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Abstract

What are the effects of financial market imperfections on unemployment and vacancies? Since standard DSGE models do not typically model unemployment, they abstract from this issue. In this paper I augment a standard monetary DSGE model with explicit financial and labour market frictions and estimate the model using US data for the period 1964:Q1-2010:Q3. I find that the estimated degree of financial frictions is higher when financial data and shocks are included. The model matches the aggregate volatility in the data reasonably well. In particular, for the labour market, the model is able to generate highly volatile unemployment and vacancies, and a relatively rigid real wage. Further, I find that the financial accelerator mechanism plays an important role in amplifying the effects of financial shocks on unemployment and vacancies. Overall, financial shocks explain about 37 per cent of the fluctuations in unemployment and vacancies.

JEL classification: E32, E44, J6
Bank classification: Economic models; Financial markets; Labour markets

Résumé

Quels effets les imperfections des marchés financiers ont-elles sur le chômage et l'offre d'emplois? Cette question est absente des études qui s'appuient sur les modèles d'équilibre général dynamiques et stochastiques (EGDS) courants, puisque ceux-ci formalisent rarement le phénomène du chômage. L'auteure incorpore des frictions financières et un marché du travail soumis à des frictions dans un modèle monétaire EGDS type, qu'elle estime sur des données américaines s'étalant du 1er trimestre de 1964 au 3e trimestre de 2010. Les frictions financières estimées sont plus intenses lorsque des données et des chocs de nature financière sont ajoutés. Le modèle restitue assez bien la volatilité globale observée dans les données. Plus précisément, la forte volatilité du chômage et de l'offre d'emplois est reproduite, de même que la relative rigidité des salaires réels. L'auteure constate en outre que le mécanisme d'accélérateur financier joue un rôle important car il amplifie l'incidence des chocs financiers sur le chômage et l'offre d'emplois. Dans l'ensemble, ces chocs expliquent environ 37 % des fluctuations du chômage et de l'offre d'emplois.

Classification JEL : E32, E44, J6
Classification de la Banque : Modèles économiques; Marchés financiers; Marchés du travail

1 Introduction

The recent financial crisis has been associated with a significant rise in the unemployment rate in the US. The unemployment rate more than doubled from 4.8 per cent at the beginning of the recession to peak at 10 per cent in the last quarter of 2009. Determining the extent to which financial market imperfections may have contributed to fluctuations in unemployment in the labour market and the extent to which monetary policy may have helped to alleviate those fluctuations has however proved difficult: On the one hand, models that study the effects of financial frictions on unemployment are often too stylized for making quantitative statements. On the other hand, DSGE models that are more suited to quantitative exercises have typically abstracted from modeling the interaction between financial imperfection and the labour market. The purpose of this paper is two-fold: (i) First, develop and estimate a quantitative macroeconomic model that incorporates both labour and financial market frictions using US time series data from 1964Q1 to 2010Q3; (ii) Second, explore the interaction of financial and labour market frictions, and assess quantitatively, through this interaction, how important it is to consider financial frictions and shocks when addressing labour market dynamics.

There is an important strand of literature that studies the effect of financial market imperfections on unemployment. These studies usually assume that there exists some difficulties for firms to access credit and these difficulties affect firms' hiring decisions. For example, Wasmer and Weil (2004) assume that new entrepreneurs have no wealth of their own and must raise funds in an imperfect credit market before they enter the labour market to search for workers. Acemoglu (2001) studies an environment in which an agent decides to become an entrepreneur or a worker. For entrepreneurs to be able to hire workers, they either need to borrow the necessary funds or use their own wealth. Both studies show that credit frictions lead to higher unemployment levels. Recent studies focus more on the effects of credit frictions on the dynamics of unemployment and vacancies. Petrosky-Nadeau (2009) assumes that firms must seek external funds over their net worth to finance current vacancies and the credit market is subject to costly state verification type frictions. He shows that the credit market frictions amplify and propagate the responses of unemployment and vacancies to productivity shocks. Monacelli, Quadrini and Trigari (2010) study the importance of financial markets for unemployment fluctuations, where firms can issue debt under limited enforcement of debt contracts. They indicate that in this environment credit shocks can generate large employment fluctuations.

However, the abovementioned models are stylized models that in most cases only consider the effects of productivity shocks on unemployment. Without other frictions or competing shocks, it is difficult to quantify the contribution of credit frictions or shocks to labour market fluctuations. DSGE models, in contrast, can allow for many shocks and frictions, and thus are more suited for quantitative exercises. However, although the recent literature in medium-scale DSGE models has shown a growing interest in the role of financial factors in business cycle fluctuations (Bernanke, Gertler and Gilchrist 1999, herein BGG; and Christiano, Motto and Rostagno 2007), it has largely abstracted from modeling unemployment in models where financial factors play an important role. One exception is Christiano, Trabandt and Waletin (2007) (herein CTW). CTW introduce BGG-type financial frictions and unemployment into a monetary DSGE model in a small open economy setting, and estimate their model using Swedish data. They find that financial shocks account for 10 per cent of the volatility in unemployment in the Swedish economy.

This paper augments a standard DSGE model with financial and labour market frictions along the lines of CTW. The financial market frictions are modeled as in BGG. Due to information asymmetry, there are financial frictions in the accumulation and management of capital. BGG have shown that this type of friction can amplify and propagate shocks to the macroeconomy (financial accelerator mechanism). The labour market frictions are modeled in a search and matching framework, and the wage setting frictions (staggered wage contracting) are modeled as in Gertler, Sala and Trigari (2008) (herein, GST).[1] As in CTW, the model economy is also subject to multiple shocks, including both productivity and financial shocks. But unlike CTW, this paper focuses on the transmission mechanism of financial shocks to labour market activities. In particular, this paper highlights the important role of the financial accelerator mechanism in amplifying the responses in unemployment and vacancies to financial shocks. Moreover, this paper attempts to analyze how the interaction between financial shocks and wage setting frictions affects labour market outcomes.

In this paper, financial imperfections affect unemployment and vacancies in the following way: After a negative financial shock that reduces the entrepreneurs' net worth, the worsened balance-sheet position leads entrepreneurs to face a higher risk premium on their external borrowing due to BGG-type frictions in the financial market. Since the external financing becomes more costly, the demand for capital declines. Given the constant returns to scale aggregate production function, it is optimal for entrepreneurs to keep a constant capital labour ratio. Thus, the demand for labour declines as well, leading firms to post fewer vacancies. This reduces the labour market tightness and the probability for a worker to find a job, leading fewer workers to leave the unemployment state. In this model, the financial accelerator mechanism amplifies the financial shock and generates large fluctuations in unemployment and vacancies even though firms' vacancy postings are not subject to financial frictions directly (spillover effects of the financial factors).

I estimate the model using US data including financial time series data. The main findings of the paper are the following. First, the model matches the aggregate volatility in the data reasonably well. In particular, the model is able to generate highly volatile unemployment and vacancies, and a relatively rigid real wage. Second, the financial wealth shock, the shock affecting net worth in the entrepreneurs' sector, accounts for around 37 per cent of the variations of unemployment and vacancies. The financial accelerator mechanism significantly amplifies the effect of the financial wealth shock. Reducing financial frictions by half decreases the contribution of the financial shock to the variations in the key labour market variables by one third. Third, I find that adding financial

[1]Since the staggered wage contracting in GST (2008) does not have a direct impact on on-going worker employer relations, it is not vulnerable to the Barro (1977) critique of sticky wages.

data into estimation generates a higher value for the elasticity of external finance, the key parameter capturing financial frictions, leading to a larger amplification effect from the financial accelerator. The estimation results without using financial data do not come close to generating the relative volatility of unemployment and vacancies observed in the data. Lastly, in order to examine the stability of the sample estimates, I divide the data into two subsamples: the first period is from 1966:2 -1979:2 ("Great Inflation" period), and the second period is from 1984:1-2010:3, which covers the "Great Moderation" period and the recent recession. I find that financial shocks are much more persistent and account for a higher portion of variations in unemployment and vacancies in the US in the second period.

The paper is organized as follows. In the next section, I describe the model, and then go on to discuss the data and estimation strategy. In Section 4 I present the estimation results and discuss why the financial shock is important in explaining the variations in the key labour market variables. In Section 5, I discuss several issues regarding the robustness of the results. Finally, section 6 contains concluding remarks.

2 The Model

In this section I describe the model economy. I consider an economy populated by a representative household, retailers, entrepreneurs, capital producers and employment agencies. Each member in the household consumes, holds nominal bonds, and decides whether to provide labour inelastically to employment agencies. Employment agencies hire workers from a frictional labour market, which is subject to an aggregate matching function. The nominal wage paid to an individual worker is determined by Nash bargaining. However, in each period an employment agency has a fixed probability that it may renegotiate the wage. Employment agencies make hiring decisions and supply labour services to entrepreneurs at the price of marginal productivity of the labour services. Entrepreneurs also acquire capital from capital producers. Since entrepreneurs have to obtain external finance for their capital purchasing, they are subject to financial market frictions. Retailers purchase the wholesale goods produced by entrepreneurs and differentiate at no cost and sell them to final good producers, who aggregate differentiated goods into a homogeneous good and supply it to the representative household.

2.1 Households

There is a representative household with a continuum of members of measure one. The number of family members currently employed is n_t. The employed family members earn nominal wage w_t^n. The unemployed members receive unemployment benefit $\bar{b}_t$. Each member has the following period utility function

$$u(c_t) = e_t \log(c_t),$$

where c_t is consumption of final goods in period t and where e_t is a preference shock which follows

$$\log\, e_t = \rho_e \log\, e_{t-1} + \epsilon_t^e, \quad \epsilon_t^e \sim i.i.d.\, N(0, \sigma_{\epsilon^e}^2).$$

Following Andolfatto (1996) and Merz (1995), I assume that family members are perfectly insured against the risk of being unemployed, thus consumption is the same for each family member. The representative household maximizes lifetime utility

$$E_0 \sum_{t=0}^{\infty} \beta^t u(c_t). \tag{1}$$

The wage income from the employed family members is $w_t^n n_t$, where w_t^n is determined by Nash bargaining between employment agencies and workers and n_t is determined by a search and match process in the labour market. The household also earns income from owning equity in retailers Π_t, pays tax T_t and saves by holding a one-period riskless bond B_t. Assuming that the aggregate price is p_t, the representative household is subject to the following budget constraint

$$c_t = \frac{w_t^n}{p_t} n_t + \bar{b}_t(1 - n_t) + \Pi_t - T_t + \frac{B_t - r_{t-1}^n B_{t-1}}{p_t}, \tag{2}$$

where r_{t-1}^n is the nominal rate of return on the riskless bond.

The household maximizes its expected lifetime utility equation (1) subject to equation (2). The first-order condition for consumption is

$$\frac{e_t}{c_t} \frac{1}{r_t^n} = \beta E_t \left[\frac{e_{t+1}}{c_{t+1}} \frac{p_t}{p_{t+1}} \right].$$

2.2 Wholesale Firms (Entrepreneurs)

As in BGG, firms are risk-neutral and manage the production of wholesale goods. The production function for wholesale goods is given by

$$y(j) = f(k_t(j), l_t(j)) = \omega_t(j)(k_t(j))^{\alpha}(z_t l_t(j))^{1-\alpha}.$$

At the end of period $t-1$, entrepreneurs purchase capital $k_t(j)$ from capital producers and use it in period t to produce wholesale goods with labour service $l_t(j)$, which is supplied by employment agencies in a competitive labour market. Production is subject to two type of shocks: ω_t is the idiosyncratic shock, which is private information to the entrepreneur and is *i.i.d* across entrepreneurs and time, with mean $E[\omega_t(j)] = 1$; z_t is an exogenous technology shock that is common to all the entrepreneurs, and it follows

$$\log\, z_t = \rho_z \log\, z_{t-1} + \epsilon_t^z, \quad \epsilon_t^z \sim i.i.d.\, N(0, \sigma_{\epsilon^z}^2).$$

Capital purchased at the end of period t, $k_{t+1}(j)$, is partly financed from the entrepreneur's net worth, $N_{t+1}(j)$, and partly from issuing nominal debt, $B_t(j)$:

$$q_t k_{t+1}(j) = N_{t+1}(j) + \frac{B_t(j)}{p_t}, \tag{3}$$

where q_t is the price of capital relative to the aggregate price p_t. Note that, unlike in BGG, the debt contract in this model is in nominal terms. That is, entrepreneurs sign a debt contract that specifies a nominal interest rate. To ensure that entrepreneurs will never accumulate enough funds to finance capital acquisitions entirely out of net worth, following BGG, I assume that they have finite lives. The probability that an entrepreneur survives until the next period is η^e.

