest that uncertainty has a substantial impact on the performance of simple rules. Although simple rules perform better in an uncertain environment than more complex policies, their performance can still deteriorate substantially. It is therefore critical to account for uncertainty in designing rules to ensure that their performance is satisfactory irrespective of the state of the world.

Work with ToTEM suggests that a price-level-forecast rule is more robust to uncertainty than an inflation-forecast rule.

Our work with ToTEM suggests that a price-level-forecast rule is more robust to uncertainty than an inflation-forecast rule. While more research in this area is required, these results suggest that greater insulation from the effects of economic uncertainty may be an additional rationale for considering price-level targeting over inflation targeting. Finally, based on the literature, other rules shown to have good robustness properties, which also warrant further research, include a difference rule, where the change in the interest rate responds to output growth, as well as a wage-inflation rule.

Literature Cited

Ambler, S. 2009. "Price-Level Targeting and Stabilization Policy: A Review." *Bank of Canada Review* (Spring): 19–29.

Bank of Canada. 2006. *Renewal of the Inflation-Control Target*. Ottawa: Bank of Canada.

Blinder, A. S. 1998. *Central Banking in Theory and Practice.* Cambridge, MA: MIT Press.

Brainard, W. 1967. "Uncertainty and the Effectiveness of Policy." *American Economic Review* 57 (2): 411–25.

Brock, W. A., S. Durlauf, and K. West. 2007. "Model Uncertainty and Policy Evaluation: Some Theory and Empirics." *Journal of Econometrics* 136 (2): 629–64.

Butler, L. 1996. "The Bank of Canada's New Quarterly Projection Model Part 4: A Semi-Structural Method to Estimate Potential Output: Combining Economic Theory with a Time-Series Filter." Bank of Canada Technical Report No. 77.

Cateau, G. 2007. "Monetary Policy under Model and Data-Parameter Uncertainty." *Journal of Monetary Economics* 54 (7): 2083–2101.

Cateau, G., H. Desgagnés, and S. Murchison. Forthcoming. "Robustness of Policy Rules." Bank of Canada Working Paper.

Cayen, J.-P., A. Corbett, and P. Perrier. 2006. "An Optimized Monetary Policy Rule for ToTEM." Bank of Canada Working Paper No. 2006-41.

Clarida, R., J. Galí, and M. Gertler. 2000. "Monetary Policy Rules and Macroeconomic Stability: Evidence and Some Theory." *Quarterly Journal of Economics* 115 (1): 147–80.

Coletti, D. and S. Murchison. 2002. "Models in Policy-Making." *Bank of Canada Review* (Summer): 19–26.

Côté, D., J.-P. Lam, Y. Liu, and P. St- Amant. 2002. "The Role of Simple Rules in the Conduct of Canadian Monetary Policy." *Bank of Canada Review* (Summer): 27–35.

Edge, R., T. Laubach, and J. Williams. 2010. "Welfare-Maximizing Monetary Policy under Parameter Uncertainty." *Journal of Applied Econometrics* 25 (1): 129–43.

Engert, W. and J. Selody. 1998. "Uncertainty and Multiple Paradigms of the Transmission Mechanism." Bank of Canada Working Paper No. 98-7.

Giannoni, M. 2002. "Does Model Uncertainty Justify Caution? Robust Optimal Monetary Policy in a Forward-Looking Model," *Macroeconomic Dynamics* 6 (1): 111–44.

Hansen, L. P. and T. Sargent. 2008. *Robustness*. Princeton, N.J.: Princeton University Press.

Levin, A., A. Onatski, J. Williams, and N. Williams. 2006. "Monetary Policy under Uncertainty in Micro-Founded Macroeconomic Models." In *NBER Macroeconomics Annual 2005*, edited by M. Gertler and K. Rogoff, 229–87. Cambridge, MA: MIT Press.

Levin, A., V. Wieland, and J. Williams. 2003. "The Performance of Forecast-Based Monetary Policy Rules under Model Uncertainty." *American Economic Review* 93 (3): 622–45.

Murchison, S. and A. Rennison. 2006. "ToTEM: The Bank of Canada's New Quarterly Projection Model." Bank of Canada Technical Report No. 97.

Onatski, A. and J. Stock. 2002. "Robust Monetary Policy under Model Uncertainty in a Small Model of the U.S. Economy." *Macroeconomic Dynamics* 6 (1): 85–110.

Orphanides, A. 2003. "Monetary Policy Evaluation with Noisy Information." *Journal of Monetary Economics* 50 (3): 605–31.

Literature Cited (cont'd)

Orphanides, A., R. Porter, D. Reifschneider, R. Tetlow, and F. Finan. 2000. "Errors in the Measurement of the Output Gap and the Design of Monetary Policy." *Journal of Economics and Business* 52 (1/2): 117–41.

Orphanides, A. and J. Williams. 2002. "Robust Monetary Policy Rules with Unknown Natural Rates." *Brookings Papers on Economic Activity* 2: 63–18.

———. 2008. "Learning, Expectations Formation, and the Pitfalls of Optimal Control Monetary Policy." *Journal of Monetary Economics* 55 (Supplement 1): S80–S96.

Sargent, T. 1999. "Comment." In *Monetary Policy Rules*, edited by J.B. Taylor, 144–54. Chicago: University of Chicago Press and NBER.

Smets, F. 1999. "Output Gap Uncertainty: Does It Matter for the Taylor Rule?" In *Monetary Policy under Uncertainty*, edited by B. Hunt and A. Orr, 10–29. Wellington, New Zealand: Reserve Bank of New Zealand.

Svensson, L. 1999. "Inflation Targeting as a Monetary Policy Rule." *Journal of Monetary Economics* 43 (3): 607–54.

Taylor, J. B. 1993. "Discretion versus Policy Rules in Practice," *Carnegie-Rochester Conference Series on Public Policy* 39: 195–214.

Tetlow, R. 2010. "Real-time Model Uncertainty in the United States: 'Robust' policies put to the test." Board of Governors of the Federal Reserve System Finance and Economics Discussion Series No. 2010-15.

Tetlow, R. and P. von zur Muehlen. 2001. "Robust Monetary Policy with Misspecified Models: Does Model Uncertainty Always Call for Attenuated Policy?" *Journal of Economic Dynamics and Control* 25 (6/7): 911–49.

Woodford, M. 1999. "Optimal Monetary Policy Inertia." National Bureau of Economic Research (NBER) Working Paper No. 7261.