The financial market imperfections are similar to those in BGG: because the idiosyncratic shock $\omega_t(j)$ is private information for the borrowers (entrepreneurs), there exists information asymmetry between borrowers and lenders (financial intermediaries). Due to costly state verification, lenders have to pay an auditing cost to observe the output of the borrowers. In BGG the optimal contract is a standard debt with costly bankruptcy: if the entrepreneur does not default, the lender receives a fixed payment independent of $\omega_t(j)$ but contingent upon the aggregate state; if the entrepreneur defaults, the lender audits and seizes the realized return (net of monitoring costs). The risk premium associated with external funds, $s(.)$, is defined as the ratio of the entrepreneur's cost of external funds to the cost of internal funds

$$s_t = \frac{E_t r^k_{t+1}}{E_t\left[r^n_t \frac{p_t}{p_{t+1}}\right]}, \tag{4}$$

where $E_t r^k_{t+1}$ is the expected rate of return of capital (defined in the next section), which is equal to the expected cost of external funds in equilibrium, and $E_t[r^n_t \frac{p_t}{p_{t+1}}]$ is the cost of internal funds. BGG solve a financial contract that maximizes the payoff to the entrepreneur, subject to the lender earning the required rate of return. BGG shows that this contract implies that the external finance premium, $s(.)$, depends on the entrepreneur's balance sheet position and it can be characterized by

$$s_t = s\left(\frac{q_t k_{t+1}(j)}{N_{t+1}(j)}\right), \tag{5}$$

where $s'(.) > 0$ and $s(1) = 1$.[2] Equation (5) expresses that the external finance premium increases with leverage, or decreases with the share of entrepreneurs' capital investment that is financed by the entrepreneur's own net worth. This is because when entrepreneurs rely more on external financing, the riskiness of loans increases. Lenders' expected loss increases and thus they charge a higher risk premium.

[2] See Appendix A in BGG for details.

2.2.1 Entrepreneurs' problem

The entrepreneur j's net worth, wealth accumulated by entrepreneurs from operating the firms, can be written as

$$N_{t+1}(j) = p_t^w(j)y^j + q_t(1-\delta)k_t(j) - p_t^l l_t(j) - \frac{r_{t-1}^n s_{t-1}}{1+\pi_t} b_{t-1}(j), \tag{6}$$

where p_t^w is the relative price of wholesale goods, p_t^l is the relative price of labour service which is provided by employment agencies, and b_t is the real debt ($b_t = B_t/P_t$). Thus the net worth is the entrepreneurs' earnings: $p_t^w(j)y^j + q_t(1-\delta)k_t(j)$ net of labour payment $p_t^l l_t(j)$ and interest payments to lenders $\frac{r_{t-1}^n s_{t-1}}{1+\pi_t} b_{t-1}(j)$. The profit for the entrepreneur j is given by

$$\begin{aligned}
\pi_t(j) &= b_t(j) + N_{t+1}(j) - q_t k_{t+1}(j) \\
&= b_t(j) + p_t^w(j)y^j - p_t^l l_t(j) + q_t(1-\delta)k_t(j) - \frac{r_{t-1}^n s_{t-1}}{1+\pi_t} b_{t-1}(j) - q_t k_{t+1}(j).
\end{aligned}$$

The entrepreneur j chooses $l_t(j)$, $k_{t+1}(j)$, and $b_t(j)$ to maximize

$$E_0 \sum_{t=0}^{\infty} \beta^t \pi_t(j).$$

The first order conditions yields:

$$l_t(j) : p_t^w \frac{\partial y_t(j)}{\partial l_t(j)} = p_t^l, \tag{7}$$

$$k_{t+1}(j) : -q_t + E_t\beta[p_{t+1}^w \frac{\partial y_{t+1}(j)}{\partial k_{t+1}(j)} + q_{t+1}(1-\delta)] = 0, \tag{8}$$

and

$$b_t(j) : 1 - E_t\beta[\frac{r_t^n s_t}{1+\pi_{t+1}}] = 0. \tag{9}$$

Equation (7) shows in the equilibrium the price for labour service is equal to its marginal productivity. Combining equation (8) and (9) yields

$$\frac{E_t[p_{t+1}^w \frac{\partial y_{t+1}(j)}{\partial k_{t+1}(j)} + q_{t+1}(1-\delta)]}{q_t} = E_t[\frac{r_t^n s_t}{1+\pi_{t+1}}]. \tag{10}$$

The left hand side of the equation (10) is the expected return of capital, which depends on the marginal productivity of capital $p_{t+1}^w \frac{\partial y_{t+1}(j)}{\partial k_{t+1}(j)}$ and the capital gain $\frac{q_{t+1}(1-\delta)}{q_t}$. The right hand of the equation is the expected cost of external funds, which is a product of risk premium s_t and the expected cost of internal funds $\frac{r_t^n}{1+\pi_{t+1}}$. Expected return of capital is defined as

$$E_t r_{t+1}^k(j) = \frac{E_t[p_{t+1}^w(j) \frac{\partial y_{t+1}(j)}{\partial k_{t+1}(j)} + q_{t+1}(1-\delta)]}{q_t}. \tag{11}$$

2.2.2 Aggregate Demand for labour Services, Capital and Financial Frictions

In this section I characterize the key equations that describe the aggregate behaviour for the entrepreneurial sector: equations for the aggregate demand curves for labour and capital, the equation for the aggregate stock of entrepreneurial net worth. I also address how the financial shock affects the demand for labour services in the model.[3]

Aggregate Demand for labour and Capital

Since production is constant returns to scale, aggregate production is

$$y_t = {k_t}^{\alpha}(z_t l_t)^{1-\alpha}.$$

Aggregating over equation (7) and equation (11) yields the following equations: the aggregate labour demand equation

$$p_t^w(1-\alpha)\frac{y_t}{l_t} = p_t^l, \tag{12}$$

and the equation of aggregate expected gross return on capital from periods t to $t+1$

$$E_t r_{t+1}^k = \frac{E_t[p_{t+1}^w \alpha \frac{y_{t+1}}{k_{t+1}} + q_{t+1}(1-\delta)]}{q_t}. \tag{13}$$

Thus, the equilibrium labour services is determined by the demand from the entrepreneurs (equation 12) and the supply from the employment agencies; the equilibrium capital demand depends on equation (13) and

$$E_t r_{t+1}^k = s_t r_t^n E_t\left[\frac{p_t}{p_{t+1}}\right], \tag{14}$$

which is the aggregate supply curve for external financing derived from equation (4).

Aggregate Net Worth

Aggregating over equation (6) yields the aggregate net worth equation

$$N_{t+1} = \eta^e \gamma_t (r_t^k q_{t-1} k_t - \frac{r_{t-1}^n s_{t-1}}{1+\pi_t} b_{t-1}).$$

The aggregate net worth of entrepreneurs at the end of period t, N_{t+1}, is the sum of equity held by entrepreneurs surviving from period $t-1$. Following Christiano, Motto and Rostagno (2007), I assume that there is a financial wealth shock, an exogenous shock to the survival probability of entrepreneurs, γ_t, which follows an AR(1) process:

$$\log\,\gamma_t = \rho_\gamma \log\,\gamma_{t-1} + \epsilon_t^\gamma, \quad \epsilon_t^\gamma \sim i.i.d.\,N(0, \sigma_{\epsilon^z}^2).$$

The reason why the shock on the survival probability of entrepreneurs has effects on their financial wealth is as follows: in the model, the number of entrepreneurs exiting is balanced by the number that enter. Since those who exit usually have more net worth than those who enter, when a positive

[3]See Appendix A for a more detailed derivation for this section's equations.

shock occurs, the aggregate net worth of entrepreneurs increases. This drives down the external finance premium, leading entrepreneurs to purchase more capital, which drives up asset price and increases entrepreneurs' net worth even more.

Entrepreneurs going out of business will consume their residual equity,

$$c_t^e = (1-\eta^e)\left(r_t^k q_{t-1} k_t - \frac{r_{t-1}^n s_{t-1}}{1+\pi_t}(q_{t-1}k_t - N_t)\right), \tag{15}$$

where c_t^e is the aggregate consumption of the entrepreneurs who exit in period t.

Demand for labour Services and the Financial Shock (spillover effect)

As equation (5) suggests, the external finance premium s_t depends on net worth N_t. After a positive financial wealth shock (an increase in the survival probability of entrepreneurs), aggregate net worth increases and the leverage falls. Since the entrepreneurs' balance-sheet position improves, the external finance premium falls. As a result, the demand for capital increases after a positive financial shock. The demand for labour services increases as well after the shock. To understand this, I rewrite equation (12)

$$p_t^w(1-\alpha)\frac{y_t}{l_t} = p_t^l,$$

as

$$p_t^w(1-\alpha)z_t^{1-\alpha}(\frac{k_t}{l_t})^\alpha = p_t^l. \tag{16}$$

Equation (16) suggests that given the relative wholesale goods price p_t^w, the price for labour services p_t^l, a constant capital labour ratio $\frac{k_t}{l_t}$ is optimal for entrepreneurs if the technology shock is absent. Thus, if a financial shock drives up the demand for capital, it will drive up the demand for labour services as well.

2.3 Employment Agencies

Following CTW, I assume that the key labour market activities–vacancy postings, wage bargaining–are all carried out by employment agencies instead of entrepreneurs themselves.[4] I assume that entrepreneurs obtain labour services supplied by employment agencies in a competitive labour market. Each employment agency i supplies labour services $n_t(i)$. The labour market is modeled using a search framework. The employment agencies make vacancy posting decisions and bargain with workers over nominal wages. I follow GST assuming a staggered multiple period nominal wage contracting.

In the next subsections, I describe the matching function, employment agencies' and workers' problem, and wage dynamics under this staggered Nash bargaining mechanism.

[4]Assuming that entrepreneurs face a frictional labour market will complicate aggregation.

2.3.1 Unemployment, Vacancies and Matching

At the beginning of period t, each employment agency i posts $v_t(i)$ vacancies in order to attract new workers and employs $n_t(i)$ workers. The total number of vacancies and employed workers are $v_t = \int v_t(i)di$ and $n_t = \int n_t(i)di$. The number of unemployed workers at the beginning of period t is

$$u_t = 1 - n_t.$$

The number of new hires or "matches", m_t, is governed by a standard Cobb-Douglas aggregate matching technology

$$m_t = \sigma_m u_t^{\sigma} v_t^{1-\sigma},$$

where σ_m is a parameter governing the matching efficiency. The probability a firm fills a vacancy in period t, q_t^l, is given by

$$q_t^l = \frac{m_t}{v_t}.$$

Similarly, the probability that a searching worker finds a job, s_t^l, is given by

$$s_t^l = \frac{m_t}{u_t}.$$

Both firms and workers take q_t^l and s_t^l as given. In each period, a fraction $1-\rho$ of existing workforce n_t exogenously separates from the firms. Thus, the total labour force is the sum of the number of surviving workers and the new matches:

$$n_{t+1} = \rho n_t + m_t. \tag{17}$$

2.3.2 Employment Agencies' Problem

To maximize comparability with the rest of the model, I assume that there are many employment agencies that supply labour services at a competitive price p_t^l. These agencies combine labour supplied by households into homogeneous labour services $n_t = \int n_t(i)di$ and supply them to entrepreneurs. This leaves the equilibrium conditions associated the production of wholesale goods unaffected even though the labour market is frictional. Define the hiring rate, $x_t(i)$, as the ratio of new hires, $q_t^l v_t(i)$, to the existing workforce, $n_t(i)$:

$$x_t(i) = \frac{q_t^l v_t(i)}{n_t(i)}.$$

Due to the law of large numbers the employment agency knows the likelihood q_t^l that each vacancy will be filled. The hiring rate is thus the employment agency's control variable. The total labour force can be also written as

$$n_{t+1} = \int n_{t+1}(i)di = \int (\rho n_t(i) + x_t(i)n_t(i))di,$$

which gives

$$m_t = \int x_t(i) n_t(i) di.$$

The value of the employment agency $F_t(i)$ is

$$F_t(i) = p_t^l n_t(i) - \frac{w_t^n(i)}{p_t} n_t(i) - \frac{\kappa}{2} x_t(i)^2 n_t(i) + \beta E_t \Lambda_{t,t+1} F_{t+1}(i),$$

where $\frac{\kappa}{2} x_t(i)^2 n_t(i)$ is the quadratic labour adjustment costs of posting vacancies, and $\beta E_t \Lambda_{t,t+1}$ is the employment agency's discount rate with $\Lambda_{t,t+1} = c_{t+1}/c_t$. At any time, the employment agency chooses the hiring rate $x_t(i)$ to maximize $F_t(i)$, given the existing employment stock, $n_{t,}$ the probability of filling a vacancy, $q_{t,}^l$ and the current and expected path of wages, $w_{t\cdot}^n$. $J_t(i)$, the value to the employment agency of adding another worker at time t, can be obtained by differentiating $F_t(i)$ with respect to $n_t(i)$:

$$J_t(i) = p_t^l - \frac{w_t^n(i)}{p_t} - \frac{\kappa}{2} x_t(i)^2 + (\rho + x_t(i)) \beta E_t \Lambda_{t,t+1} J_{t+1}(i). \tag{18}$$

The first order condition for vacancy posting equates the marginal cost of adding a worker with the discounted marginal benefit:

$$\kappa x_t(i) = \beta E_t \Lambda_{t,t+1} J_{t+1}(i). \tag{19}$$

Substituting equation (19) into equation (18):

$$J_t(i) = p_t^l - \frac{w_t^n(i)}{p_t} + \frac{\kappa}{2} x_t(i)^2 + \rho \beta E_t \Lambda_{t,t+1} J_{t+1}(i). \tag{20}$$

Combining equations yields the following forward looking difference equation for the hiring rate:

$$\kappa x_t(i) = \beta E_t \Lambda_{t,t+1} (p_{t+1}^l - \frac{w_{t+1}^n(i)}{p_{t+1}} + \frac{\kappa}{2} x_{t+1}(i)^2 + \rho \kappa x_{t+1}(i)).$$

Using the hiring rate condition and the evolution of the workforce, $J_t(i)$ can be written as

$$J_t(i) = p_t^l - \frac{w_t^n(i)}{p_t} + \frac{\kappa}{2} x_t(i)^2 + \rho \kappa x_t(i).$$

2.3.3 Workers' Problem

The value to a worker of employment at agency i, $V_t(i)$, is,

$$V_t(i) = w_t(i) + \beta E_t \Lambda_{t,t+1} [\rho V_{t+1}(i) + (1 - \rho) U_{t+1}].$$

The average value of employment on being a new worker at time t, V_t, is[5]

$$V_t = w_t + \beta E_t \Lambda_{t,t+1}[\rho V_{t+1} + (1-\rho)U_{t+1}],$$

where

$$V_t = \int V_t(i)\frac{x_t(i)n_t(i)}{x_t n_t}di.$$

The value of unemployment, U_t, depends on the unemployment benefit $\bar{b}$ and the probability of being employed versus unemployed next period:

$$U_t = \bar{b} + \beta E_t \Lambda_{t,t+1}[s^l_{t+1}V_{t+1} + (1-s^l_{t+1})U_{t+1}].$$

The worker surplus at firm i, $H_t(i)$, and the average worker surplus, H_t, are given by:

$$H_t(i) = V_t(i) - U_t,$$

and

$$H_t = V_t - U_t.$$

It follows that:

$$H_t(i) = w_t(i) - \bar{b} + \beta E_t \Lambda_{t,t+1}[\rho H_{t+1}(i) - s^l_{t+1}H_{t+1}]. \tag{21}$$

2.3.4 Nash Bargaining and Wage Dynamics

In this section, I introduce the staggered multi-period wage contracting and describe wage dynamics. A more explicit derivation is provided in Appendix B. Every period, each employment agency has a fixed probability $1-\lambda$ that it may renegotiate the nominal wage w^n_t (real wage $w_t = \frac{w^n_t}{p_t}$). At the beginning of period t, for employment agencies that are allowed to renegotiate the wage, they negotiate with the existing workforce, including the new hires. Due to constant returns, all workers are the same at the margin. For employment agencies that are not allowed to renegotiate the wage, all existing and newly hired workers receive the wage paid in the previous period. This simple Poisson adjustment process implies that it is not necessary to keep track of individual firms' wage histories, which simplifies aggregation. Given constant returns, all sets of renegotiating employment agencies and workers at time t face the same problem, and set the same nominal wage, w^{n*}_t. Thus, the renegotiating employment agency i solves the following problem:

$$\max H_t(i)^\eta J_t(i)^{1-\eta},$$

[5] See Gertler and Trigari (2009) for details about the average value of employment.

s.t.

$$\begin{aligned} w_t^n(i) &= w_t^{n*} \text{ with probability } 1-\lambda \\ &= w_{t-1}^n \pi \text{ with probability } \lambda, \end{aligned}$$

where π is the steady-state inflation rate. The first order condition for the Nash bargaining solution is given by

$$\eta \frac{\partial H_t(i)}{\partial w_t^n(i)} J_t(i) = (1-\eta) \frac{\partial J_t(i)}{\partial w_t^n(i)} H_t(i), \tag{22}$$

with

$$\frac{\partial H_t(i)}{\partial w_t^n(i)} = 1/p_t + \rho\lambda\pi\beta E_t \Lambda_{t,t+1} \frac{\partial H_{t+1}(i)}{\partial w_{t+1}^n(i)},$$

and

$$\frac{\partial J_t(i)}{\partial w_t^n(i)} = -1/p_t + \rho\lambda\pi\beta E_t \Lambda_{t,t+1} \frac{\partial J_{t+1}(i)}{\partial w_{t+1}^n(i)}.$$

Let $\epsilon_t = p_t \frac{\partial H_t(i)}{\partial w_t^n(i)}$ and $\mu_t = -p_t \frac{\partial J_t(i)}{\partial w_t^n(i)}$ and it can be shown that

$$\epsilon_t = \mu_t.$$

Given this, the first order condition for wages (equation 22) becomes the conventional sharing rule:[6]

$$\eta J_t(i) = (1-\eta) H_t(i). \tag{23}$$

However, due to the staggered wage contracting, $J_t(i)$ and $H_t(i)$ are different from the period-by-period Nash bargaining. To examine this, I first use $W_t(i)$ to denote the sum of expected future wage payments over the existing contract and subsequent contracts

$$\begin{aligned} W_t(i) &= E_t \sum_{s=0}^{\infty} (\rho\beta)^s \Lambda_{t,t+s} w_{t+s}(i) \\ &= w_t(i) + E_t \rho\beta \Lambda_{t,t+1} w_{t+1}(i) + E_t \rho^2 \beta^2 \Lambda_{t,t+2} w_{t+2}(i) + ..., \end{aligned}$$

which can be written as

$$W_t(i) = \Delta_t w_t^* + (1-\lambda) E_t \sum_{s=1}^{\infty} (\rho\beta)^s E_t \Lambda_{t,t+s} \Delta_{t+s} w_{t+s}^* \tag{24}$$

where

$$\Delta_t = E_t \sum_{s=0}^{\infty} (\rho\beta\lambda)^s \Lambda_{t,t+s} \frac{p_t}{p_{t+s}} \pi^s.$$

[6]Gertler and Trigari (2009) and Gertler, Sala and Trigari (2008) suggest $\mu_t > \epsilon_t$. This means that firms place a greater weight on the future than the workers do since firms have a longer horizon. This horizon effect makes firms more patient than workers and thus reduces the workers' bargaining power. However, it is not the case here.

Using $W_t(i)$, $H_t(i)$ and $J_r(i)$ can be written as

$$H_t(i) = W_t(i) - E_t \sum_{s=0}^{\infty} (\rho\beta)^s \Lambda_{t,t+s}[b + s_{t+s+1}\beta\Lambda_{t+s,t+s+1}H_{t+s+1}],$$

and

$$J_t(i) = E_t \sum_{s=0}^{\infty} (\rho\beta)^s \Lambda_{t,t+s}[p^l_{t+s} + \frac{\kappa}{2}x^2_{t+s}] - W_t(i).$$

Substituting equation (24) into $H_t(i)$ and $J_t(i)$, we have

$$\begin{aligned} H_t(i) &= \Delta_t w^*_t - E_t \sum_{s=0}^{\infty} (\rho\beta)^s \Lambda_{t,t+s}[\bar{b} + s_{t+s+1}\beta\Lambda_{t+s,t+s+1}H_{t+s+1} \\ &\quad -(1-\lambda)(\rho\beta)\Lambda_{t+s,t+s+1}\Delta_{t+s+1}w^*_{t+s+1}], \end{aligned} \tag{25}$$

and

$$\begin{aligned} J_t(i) &= E_t \sum_{s=1}^{\infty} (\rho\beta)^s \Lambda_{t,t+s}[p^l_{t+s} + \frac{\kappa}{2}x^2_{t+s}(i) \\ &\quad -(1-\lambda)(\rho\beta)\Lambda_{t+s,t+s+1}\Delta_{t+s+1}w^*_{t+s+1}] - \Delta_t w^*_t. \end{aligned} \tag{26}$$

Equations (25) and (26) suggest that with multi-period contracting, $H_t(i)$ and $J_t(i)$ will depend on λ. In the limiting case of $\lambda = 0$, $H_t(i)$ and $J_t(i)$ collapse to the values in the conventional period-by-period Nash bargaining. Substituting equations (25) and (26) into the Nash bargaining first-order condition,

$$\eta J_t(i) = (1-\eta)H_t(i),$$

it yields the following equation for the contract wage in real term w^*_t:

$$\begin{aligned} \Delta_t w^*_t &= \eta(p_t + \frac{\kappa}{2}x^2_t(i)) + (1-\eta)(\bar{b} + s_{t+1}\beta\Lambda_{t,t+1}H_{t+s+1}) \\ &\quad +\lambda\rho\beta E_t\Lambda_{t,t+1}\Delta_{t+1}w^*_{t+1}. \end{aligned} \tag{27}$$

The first two terms of equation (27) are conventional components for Nash bargaining solutions for wages: the first term is the worker's contribution to the match and the second is the workers' opportunity cost. The third term is from the staggered multi-period contracting. Following Gertler and Trigari (2009), I define a target wage $w^{tar}_t(i)$ as the sum of the first two terms:

$$w^{tar}_t(i) = \eta(p_t + \frac{\kappa}{2}x^2_t(i)) + (1-\eta)(\bar{b} + s^l_{t+1}\beta\Lambda_{t,t+1}H_{t+s+1}).$$

The target wage is computed as the wage that would arise under period-by-period Nash bargaining for the employment agency i, taking as given that all other employment agencies and workers operates on multi-period wage contracts. It is different from the conventional Nash bargaining wage w^{flex}_t, which would arise if all employment agencies and workers were operating on period-by-period wage contract:

$$w_t^{flex} = \eta(p_t^l + \frac{\kappa}{2}x_t^2 + s_{t+1}^l \kappa x_t) + (1-\eta)\bar{b}.$$

To examine the difference, I use the following equation

$$\beta E_t \Lambda_{t,t+1} H_{t+1} = \frac{\eta}{1-\eta}\kappa x_t(i) + \beta E_t \Lambda_{t,t+1} \lambda \pi \frac{p_t}{p_{t+1}} \Delta_{t+1}(w_t - w_t^*),$$

and rewrite the target wage equation as

$$\begin{aligned} w_t^{tar}(i) &= \eta(p_t^l + \frac{\kappa}{2}x_t^2(i) + \kappa s_{t+1}^l x_t(i)) + (1-\eta)\bar{b} \\ &\quad + (1-\eta)E_t s_{t+1}^l \beta \Lambda_{t,t+1} \lambda \pi \frac{p_t}{p_{t+1}} \Delta_{t+1}(w_t - w_t^*) \\ &= \eta(p_t^l + \frac{\kappa}{2}x_t^2 + \kappa s_{t+1}^l x_t) + (1-\eta)\bar{b} \\ &\quad + \eta[\frac{\kappa}{2}(x_t^2(i) - x_t^2) + \kappa s_{t+1}^l (x_t(i) - x_t)] \\ &\quad + (1-\eta)E_t s_{t+1}^l \beta \Lambda_{t,t+1} \lambda \pi \frac{p_t}{p_{t+1}} \Delta_{t+1}(w_t - w_t^*), \end{aligned} \tag{28}$$

where the first term is w_t^{flex} and w_t is the aggregate real wage, which is defined below. As suggested in Gertler and Trigari (2009), equation (28) reflects the impact of spillovers of economy-wide average wages on the individual bargaining wage between the employment agency and worker. When w_t exceeds w_t^*, everything else equal, it suggests workers' outside options are good. This will raise the target wage. The reverse happens if w_t is below w_t^*. The stickiness in the aggregate wage affects the individual wage bargain by this type of spillover, adding more inertia to the individual wages. In addition, the relative hiring rate, $x_t(i) - x_t$, can generate spillovers as well.

Finally, the aggregate nominal wage w_t^n is

$$w_t^n = (1-\lambda)w_t^{n*} + \lambda \pi w_{t-1}^n.$$

Thus in real terms we have

$$w_t = (1-\lambda)w_t^* + \lambda \pi \frac{1}{\pi_t} w_{t-1}.$$

2.4 Capital Producers

Capital production is assumed to be subject to an investment-specific shock, τ_t. Capital producers purchase the final goods from retailers as investment goods, i_t, and produce efficient investment goods, $\tau_t i_t$. Efficient investment goods are then combined with the existing capital stock to produce new capital goods, k_{t+1}. The aggregate capital stock evolves according to:

$$k_{t+1} = \tau_t i_t + (1-\delta)k_t.$$

The shock τ_t follows the first-order autoregressive process:

$$\log \tau_t = \rho_x \log \tau_{t-1} + \epsilon_t^{\tau}, \epsilon_t^{\tau} \sim i.i.d.N(0, \sigma_{\epsilon^{\tau}}^2).$$

Capital producers are also subject to a quadratic capital adjustment cost , $\frac{\xi}{2}(\frac{i_t}{k_t} - \delta)^2 k_t$. The profit of capital producers is

$$\Pi_t^k = E_t \left[q_t \tau_t i_t - i_t - \frac{\xi}{2} \left(\frac{i_t}{k_t} - \delta \right)^2 k_t \right], \tag{29}$$

and the first-order condition is

$$E_t \left[q_t \tau_t - 1 - \xi \left(\frac{i_t}{k_t} - \delta \right) \right] = 0. \tag{30}$$

2.5 Retailers

There is a continuum of monopolistically competitive retailers of measure 1. Retailers buy wholesale goods from entrepreneurs and produce a good of variety j. Let $y_t(j)$ be the retail good sold by retailer j to households and let $p_t(j)$ be its nominal price. The final good, y_t, is the composite of individual retail goods,

$$y_t = \left[\int_0^1 y_t(j)^{\frac{\varepsilon-1}{\varepsilon}} dj \right]^{\frac{\varepsilon}{\varepsilon-1}}.$$

Following the household's expenditure minimization problem, the corresponding price index, p_t, is given by

$$p_t = \left[\int_0^1 p_t(j)^{1-\varepsilon} dj \right]^{\frac{1}{1-\varepsilon}},$$

and the demand function faced by each retailer is given by

$$y_t(j) = \left(\frac{p_t(j)}{p_t} \right)^{-\varepsilon} y_t. \tag{31}$$

Following Calvo (1983), each retailer cannot change prices unless it receives a random signal. The probability of receiving such a signal is $1 - \nu$. Thus, in each period, only a fraction of $1 - \nu$ of retailers reset their prices, while the remaining retailers keep their prices unchanged. Given the demand function equation (31), the retailer chooses $p_t(j)$ to maximize its expected real total profit over the periods during which its prices remain fixed:

$$E_t \Sigma_{i=0}^{\infty} \nu \Delta_{i,t+i}^p \left[\left(\frac{p_t(j)}{p_{t+i}} \right) y_{t+i}(j) - mc_{t+i} y_{t+i}(j) \right],$$

where $\Delta_{t,i}^p \equiv \beta^i c_{t+i}/c_t$ is the stochastic discount factor and the real marginal cost, mc_t, is the price of wholesale goods relative to the price of final goods $(p_{w,t}/p_t)$. Let p_t^* be the optimal price chosen by all firms adjusting at time t. The first order condition is:

$$p_t^* = \left(\frac{\varepsilon}{\varepsilon - 1}\right) \frac{E_t \sum_{i=0}^{\infty} \nu^i \Delta_{i,t+i}^p mc_{t+1} y_{t+i} (\frac{1}{p_{t+i}})^{-\varepsilon}}{E_t \sum_{i=0}^{\infty} \nu^i \Delta_{i,t+i}^p y_{t+i} (\frac{1}{p_{t+i}})^{1-\varepsilon}}.$$

The aggregate price evolves according to:

$$p_t = [\nu p_{t-1}^{1-\varepsilon} + (1-\nu)(p_t^*)^{1-\varepsilon}]^{\frac{1}{1-\varepsilon}}.$$

2.6 Government

I assume that the government spending is g_t and it balances its budget,

$$g_t = T_t,$$

where g_t follows an AR(1) process,

$$\log g_t = (1-\rho_x) \log g_{ss} + \rho_x \log g_{t-1} + \epsilon_t^g, \epsilon_t^g \sim i.i.d.N(0, \sigma_{\epsilon^g}^2).$$

2.7 Monetary Policy Rules

The central bank is assumed to operate according to the standard Taylor Rule. The central bank adjusts the nominal interest rate, r_t^n, in response to deviations of inflation, π_t, from its steady-state value, π, and output, y_t, from its steady-state level, y.

$$\frac{r_t^n}{r^n} = (\frac{r_{t-1}^n}{r^n})^{\rho_r} ((\frac{\pi_t}{\pi})^{\rho_\pi} (\frac{y_t}{y})^{\rho_y})^{1-\rho_r} e^{\epsilon_t^m},$$

where r^n, π and y are the steady-state values of r_t^n, π_t and y_t, and ε_t^m is a monetary policy shock which follows

$$\varepsilon_t^m \sim i.i.d.\, N(0, \sigma_{\varepsilon^m}).$$

ρ_π, ρ_y and ρ_r are policy coefficients chosen by the central bank.

2.8 Aggregation and Equilibrium

The resource constraint for final goods is

$$z_t k_t^\alpha l_t{}^{1-\alpha} = c_t + c_t^e + i_t + g_t + \frac{\xi}{2}\left(\frac{i_t}{k_t} - \delta\right)^2 k_t + \frac{\kappa}{2} x_t^2 n_t.$$

Furthermore, for the labour market we have

$$l_t = n_t.$$

3 Data and Estimation

3.1 Data

I first log-linearize the model around the steady-state. Appendix D and E contain the complete log-linear model, as well as the steady-state conditions. I then adopt a Bayesian approach to estimate the model. I use six series of quarterly US data: output, consumption, investment, nominal interest rate, inflation and external finance cost. The sample spans from $1964Q1$ to $2010Q3$. Data on output, consumption and investment are expressed in per capita terms using the civilian population aged 15 and up. Output is measured by real GDP. Consumption is measured by real expenditures of non-durable goods, services and durable goods. Investment is measured by real private investment. The nominal interest rate is measured by Federal Funds rate expressed in quarterly terms. Inflation is the quarter-to-quarter growth rate of the GDP deflator. External finance costs are measured by U.S. business prime lending rate in real terms. All the series are detrended using an HP filter with smoothing parameter 1600.

Table 1: Calibrated Values

β	discount factor	0.99
σ	inverse of intertemporal substitution of consumption	2
α	capital share	0.33
δ	capital depreciation rate	0.025
ϵ	intermediate-good elasticity of substitution	11
N/k	steady-state ratio of net worth to capital	0.6
η^e	survivor rate of entrepreneurs	0.985
ρ	survival rate of firms	0.90
s^l	job finding rate	0.95
q^l	job filling rate	0.75
η	bargaining power of workers	0.5
$\tilde{b}$	parameter for unemployment flow value	0.4
σ_m	elasticity in matches to unemployment	0.5

3.2 Calibrated Values

As is standard when taking DSGE models to the data, the parameters for which the data used contain only limited information are calibrated to match salient features of the U.S. economy. Table 1 reports the calibrated values. There are 13 parameters. Two of them are for financial market, six of them are for labour market, and the rest of the parameters are "conventional" parameters. Financial market parameters include the survival rate of entrepreneurs, η^e, and the steady-state ratio of net worth to capital N/k. I set $\eta^e = 0.985$ so that the steady-state external risk premium is 200 basis points, which is the sample average spread between the prime lending rate and Federal Funds rate.

I also set N/k to 0.6, which is close to the value used in Christensen and Dib (2008). In calibration, I adopt the following functional form for the external finance premium:

$$s_t = \left(\frac{q_t k_{t+1}}{N_{t+1}}\right)^{\chi}, \tag{32}$$

where χ is the elasticity of external risk premium with respect to leverage and $\chi > 0$. χ is a "reduced form" parameter capturing financial market frictions.

For the labour market parameters, I set the bargaining power parameter, η, to be 0.5, which is commonly used in the literature. The elasticity of matches to unemployment, σ_m, is set to to be 0.5, the midpoint of values typically used. The job separation rate, $1-\rho$, is set to be 0.1, matching the average job duration of two and a half years in the US. The job finding rate s^l is set to be 0.95 as in Shimer (2005). The average job filling rate q^l is set to 0.75, which is suggested by den Haan, Ramey and Watson (2000). Following GST, I express $\bar{b}$, the steady state flow value of unemployment as

$$\bar{b} = \tilde{b}(p^l + \frac{\kappa}{2}x^2), \tag{33}$$

where $\tilde{b}$ is the fraction of the contribution of the worker to the job. I choose $\tilde{b}$ to be 0.4, following Shimer (2005).

I use conventional values for the five "conventional" parameters. The discount factor β is set to be 0.99, which corresponds to an annual real interest rate in the steady-state at four per cent. The curvature parameter in the utility function, σ, is set to 2, implying an elasticity of intertemporal substitution of 0.5. The steady-state depreciation rate, δ, is set to 0.025, which implies an annual rate of depreciation of ten per cent. The parameter of the Cobb-Douglas function, α, is set to be $1/3$. The steady-state price mark up $\varepsilon/(\varepsilon-1)$ is 1.1 by setting $\varepsilon = 11$.

Table 2: Prior and Posterior Distribution of Structural Parameters: Baseline

		Prior distribution	Posterior distribution			
			Mode	Mean	5%	95%
Risk premium elasticity	χ	gamma (0.05,0.02)	0.230	0.240	0.203	0.288
Calvo wage parameter	λ	beta (0.67, 0.05)	0.810	0.806	0.777	0.833
Calvo price parameter	ν	beta (0.67, 0.05)	0.538	0.530	0.470	0.590
Capital adj. cost parameter	ξ	norm (0.25, 0.05)	0.217	0.216	0.144	0.292
Taylor rule inertia	ρ_r	beta (0.75, 0.1)	0.275	0.292	0.213	0.372
Taylor rule inflation	ρ_π	gamma(1.5, 0.1)	1.675	1.685	1.562	1.782
Taylor rule output gap	ρ_y	norm (0.125, 0.15)	-0.006	-0.007	-0.022	0.008

Table 3: Prior and Posterior Distribution of Shock Parameters: Baseline

		Prior distribution	Posterior distribution: Mode	Mean	5%	95%
Panel A: Autoregressive parameters						
Technology	ρ_z	beta (0.6,0.2)	0.896	0.891	0.867	0.914
Preference	ρ_e	beta (0.6,0.2)	0.598	0.591	0.471	0.709
Investment	ρ_τ	beta (0.6,0.2)	0.834	0.813	0.741	0.882
Government	ρ_g	beta (0.6,0.2)	0.692	0.687	0.623	0.759
Financial	ρ_γ	beta (0.6,0.2)	0.242	0.270	0.095	0.444
Panel B: Standard deviations						
Technology	σ_{ϵ^z}	invg (0.005,2)	0.83	0.83	0.76	0.90
Monetary	σ_{ϵ^m}	invg (0.005,2)	0.34	0.34	0.30	0.38
Investment	σ_{ϵ^τ}	invg (0.005,2)	1.66	1.57	0.98	1.99
Preference	σ_{ϵ^e}	invg (0.005,2)	1.03	1.06	0.96	1.15
Government	σ_{ϵ^g}	invg (0.005,2)	1.02	1.02	0.95	1.11
Financial	σ_{ϵ^γ}	invg (0.005,2)	0.55	0.54	0.41	0.67

3.3 Priors

I estimate the remaining parameters: the elasticity of external risk premium, χ; the capital adjustment cost parameter ξ; the Calvo price and wage parameters ν and λ; and the Taylor rule parameters, ρ_π, ρ_y, and ρ_r. I also estimate the first-order autocorrelations of all the exogenous shocks and their respective standard deviations. Tables 2 and 3 report the prior and the posterior distributions for each of them. Among the behavioural parameters listed in Table 2, the Taylor rule parameters, the Calvo price and capital adjustment cost parameters are rather conventional. For the priors of these parameters, I closely follow the existing literature. The elasticity of external risk premium χ and Calvo wage parameter λ are less conventional. In the literature, χ is typically calibrated at 0.05 as in BGG. Thus, I assume that χ follows a gamma distribution with mean 0.05 and standard deviation 0.02. Since there is not much guidance for the average wage contracting duration, I assume that Calvo wage parameter λ follows the same prior distribution as Calvo price parameter ν, which suggests that firms negotiate wage contract with workers every 3 quarters on average.

The priors of the shock processes are presented in Table 3. I follow Smets and Wouters (2007), the priors on the shock processes are harmonized as much as possible. The standard deviation of the shocks are assumed to follow an Inverted Gamma distribution with a mean of 0.5 per cent and two degrees of freedom. The persistence of the shock processes is beta distributed with mean 0.6 and standard deviation 0.2.

I use Dynare 3.065 to estimate the model and use Metropolis-Hastings algorithm to perform simulations. The total number of draws is 20,000 and the first 20 per cent draws are neglected. A step size of 0.4 results in a rejection rate of 0.38.

4 Estimation Results

4.1 Posterior Estimates of the Parameters

Table 2 gives the mode, the mean and the 5 and 95 percentiles of the posterior distribution of the behavioral parameters. The risk premium elasticity parameter, χ, is estimated to be around 0.24 (mean 0.24, mode 0.23). Christensen and Dib (2008) use maximum likelihood procedure to estimate a sticky-price model with a financial accelerator on U.S. data and suggest that χ is around 0.042. Compared to their value, $\chi = 0.24$ is much higher. Since Christensen and Dib (2008) do not use any financial data in their estimation, this much larger elasticity might have resulted from the information contained in the financial data I used in the estimation. Calvo wage contract parameter, λ, is estimated to be around 0.81, suggesting a mean of five quarters between wage contracting periods. This value is higher than the estimate of the same parameter in GST, which is $\lambda = 0.72$. This might because the flow value of unemployment, $\tilde{b}$, of 0.73 used in their paper is much higher than the calibrated value of 0.4 in this paper. A higher $\tilde{b}$ helps generate higher volatility in unemployment and vacancies and lower volatility in real wages. Thus, with a higher $\tilde{b}$, a lower λ is able to generate the same degree of wage rigidity. The estimates of the "conventional" parameters are consistent with other studies. The degree of price stickiness, ν, is estimated to be 0.53, which implies average price adjustment duration of half a year. The capital adjustment cost parameter, ξ, is estimated to be around 0.22. For the monetary policy reaction function parameters, ρ_π, the Taylor rule inflation parameter, is estimated to be 1.68, and the reaction coefficient to output gap, ρ_y, is estimated to be -0.007, suggesting that policy respond very little to output gap. There is a relatively low degree of interest rate smoothing, as the coefficient on the lagged interest rate is estimated to be 0.29.

Table 3 presents the estimates of the shock processes. The new shock, financial wealth shock, appears to be the least persistent shock, with an AR(1) coefficient of 0.27. The technology and investment shocks are estimated to be most persistent, with a coefficient of 0.89 and 0.81, respectively. The mean of the standard error of the shock to investment is 1.57, suggesting it is the most volatile shock. In contrast, the standard deviation of the new financial shock is relatively low at 0.54.

4.2 Empirical Fit

One way to assess how the model captures the data is to compare the volatilities of the model against the data. Table 4 reports this information. Overall the model does a decent job in matching the data. It does particularly well in matching the volatility in consumption, investment and inflation. For the key labour market variables, the model is able to capture the fact that both unemployment and vacancies are highly volatile and real wages are relatively rigid, although the model predicted volatility for each of them is higher than that in the data. For financial variables, the model is able to capture 50% of the relative volatility in external finance cost fc.

Table 4: Relative Standard Deviations: Model vs Data

	y	c	i	w	v	u	r_n	π	fc
Data	1	0.82	5.02	0.44	9.29	8.10	0.28	0.20	0.24
Baseline	1	0.83	4.19	0.89	14.00	11.32	0.36	0.23	0.13

4.3 Sources of labour Market Fluctuations

Given the estimation results of the shock processes, I next simulate the model to examine the contribution of each shock to the variations in the key labour market variables. Table 5 presents the results. The financial shock appears to be the most important shock determining the variations in unemployment and vacancies. It accounts for 37% of the variations in these two variables. Investment-specific and technology shocks are next in importance, accounting for roughly 33% and 26% of the variations in unemployment and vacancies, respectively. For real wages, the technology shock is the main driving force, and it accounts for 43% of the variations. The financial shock accounts for 30% and investment- specific shock accounts for 23% for the real wages. The result that the financial shock is the main driving force for both unemployment and vacancies is somewhat surprising, given that it is the least persistent among the six shocks and has a low standard deviation. This suggests that the financial accelerator mechanism might have played an important role in amplifying the shock internally.

Table 5: Variance Decomposition of the Key labour Market Variables

	Technology	Monetary	Financial	Investment	Preference	Government
u	26.1	1.2	37.2	33.1	0.3	2.0
v	26.2	1.3	37.4	32.9	0.3	2.0
w	43.7	1.8	29.6	23.4	0.2	1.4

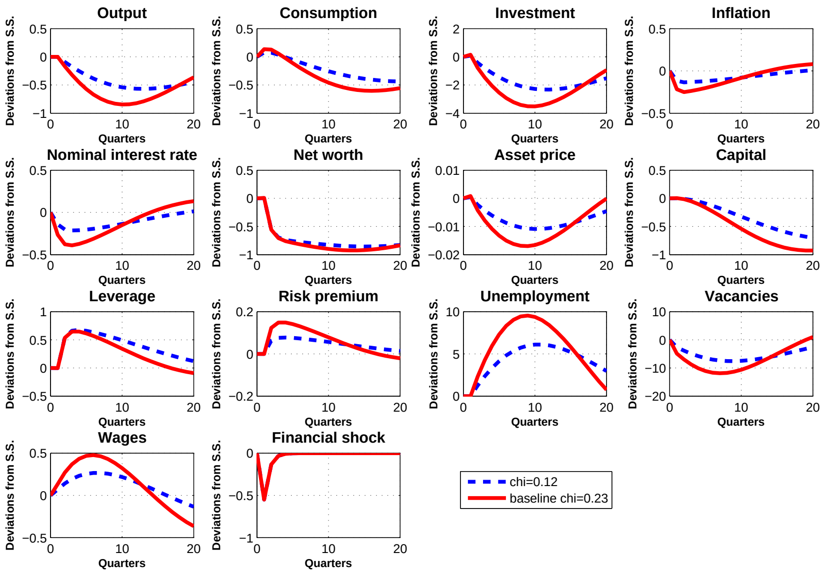
4.4 Amplification Effect of the Financial Accelerator

I examine this issue by simulating the response of several key variables after the financial shock. I analyze the role of financial frictions by examining both the baseline model and the same model with the financial frictions reduced by half (χ is reduced to 0.12).[7] Figure 1 illustrates the response of the model economy to a negative financial shock. The solid line is the baseline model. The dotted line is the model with $\chi = 0.12$. In both cases, following a negative financial wealth shock, the survivor rate of the entrepreneurs decreases, causing the aggregate net worth to fall. This drives up the external finance premium, forcing entrepreneurs to reduce their demand for capital by reducing investment. The fall in demand for capital is accompanied by the fall in demand for labour. Asset

[7]The rest of the parameters are the same for both models.

price falls with the reduced demand for capital, and this further decreases entrepreneurs' net worth (Financial accelerator effect). Due to the fall in the aggregate demand for labour, employment agencies post fewer vacancies. This reduces the probability for a worker to find a job, and leads the unemployment rate to rise. Notice that after the shock, the initial responses of net worth and leverage ratio are similar for both cases; however, the response of risk premium is significantly greater in the baseline than in the alternative model due to the higher value of χ. The stronger rise in the risk premium leads to a stronger response in demand for capital. Asset price declines further, driving net worth further down. The amplification effect of the financial accelerator is more significant in the baseline model and this leads to stronger responses of the other variables to the financial shock.

Figure 1: Effects of an Negative Financial Shock



I further simulate the alternative model to examine the contribution of the financial shock to the labour market fluctuation, and compare the results with that from the baseline model in Table 6. The results suggest that with a weaker financial accelerator effect, the financial shock becomes less important, contributing only 23% of the variations in unemployment and vacancies, which is a 32 per cent drop from the baseline case.

Table 6: Variance Decomposition for labour Market Variables: a Comparison

	Less friction ($\chi = 0.12$)			Baseline		
Shocks	Unemployment	Vacancy	Real wage	Unemployment	Vacancy	Real wage
Technology	33.04	33.19	49.46	25.97	26.06	43.54
Monetary	1.39	1.45	1.75	1.24	1.3	1.77
Investment	40.15	40.06	28.84	32.99	32.71	23.27
Preference	0.24	0.23	0.16	0.3	0.3	0.22
Government	1.57	1.56	1.07	1.97	1.96	1.37
Financial	23.6	23.52	18.71	37.51	37.67	29.82

4.5 Financial shock and financial data

In order to further identify the importance of the financial shock and financial data, I re-estimate the model but without the financial shock and without the financial time series. I compare the results from the alternative model (NoFS model) with the baseline model in Tables 7 and 8. The estimates of the behavioral parameters and the shock processes do not change much. There is, however, a large change in the estimate of the elasticity of external financing: χ falls to 0.009 from 0.23. This significant change might reflect the fact that it is important to include financial time series to identify financial frictions. I next explore how well the NoFS model is able to account for the overall volatility in the data compared to the baseline model. Table 9 presents the results. Overall, NoFS model matches the data less well. In particular, the NoFS model does not come close to generating the relative volatility of unemployment and vacancies in the data. Table 10 presents the comparison of the variance decomposition of the key labour market variables for the two models. Without the financial shock, the technology shock becomes the most important shock: it explains 78% of the variance of unemployment and vacancies, and 96% of the variance of real wage.

Table 7: Posterior Mode of Structural Parameters: No FS vs. Baseline

		Posterior			
		NoFS		Baseline	
		Mode	SD	Mode	SD
Risk premium elasticity	χ	0.009	0.004	0.230	0.023
Calvo wage parameter	λ	0.628	0.039	0.810	0.014
Calvo price parameter	ν	0.703	0.054	0.538	0.038
Capital adj. cost parameter	ξ	0.230	0.050	0.217	0.050
Taylor rule inertia	ρ_r	0.238	0.047	0.275	0.046
Taylor rule inflation	ρ_π	1.841	0.086	1.675	0.062
Taylor rule output gap	ρ_y	0.021	0.015	-0.006	0.009

Table 8: Posterior Mode of Shock Parameters: No FS vs. Baseline

		Posterior			
		NoFS		Baseline	
		Mode	SD	Mode	SD
Panel A: Autoregressive parameters					
Technology	ρ_z	0.897	0.012	0.896	0.015
Preference	ρ_e	0.547	0.074	0.598	0.077
Investment	ρ_τ	0.816	0.033	0.834	0.027
Government spending	ρ_g	0.720	0.036	0.692	0.040
Financial	ρ_γ	-	-	0.242	0.110
Panel B: Standard deviations					
Technology	σ_{ϵ^z}	0.81	0.04	0.83	0.04
Monetary	σ_{ϵ^m}	0.37	0.03	0.34	0.02
Investment	σ_{ϵ^τ}	0.62	0.08	1.66	0.25
Preference	σ_{ϵ^e}	1.07	0.06	1.03	0.06
Government spending	σ_{ϵ^g}	0.98	0.05	1.02	0.05
Financial	σ_{ϵ^γ}	-	-	0.55	0.07

Table 9: Relative Standard Deviations: Model vs Data

	y	c	i	w	v	u	r_n	π	fc
Data	1	0.82	5.02	0.44	9.29	8.10	0.28	0.20	0.24
NoFS	1	1.39	3.18	0.83	2.10	1.62	0.17	0.14	0.09
Baseline	1	0.83	4.19	0.89	14.00	11.32	0.36	0.23	0.13

Table 10: Variance Decomposition for labour Market Variables

	No FS			Baseline		
Shocks	Unemployment	Vacancy	Real wage	Unemployment	Vacancy	Real wage
Technology	78.05	78.81	96.25	25.97	26.06	43.54
Monetary	0.7	0.7	0.31	1.24	1.3	1.77
Investment	20.94	20.21	2.13	32.99	32.71	23.27
Preference	0.03	0.03	0.14	0.3	0.3	0.22
Government	0.28	0.25	1.17	1.97	1.96	1.37
Financial	-	-	-	37.51	37.67	29.82

5 Issues

In this section I address several issues involving the robustness of the results.

5.1 Staggered wage contracting

The previous section suggests that without the significant amplification effect from the financial accelerator mechanism ($\chi = 0.009$), staggered wage contracting alone cannot generate enough volatility in unemployment and vacancies. This result seems to contradict the finding in GST that a model with wage rigidity provides a better fit for the dynamics of the labour market variables. Their results do not rely on the amplification effect of the financial accelerator. However, as noticed in the previous section, the flow value of unemployment in GST, $\tilde{b}$, is much higher. As suggested in Hagedorn and Manovskii (2008), when $\tilde{b}$ is close to unity, the value of unemployment is very close to that of employment to the worker. Since labour supply is very elastic in this case, this high $\tilde{b}$ might have helped their model to capture unemployment and wage dynamics in the data. Moreover, the workers' bargaining power parameter, η, used in GST, is 0.9, which lies well above the range considered in the literature, 0.5-0.7. In order to examine the effects of these "unconventional" values on the labour market dynamics, I simulate the model using their values for $\tilde{b}$ and η while keeping the rest of the parameters the same as in the NoFS case

Table 11: Relative Standard Deviations: Model Comparison

	y	c	i	w	v	u	r_n	π	fc
Data	1	0.82	5.02	0.44	9.29	8.10	0.28	0.20	0.24
Baseline	1	0.83	4.19	0.89	14.00	11.32	0.36	0.23	0.13
NoFS	1	1.39	3.18	0.83	2.10	1.62	0.17	0.14	0.09
No FS with high $\tilde{b}$ and η	1	0.53	4.59	0.80	13.14	10.38	0.15	0.10	0.08
Baseline w/ flexible wages	1	1.97	3.40	0.88	1.79	1.45	0.46	0.29	0.21

Table 11 shows that with the higher values for the flow value of unemployment and bargaining power, the NoFS model generates similar variability in unemployment and vacancies as the baseline model, suggesting that mechanically these unconventional values play the same role in amplifying the responses in unemployment and vacancies to shocks as the financial accelerator mechanism in the baseline model.

Although the staggered wage contracting is not a sufficient condition to generate enough variability in unemployment and vacancies, it is a necessary condition for the model in this paper to match the data. To examine how important this type of wage setting friction is in the model, I simulate the baseline model again but assume period-by-period wage contracting (by setting $\lambda = 0$), and compare the relative volatilities of the key variables generated by the alternative model (the last row in Table 11) with the baseline case. As Table 11 makes clear, the flexible wage case is not able to generate enough variability in unemployment and vacancies even though the external finance premium stays very elastic ($\chi = 0.23$). This result confirms the argument of recent studies that the conventional search models cannot account for the key cyclical movements of unemployment and vacancies in the labour market.

Overall, Table 11 suggests that the interaction of the financial accelerator mechanism and wage setting frictions is the key for the model to match the data.

5.2 Elasticity of External Risk Premium

The elasticity of the external risk premium χ is the key parameter of the financial accelerator mechanism. In BGG, χ is a "reduced form" parameter that captures financial frictions. It is determined by the "deep" parameters in the original BGG model: the variance of idiosyncratic shocks to the return on capital, the bankruptcy costs, and entrepreneurs' survival rate. In the literature, it is typically calibrated at 0.05.[8] The examples of estimated models based on BGG for US data are Christensen and Dib (2008), De Graeve (2008) and Queijo (2009). None of these paper use financial data. Christensen and Dib (2008) and Queijo (2009) estimate χ to be around 0.04.[9] De Graeve (2008) estimates χ to be at 0.1. Compared to these values, the estimate of $\chi = 0.23$ in the baseline model is substantially higher. As suggested in the previous section, the high value of χ might be due to the inclusion of the financial data. In other words, the non financial variables used in the estimation in those studies contain very limited information on financial frictions, thus χ is underestimated. Another example is CTW. CTW estimate a model with financial shocks and use two financial time series. They have not estimated χ explicitly, however, the estimated monitoring cost parameter in their model, which is one of "deep" parameters determining χ, is almost 4 times higher than the one BGG proposed.[10]

5.3 Estimations of Subsamples

Since the importance of the financial shock depends χ and χ can be sensitive to the sample period, I follow Smets and Wouters (2007) and divide the data into two subsamples: the first one is the "Great Inflation" period 1966:2-1979:2, and the second subsample is from 1984:1-2010:3, which includes mostly the "Great Moderation" period and the recent recession. I estimate the model over these two subsamples. It is well-known that output and inflation volatility fall considerably during the "Great Moderation" period. Table 12 presents the standard deviations of the key macro

[8] See for examples, Bernanke, Gertler and Gilchrist (1999) and Bernanke and Gertler (2000).

[9] Queijo (2009) does not estimate χ directly; instead she estimates parameters for bankruptcy costs and survival rate of entrepreneurs. The estimates of these parameters imply that χ is 0.04.

[10] The estimated value of $\chi = 0.009$ in the NoFS case is lower than conventional wisdom. To investigate why χ is low in the NoFS model. I re-estimate the NoFS model using the data from the same time period from 1979Q3 to 2004Q3 as Christensen and Dib (2008). I also reset the quarterly survival rate for entrepreneurs, η^e, to be the value of 0.9728 used in their paper in order to capture the fact that during this period the average spread is higher. The estimate of χ is around 0.02, which is closer to the estimated value in Christensen and Dib (2008), although it is still lower. This suggests that the difference in sample periods might be one factor contributing the difference in the estimation results.

variables and the key labour market variables. It shows that the volatilities of the unemployment and vacancies fall as well, although the volatility in real wage appears to stay the same. Table 13

Table 12: Standard Deviations of Subsamples

	66:1-79:2	84:1-10:3
Output	0.016	0.010
Consumption	0.014	0.008
Investment	0.076	0.058
Real wage	0.005	0.005
Vacancy	0.138	0.112
Unemployment	0.132	0.103
Nom. Interest rate	0.005	0.003
Inflation	0.004	0.002

compares the modes of the posterior distribution of the model parameters over these two periods. Similar to Smets and Wouters (2007), the most significant differences between the two subsamples are in the variances of the shock processes. The standard deviations of all the shocks considered in the model fall in the second period, including the financial shock, which falls by half from 0.56 to 0.23. The persistence of these shocks changes much less for the most part; however, interestingly the persistence of the financial shock increases significantly during the second period from 0.29 to 0.62. Table 14 shows that due to this increased persistence, the financial shock accounts for a higher portion of the variability in unemployment, vacancies and real wages.[11]

6 Concluding Remarks

In this paper, I argue that the spillover effects of financial market shocks are important for labour market fluctuations. Although the estimation results suggest that the financial disturbance is neither persistent nor volatile, the financial accelerator mechanism amplifies the financial shock and generates large fluctuations in the labour market. Overall, I find that more than 30 per cent of the variations in unemployment and vacancies is explained by the financial shock. I show that ignoring these financial shocks and financial data can reduce the model's explanatory power. In particular, the model without these financial factors has difficulties matching the observed volatility in unemployment and vacancies.

However, the fit of the model largely relies on the value of one key parameter: χ, the parameter that summarizes the frictions in the financial market. I show that the estimate of χ lies above conventional wisdom when the financial data is included in the estimation. To ensure that this key parameter is well identified, more work on checking the robustness of the model is necessary. Another line of future research would be to incorporate financial frictions that explicitly affect firms'

[11] I have also estimated the model for the "Great Moderation" period only (1984:1-2004:4). The results are similar to those from the second subsample. This might be due to that the recent recession period is relatively short, and the "Great Moderation" periods dominate the results for the second subsample.

Table 13: Subsample Estimates

Structural parameters					Shock process				
	1966:1-1979:2		1984:1-2010:3			1966:1-1979:2		1984:1-2010:3	
	Mode	SD	Mode	SD		Mode	SD	Mode	SD
χ	0.206	0.0296	0.2139	0.0203	ρ_z	0.9039	0.0267	0.8491	0.0263
λ	0.7704	0.0312	0.8107	0.0162	ρ_e	0.5499	0.1531	0.6645	0.073
ν	0.5778	0.0412	0.5678	0.0406	ρ_τ	0.8668	0.0317	0.8553	0.022
ξ	0.2383	0.05	0.2202	0.0502	ρ_g	0.6944	0.0768	0.6253	0.0568
ρ_r	0.317	0.0658	0.4976	0.0554	ρ_γ	0.29	0.1716	0.6203	0.1085
ρ_π	1.5068	0.0822	1.8207	0.0839	σ_{ϵ^z}	0.99	0.09	0.56	0.04
ρ_y	0.017	0.0208	-0.0165	0.0146	σ_{ϵ^m}	0.35	0.04	0.2	0.02
					σ_{ϵ^τ}	2.16	0.47	1.54	0.22
					σ_{ϵ^e}	1.39	0.17	0.83	0.06
					σ_{ϵ^g}	1.12	0.11	0.91	0.06
					σ_{ϵ^γ}	0.56	0.11	0.23	0.04

Table 14: Variance Decomposition: Subsamples

	1966:1-1979:2			1984:1-2010:3		
	Unemployment	Vacancy	Real wage	Unemployment	Vacancy	Real wage
Technology	30.56	30.6	58.76	13.71	13.89	25.72
Monetary	1.32	1.41	1.73	1.82	1.9	2.84
Investment	33.05	32.65	17.56	41.97	41.62	33
Preference	0.4	0.4	0.21	0.44	0.44	0.36
Government	2.09	2.09	1.02	1.86	1.86	1.56
Financial	32.58	32.86	20.71	40.2	40.3	36.52

hiring decisions into a monetary DSGE model. As the existing literature has already shown, firms' hiring activities can be directly related to how easily firms are able to access external funds. It would be interesting to evaluate the impact of this type of financial frictions on unemployment and vacancies at the aggregate level.

7 Appendix

7.1 Appendix A: Aggregation of labour and Capital Demand, Aggregation of Net Worth

For each entrepreneur j, the labour demand is determined by

$$\frac{\partial y_t(j)}{\partial l_t^h(j)} = (1-\alpha)\frac{y_t(j)}{l_t(j)} = p_t^l.$$

Since all the firm j has the same output labour ratio, aggregating this first order condition yields

$$p_t^w(1-\alpha)\frac{y_t}{l_t} = p_t^l.$$

Define the expected return on capital as

$$E_t r_{t+1}^k(j) = \frac{E_t[p_{t+1}^w(j)\alpha\frac{\partial y_{t+1}(j)}{\partial k_{t+1}(j)} + q_{t+1}(1-\delta)]}{q_t},$$

and aggregate over this first order condition, we have

$$\begin{aligned} E_t R_{t+1}^k &= \int \frac{E_t[p_{t+1}^w(j)\alpha\frac{\partial y_{t+1}(j)}{\partial k_{t+1}(j)} + q_{t+1}(1-\delta)]}{q_t} dj \\ &= \frac{E_t[p_{t+1}^w(j)\alpha\frac{\partial y_{t+1}}{\partial k_{t+1}} + q_{t+1}(1-\delta)]}{q_t}. \end{aligned}$$

The entrepreneur j's net worth is given by

$$N_{t+1}(j) = p_t^w(j)y^j - p_t^l l_t(j) + q_t(1-\delta)k_t(j) - \frac{r_{t-1}^n s_{t-1}}{1+\pi_t} b_{t-1}(j),$$

which can be written as

$$\begin{aligned} n_{t+1}(j) &= p_t^w(j)y_t(j) - p_t^l l_t(j) - \frac{r_{t-1}^n s_{t-1}(j)}{1+\pi_t} b_{t-1}(j) + q_t(1-\delta)k_t(j) \\ &= p_t^w(j)y_t(j) - p_t^w(1-\alpha)\frac{y_t(j)}{l_t(j)} l_t(j) - \frac{r_{t-1}^n s_{t-1}(j)}{1+\pi_t} b_{t-1}(j) + q_t(1-\delta)k_t(j) \\ &= p_t^w\alpha\frac{y_t(j)}{k_t(j)} k_t(j) - \frac{r_{t-1}^n s_{t-1}(j)}{1+\pi_t} b_{t-1}(j) + q_t(1-\delta)k_t(j) \\ &= \frac{[p_t^w\alpha\frac{y_t(j)}{k_t(j)} + q_t(1-\delta)]}{q_{t-1}} q_{t-1}k_t(j) - \frac{r_{t-1}^n s_{t-1}(j)}{1+\pi_t} b_{t-1}(j). \end{aligned}$$

Given that $r_t^k(j) = \frac{[p_t^w \alpha \frac{y_t(j)}{k_t(j)} + q_t(1-\delta)]}{q_{t-1}}$, the aggregate net worth is

$$n_{t+1} = \eta(r_t^k q_{t-1} k_t - \frac{r_{t-1}^n s_{t-1}}{1+\pi_t} b_{t-1}).$$

7.2 Appendix B: Contract Wage

7.2.1 Period-by-period Nash Bargaining Contract Wage

For standard period-by-period Nash bargaining, the first order condition is[12]

$$\eta J_t = (1-\eta) H_t.$$

This implies that the bargaining wage is

$$\begin{aligned} w_t \quad = \quad & \frac{w_t^n}{p_t} = \eta(p_t^l + \frac{\kappa}{2} x_t^2 + \rho\beta E_t \Lambda_{t,t+1} J_{t+1}) \\ & +(1-\eta)(\bar{b} - \beta E_t \Lambda_{t,t+1}(\rho - s_{t+1}) H_{t+1}). \end{aligned} \tag{34}$$

Substituting the following two equations

$$\kappa x_t = \beta E_t \Lambda_{t,t+1} J_{t+1},$$

and

$$H_{t+1} = \frac{\eta J_{t+1}}{1-\eta},$$

into equation (34) gives the period-by-period wage w_t^{flex} as

$$w_t^{flex} = w_t = \frac{w_t^n}{p_t} = \eta(p_t^l + \frac{\kappa}{2} x_t^2 + s_{t+1}^l \kappa x_t) + (1-\eta)\bar{b}.$$

7.2.2 Solving for Staggered Wage Contract

First Order Condition for Wage Now turning to the staggered wage contract. The first order condition with respect to $w_t^n(i)$ is

$$\eta \frac{\partial H_t(i)}{\partial w_t^n(i)} J_t(i) = (1-\eta) \frac{\partial J_t(i)}{\partial w_t^n(i)} H_t, \tag{35}$$

where

$$\frac{\partial H_t(i)}{\partial w_t^n(i)} = 1/p_t + \rho\lambda\pi\beta E_t \Lambda_{t,t+1} \frac{\partial H_{t+1}(i)}{\partial w_{t+1}^n(i)},$$

[12] Since it is period-by-period Nash bargaining, there is no need to have subscript i to indicate for the firm that renegotiate wages in the current period.

and

$$\frac{\partial J_t(i)}{\partial w_t^n(i)} = -1/p_t + \rho\lambda\pi\beta E_t \Lambda_{t,t+1} \frac{\partial J_{t+1}(i)}{\partial w_{t+1}^n(i)}.$$

Define

$$\epsilon_t = p_t \frac{\partial H_t(i)}{\partial w_t^n(i)},$$

and we have

$$\epsilon_t = 1 + \rho\lambda\beta\pi E_t \Lambda_{t,t+1} \frac{p_t}{p_{t+1}} \epsilon_{t+1},$$

with the steady-state ϵ_{ss}

$$\epsilon_{ss} = \frac{1}{1 - \rho\lambda\beta\pi}.$$

Define

$$\mu_t = -p_t \frac{\partial J_t(i)}{\partial w_t^n(i)},$$

and we have

$$\mu_t = 1 + \rho\lambda\beta\pi E_t \Lambda_{t,t+1} \frac{p_t}{p_{t+1}} \mu_{t+1},$$

with the steady-state μ_{ss}

$$\mu_{ss} = \frac{1}{1 - \rho\lambda\beta\pi}.$$

Thus,

$$\epsilon_t = \mu_t$$

Derivation of $H_t(i)$ and $J_t(i)$ Define

$$\begin{aligned} W_t(i) &= E_t \sum_{s=0}^{\infty} (\rho\beta)^s \Lambda_{t,t+s} w_{t+s}(i) \\ &= w_t(i) + E_t w_{t+1}(i) + E_t w_{t+2}(i) + ... \end{aligned} \tag{36}$$

Considering a renegotiating firm i at time t, we have

$$w_t(i) = \frac{w_t^{n*}}{p_t},$$

$$E_t w_{t+1}(i) = E_t \frac{1}{p_{t+1}} [\lambda\pi w_t^{n*} + (1-\lambda) w_{t+1}^{n*}],$$

$$
\begin{aligned}
E_t w_{t+2}(i) &= E_t \frac{1}{p_{t+2}}[\lambda\pi w_{t+1}^{n*} + (1-\lambda) w_{t+2}^{n*}] \\
&= E_t \frac{1}{p_{t+2}}[\lambda\pi(\lambda\pi w_t^{n*} + (1-\lambda) w_{t+1}^{n*}) + (1-\lambda) w_{t+2}^{n*}] \\
&= E_t \frac{1}{p_{t+2}}[\lambda^2\pi^2 w_t^{n*} + \lambda(1-\lambda)\pi w_{t+1}^{n*} + (1-\lambda) w_{t+2}^{n*}],
\end{aligned}
$$

and so on. Substituting the above equations into equation (36), we can write $W_t(i)$ as

$$
\begin{aligned}
W_t(i) &= \frac{w_t^{n*}}{p_t} \\
&+\rho\beta E_t \Lambda_{t,t+1} \frac{1}{p_{t+1}}[\lambda\pi w_t^{n*} + (1-\lambda) w_{t+1}^{n*}] \\
&+(\rho\beta)^2 E_t \Lambda_{t,t+2} E_t \frac{1}{p_{t+2}}[\lambda^2\pi^2 w_t^{n*} + \lambda(1-\lambda)\pi w_{t+1}^{n*} + (1-\lambda) w_{t+2}^{n*}] \\
&+...,
\end{aligned}
$$

that is

$$
\begin{aligned}
W_t(i) &= w_t^* \\
&+\rho\beta E_t \Lambda_{t,t+1}[\lambda\pi \frac{p_t}{p_{t+1}} w_t^* + (1-\lambda) w_{t+1}^*] \\
&+(\rho\beta)^2 E_t \Lambda_{t,t+2} E_t[\lambda^2\pi^2 \frac{p_t}{p_{t+2}} w_t^* + \lambda(1-\lambda)\pi \frac{p_{t+1}}{p_{t+2}} w_{t+1}^* + (1-\lambda) w_{t+2}^*] \\
&+....
\end{aligned}
$$

Combining terms, we have

$$
\begin{aligned}
W_t(i) &= [1 + \rho\beta\lambda\Lambda_{t,t+1}\frac{p_t}{p_{t+1}}\pi + (\rho\beta\lambda)^2\Lambda_{t,t+2}\frac{p_t}{p_{t+2}}\pi^2 + ...] w_t^* \\
&+\rho\beta(1-\lambda) E_t \Lambda_{t,t+1}[1 + \rho\beta\lambda\Lambda_{t+1,t+2}\frac{p_{t+1}}{p_{t+2}}\pi] w_{t+1}^* \\
&+....
\end{aligned}
$$

Define

$$
\Delta_t = E_t \sum_{s=0}^{\infty} (\rho\beta\lambda)^s \Lambda_{t,t+s} \frac{p_t}{p_{t+s}} \pi^s,
$$

where

$$
\Delta_{ss} = \frac{1}{1-\rho\beta\lambda}.
$$

We can show that

$$\begin{aligned} W_t(i) &= \Delta_t w_t^* + (1-\lambda)\rho\beta E_t \Lambda_{t,t+1}\Delta_{t+1} w_{t+1}^* \\ &\quad +(1-\lambda)(\rho\beta)^2 E_t \Lambda_{t,t+2}\Delta_{t+2} w_{t+2}^* \\ &\quad +..., \end{aligned}$$

that is

$$W_t(i) = \Delta_t w_t^* + (1-\lambda) E_t \sum_{s=1}^{\infty} (\rho\beta)^s E_t \Lambda_{t,t+s}\Delta_{t+s} w_{t+s}^*. \tag{37}$$

For $H_t(i)$ we have

$$\begin{aligned} H_t(i) &= w_t(i) - \bar{b} + \beta E_t \Lambda_{t,t+1}[\rho H_{t+1}(i) - s_{t+1}^l H_{t+1}] \\ &= w_t(i) - \bar{b} - s_{t+1}^l \beta E_t \Lambda_{t,t+1} H_{t+1} + \rho\beta E_t \Lambda_{t,t+1} H_{t+1}(i) \\ &= w_t(i) - \bar{b} - s_{t+1}^l \beta E_t \Lambda_{t,t+1} H_{t+1} \\ &\quad +\rho\beta E_t \Lambda_{t,t+1}[w_{t+1}(i) - \bar{b} + \beta E_{t+1}\Lambda_{t+1,t+2}[\rho H_{t+2}(i) - s_{t+2}^l H_{t+2}]] \\ &= w_t(i) + \rho\beta E_t \Lambda_{t,t+1} w_{t+1}(i) - \bar{b} - \rho\beta E_t \Lambda_{t,t+1}\bar{b} \\ &\quad -s_{t+1}^l \beta E_t \Lambda_{t,t+1} H_{t+1} - \rho\beta E_t \Lambda_{t,t+1}\beta E_{t+1}\Lambda_{t+1,t+2} s_{t+2}^l H_{t+2} \\ &\quad +(\rho\beta)^2 E_t \Lambda_{t,t+2}[w_{t+2}(i) - \bar{b} \\ &\quad +\beta E_{t+2}\Lambda_{t+2,t+3}[\rho H_{t+3}(i) - s_{t+3}^l H_{t+3}]]. \end{aligned}$$

Combining terms,

$$H_t(i) = W_t(i) - E_t \sum_{s=0}^{\infty} (\rho\beta)^s \Lambda_{t,t+s}[\bar{b} + s_{t+s}^l \beta \Lambda_{t+s,t+s+1} H_{t+s+1}].$$

Substituting equation (37) into the above equation, we obtain

$$\begin{aligned} H_t(i) &= \Delta_t w_t^* + (1-\lambda) E_t \sum_{s=1}^{\infty} (\rho\beta)^s E_t \Lambda_{t,t+s}\Delta_{t+s} w_{t+s}^* \\ &\quad - E_t \sum_{s=0}^{\infty} (\rho\beta)^s \Lambda_{t,t+s}[b + s_{t+s+1}^l \beta \Lambda_{t+s,t+s+1} H_{t+s+1}] \end{aligned} \tag{38}$$

For the value of job $J_t(i)$, we have

$$\begin{aligned}
J_t(i) &= p_t^l - w_t(i) + \frac{\kappa}{2}x_t^2(i) + \rho\beta E_t\Lambda_{t,t+1}J_{t+1}(i) \\
&= p_t^l - w_t(i) + \frac{\kappa}{2}x_t^2(i) \\
&\quad +\rho\beta E_t\Lambda_{t,t+1}[p_{t+1}^l - w_{t+1}(i) \\
&\quad +\frac{\kappa}{2}x_{t+1}^2(i) + \rho\beta E_{t+1}\Lambda_{t+1,t+2}J_{t+2}(i)] \\
&= p_t^l + \rho\beta E_t\Lambda_{t,t+1}p_{t+1}^l + \frac{\kappa}{2}x_t^2(i) \\
&\quad +\rho\beta E_t\Lambda_{t,t+1}\frac{\kappa}{2}x_{t+1}^2(i) - w_t(i) \\
&\quad -\rho\beta E_t\Lambda_{t,t+1}w_{t+1}(i) \\
&\quad +\rho\beta E_t\Lambda_{t,t+1}\rho\beta\Lambda_{t+1,t+2}J_{t+2}(i) + ... \qquad (39)
\end{aligned}$$

Substituting $W_t(i)$ into equation (39), we obtain

$$\begin{aligned}
J_t(i) &= E_t\sum_{s=0}^{\infty}\rho^s\beta^s\Lambda_{t,t+s}[p_{t+s}^l + \frac{\kappa}{2}x_{t+s}^2(i)] - W_t^J(i) \\
&= E_t\sum_{s=0}^{\infty}\rho^s\beta^s\Lambda_{t,t+s}[p_{t+s}^l + \frac{\kappa}{2}x_{t+s}^2(i)] \\
&\quad -\Delta_t w_t^* - (1-\lambda)E_t\sum_{s=1}^{\infty}\rho^s\beta^s E_t\Lambda_{t,t+s}\Delta_{t+s}w_{t+s}^* \qquad (40)
\end{aligned}$$

Nash Bargaining Solution under Staggered Nominal Wage Contracting Substituting equation (38) and (40) into Nash bargaining first-order condition

$$\eta J_t(i) = (1-\eta)H_t(i),$$

we can have

$$\begin{aligned}
&\Delta_t w_t^* \\
= \ &E_t\sum_{s=0}^{\infty}(\rho\beta)^s\Lambda_{t,t+s}\{\eta(p_{t+s}^l + \frac{\kappa}{2}x_{t+s}^2(i)) + (1-\eta)(b + s_{t+s+1}\beta\Lambda_{t+s,t+s+1}H_{t+s+1})\} \\
&-E_t\sum_{s=1}^{\infty}(1-\lambda)(\rho\beta)^s\Lambda_{t,t+s}\Delta_{t+s}w_{t+s}^*.
\end{aligned}$$

That is,

$$\begin{aligned}
& \Delta_t w_t^* \\
= \; & E_t \sum_{s=0}^{\infty} (\rho\beta)^s \Lambda_{t,t+s} \{ \eta(p_{t+s}^l + \frac{\kappa}{2} x_{t+s}^2(i)) + (1-\eta)(b + s_{t+s+1}\beta\Lambda_{t+s,t+s+1}H_{t+s+1}) \\
& -(1-\lambda)\rho\beta\Lambda_{t+s,t+s+1}\Delta_{t+s+1}w_{t+s+1}^* \},
\end{aligned}$$

which can be written in a recursive form as

$$\begin{aligned}
\Delta_t w_t^* \; = \; & \eta(p_t^l + \frac{\kappa}{2} x_t^2(i)) + (1-\eta)(b + s_{t+1}\beta\Lambda_{t,t+1}H_{t+1}) \\
& -(1-\lambda)\rho\beta\Lambda_{t,t+1}\Delta_{t+1}w_{t+1}^* \\
& +\rho\beta E_t \Lambda_{t,t+1}\Delta_{t+1}w_{t+1}^*.
\end{aligned}$$

Thus, the contract wage w_t^* equation is

$$\begin{aligned}
\Delta_t w_t^* \; = \; & \eta(p_t^l + \frac{\kappa}{2} x_t^2(i)) + (1-\eta)(b + s_{t+1}\beta\Lambda_{t,t+1}H_{t+1}) \\
& +\lambda\rho\beta\Lambda_{t,t+1}\Delta_{t+1}w_{t+1}^*.
\end{aligned}$$

The aggregate wage w_t^n can be written as

$$w_t^n = (1-\lambda)w_t^{n*} + \lambda\pi w_{t-1}^n,$$

i.e.

$$p_t w_t = (1-\lambda)p_t w_t^* + \lambda\pi p_{t-1} w_{t-1}.$$

Thus in real terms we have

$$w_t = (1-\lambda)w_t^* + \lambda\pi \frac{1}{\pi_t} w_{t-1}.$$

Now we replace H_{t+1} in the target wage equation with relative hiring rate $(x_t(i) - x_t)$ and relative wage $(w_t - w_t^*)$. Given that

$$H_t(i) = W_t(i) - E_t \sum_{s=0}^{\infty} (\rho\beta)^s \Lambda_{t,t+s} [b + s_{t+s+1}^l \beta\Lambda_{t+s,t+s+1}H_{t+s+1}],$$

we have

$$
\begin{aligned}
E_t[H_{t+1} - H_{t+1}(i)] &= E_t[W_{t+1} - W_{t+1}(i)] \\
&= E_t \sum_{s=0}^{\infty} (\rho\beta)^s \Lambda_{t+1,t+s+1}(w_{t+s+1} - w_{t+s+1}(i)) \\
&= E_t\{\lambda\pi \frac{p_t}{p_{t+1}}(w_t - w_t^*) + \rho\beta\Lambda_{t+1,t+2}\lambda^2\pi^2 \frac{p_t}{p_{t+2}}(w_t - w_t^*) \\
&\quad +(\rho\beta)^2\Lambda_{t+1,t+3}\lambda^3\pi^3 \frac{p_t}{p_{t+3}}(w_t - w_t^*) + ...\} \\
&= E_t\{\lambda\pi \frac{p_t}{p_{t+1}}(1 + \rho\beta\Lambda_{t+1,t+2}\lambda\pi \frac{p_{t+1}}{p_{t+2}} + (\rho\beta)^2\Lambda_{t+1,t+3}(\lambda\pi)^2 \frac{p_{t+1}}{p_{t+3}} + ...)(w_t - w_t^*)\} \\
&= \lambda\pi \frac{p_t}{p_{t+1}}[E_t \sum_{s=0}^{\infty} (\rho\beta\lambda)^s \Lambda_{t+1,t+1+s} \frac{p_{t+1}}{p_{t+1+s}} \pi^s](w_t - w_t^*) \\
&= \lambda\pi \frac{p_t}{p_{t+1}} \Delta_{t+1}(w_t - w_t^*).
\end{aligned}
$$

Recall

$$
\Delta_t = E_t \sum_{s=0}^{\infty} (\rho\beta\lambda)^s \Lambda_{t,t+s} \frac{p_t}{p_{t+s}} \pi^s,
$$

and we also use

$$
\begin{gathered}
E_t(w_{t+1} - w_{t+1}(i)) = E_t \lambda\pi \frac{p_t}{p_{t+1}}(w_t - w_t^*), \\
E_t(w_{t+2} - w_{t+2}(i)) = E_t \lambda^2\pi^2 \frac{p_t}{p_{t+2}}(w_t - w_t^*), \\
...,
\end{gathered}
$$

in the derivation of the above equation. We can then write the first order condition of Nash bargaining as

$$
\begin{aligned}
\eta_t E_t J_{t+1}(i) &= (1 - \eta_t) E_t H_{t+1}(i) \\
&= (1 - \eta_t)(E_t H_{t+1} - \lambda\pi \frac{p_t}{p_{t+1}} \Delta_{t+1}(w_t - w_t^*)).
\end{aligned}
$$

Rewrite the above equation as

$$
E_t H_{t+1} = \frac{\eta E_t J_{t+1}(i)}{1 - \eta} + E_t \lambda\pi \frac{p_t}{p_{t+1}} \Delta_{t+1}(w_t - w_t^*).
$$

Given that

$$
\kappa x_t(i) = \beta E_t \Lambda_{t,t+1} J_{t+1}(i), \tag{41}
$$

we have

$$
\beta E_t \Lambda_{t,t+1} H_{t+1} = \frac{\eta}{1 - \eta} \kappa x_t(i) + \beta E_t \Lambda_{t,t+1} \lambda\pi \frac{p_t}{p_{t+1}} \Delta_{t+1}(w_t - w_t^*).
$$

Now the target wage can be expressed in terms of $(w_t - w_t^*)$ and $(x_t(i) - x_t)$

$$\begin{aligned}
w_t^{tar}(i) &= \eta(p_t^l + \frac{\kappa}{2}x_t^2(i)) + (1-\eta)(b + E_t s_{t+1}^l \beta \Lambda_{t,t+1} H_{t+1}) \\
&= \eta(p_t^l + \frac{\kappa}{2}x_t^2(i) + \kappa s_{t+1}^l x_t(i)) + (1-\eta)b \\
&\quad +(1-\eta)E_t s_{t+1}^l \beta \Lambda_{t,t+1} \lambda \pi \frac{p_t}{p_{t+1}} \Delta_{t+1}(w_t - w_t^*) \\
&= \eta(p_t^l + \frac{\kappa}{2}x_t^2 + \kappa s_{t+1}^l x_t) + (1-\eta)b \\
&\quad +\eta[\frac{\kappa}{2}(x_t^2(i) - x_t^2) + \kappa s_{t+1}^l (x_t(i) - x_t)] \\
&\quad +(1-\eta)E_t s_{t+1}^l \beta \Lambda_{t,t+1} \lambda \pi \frac{p_t}{p_{t+1}} \Delta_{t+1}(w_t - w_t^*).
\end{aligned}$$

If we define

$$w_t^{flex} = \eta(p_t^l + \frac{\kappa}{2}x_t^2 + \kappa s_t^l x_t) + (1-\eta)b,$$

then the targeted wage can be further written as

$$\begin{aligned}
w_t^{tar}(i) &= w_t^{flex} + \eta[\frac{\kappa}{2}(x_t^2(i) - x_t^2) + \kappa s_{t+1}^l (x_t(i) - x_t)] \\
&\quad +(1-\eta) s_{t+1}^l \beta \Lambda_{t,t+1} \lambda \pi \frac{p_t}{p_{t+1}} \Delta_{t+1}(w_t - w_t^*).
\end{aligned}$$

7.3 Appendix C: System of Equations

$$\frac{u'(e_t c_t)}{p_t} = \beta r_t^n \frac{u'(e_{t+1} c_{t+1})}{p_{t+1}}$$

$$E_t r_{t+1}^k = \frac{E_t[p_{t+1}^w \alpha \frac{y_{t+1}}{k_{t+1}} + q_{t+1}(1-\delta)]}{q_t}$$

$$E_t r_{t+1}^k = E_t \frac{r_t^n s_t}{1+\pi_{t+1}}$$

$$N_{t+1} = \eta^e \gamma_t [r_t^k q_{t-1} k_t - \frac{r_{t-1}^n s_{t-1}}{1+\pi_t}(q_{t-1}k_t - N_t)]$$

$$s_t = (\frac{q_t k_{t+1}}{N_{t+1}})^\chi$$

$$k_{t+1} = (1-\delta)k_t + \tau_t i_t$$

$$q_t \tau_t = 1 + \xi(\frac{i_t}{k_t} - \delta)$$

$$m_t = u_t^\sigma v_t^{1-\sigma}$$

$$n_{t+1} = \rho n_t + m_t$$

$$u_t = 1 - n_t$$

$$x_t = \frac{q_t^l v_t}{n_t}$$

$$\kappa x_t(i) = \beta E_t \Lambda_{t,t+1}[p_{t+1}^l a - \frac{w_{t+1}^n(i)}{p_{t+1}} + \frac{\kappa}{2} x_{t+1}(i)^2 + \rho\kappa x_{t+1}(i)]$$

$$w_t^{flex} = \eta(p_t^l + \frac{\kappa}{2}x_t^2 + \kappa s_{t+1}^l x_t) + (1-\eta)\bar{b}$$

$$\begin{aligned} w_t^{tar}(i) &= w_t^{flex} + \eta[\frac{\kappa}{2}(x_t^2(i) - x_t^2) + \kappa s_{t+1}^l (x_t(i) - x_t)] \\ &\quad + (1-\eta) s_{t+1}^l \beta \Lambda_{t,t+1} \lambda \pi \frac{p_t}{p_{t+1}} \Delta_{t+1}(w_t - w_t^*) \end{aligned}$$

$$\Delta_t w_t^* = w_t^{tar}(i) + \lambda\rho\beta E_t \Lambda_{t,t+1} \Delta_{t+1} w_{t+1}^*$$

$$\Delta_t = 1 + E_t \Lambda_{t,t+1}(\rho\lambda\beta)\frac{p_t}{p_{t+1}}\pi\Delta_{t+1}$$

$$w_t^n = (1-\lambda)w_t^{n*} + \lambda\pi w_{t-1}^n$$

$$p_t^w(1-\alpha)\frac{y_t}{l_t} = p_t^l$$

$$y_t = c_t + c_t^e + i_t + g_t + \frac{\kappa}{2}x_t^2 n_t + \frac{\xi}{2}(\frac{i_t}{k_t} - \delta)^2 k_t$$

$$y_t = z_t k_t^\alpha l_t^{1-\alpha}$$

$$p_t^* = \left(\frac{\varepsilon}{\varepsilon - 1}\right) \frac{E_t \sum_{i=0}^{\infty} \nu^i \Delta_{i,t+i} mc_{t+1} y_{t+i} (\frac{1}{p_{t+i}})^{-\varepsilon}}{E_t \sum_{i=0}^{\infty} \nu^i \Delta_{i,t+i} y_{t+i} (\frac{1}{p_{t+i}})^{1-\varepsilon}}$$

$$p_t = [\nu p_{t-1}^{1-\varepsilon} + (1-\nu)(p_t^*)^{1-\varepsilon}]^{\frac{1}{1-\varepsilon}}.$$

$$\frac{r_t^n}{r^n} = (\frac{r_{t-1}^n}{r^n})^{\rho_r}((\frac{\pi_t}{\pi})^{\rho_\pi}(\frac{y_t}{y})^{\rho_y})^{1-\rho_r} e^{\epsilon_t^m},$$

7.4 Appendix D: Log-linearized System of Equations

$$\hat{\lambda}_t = \hat{r}_t + \hat{\lambda}_{t+1} - E_t \hat{\pi}_{t+1}$$

$$\hat{\lambda}_t = \hat{u}_c + \hat{e}_t$$

$$E_t \hat{R}_{t+1}^k = \frac{mc\alpha\frac{y}{k}}{mc\alpha\frac{y}{k} + q(1-\delta)}(m\hat{c}_{t+1} + \hat{y}_{t+1} - \hat{k}_{t+1}) + \frac{(1-\delta)}{mc\alpha\frac{y}{k} + q(1-\delta)}\hat{q}_{t+1} - \hat{q}_t$$

$$E_t \hat{R}_{t+1}^k = \hat{r}_t + \hat{s}_t - E_t \hat{\pi}_{t+1}$$

$$\hat{n}w_{t+1} = \frac{k}{N}\hat{R}_t^k - (\frac{k}{N} - 1)(\hat{r}_{t-1} + \hat{s}_{t-1} - \hat{\pi}_t) + n\hat{w}_t + \hat{\gamma}_t$$

$$\hat{s}_t = \chi(\hat{q}_t + \hat{k}_{t+1} - \hat{n}_{t+1})$$

$$\hat{k}_{t+1} = (1-\delta)\hat{k}_t + \delta\hat{\imath}_t + \delta\hat{\tau}_t$$

$$\hat{q}_t = \xi\delta(\hat{\imath}_t - \hat{k}_t) - \hat{\tau}_t$$

$$\hat{m}_t = \hat{\sigma}\hat{u}_t + (1-\sigma)\hat{v}_t$$

$$\hat{n}_t = \rho\hat{n}_t + (1-\rho)\hat{m}_t$$

$$\hat{u}_t = -\frac{n}{u}\hat{n}_t$$

$$\hat{x}_t = \hat{q}^l_t + \hat{v}_t - \hat{n}w_t$$

$$\hat{x}_t = E_t\hat{\Lambda}_{t,t+1} + (\frac{\beta}{\kappa x})(pE_t\hat{p}^l_{t+1} - wE_t\hat{w}_{t+1}) + \beta(x+\rho)E_t\hat{x}_{t+1}$$

$$\hat{w}^{flex}_t = \frac{\eta p^l}{w}\hat{p}^l_t + \frac{\eta\kappa x(x+s)}{w}\hat{x}_t + \frac{\eta\kappa x s}{w}\hat{s}^l_t$$

$$\hat{w}^{tar}_t = \hat{w}^{flex}_t + (\tau_1+\tau_2)(\hat{w}_t - \hat{w}^*_t)$$

$$\hat{w}^*_t = (1-\rho\beta\lambda)\hat{w}^{flex}_t + \rho\beta\lambda E_t\hat{w}^*_{t+1} + (1-\rho\beta\lambda)(\tau_1+\tau_2)(\hat{w}_t - \hat{w}^*_t) + \frac{\rho\beta\lambda}{1-\rho\beta\lambda}E_t\hat{\pi}_{t+1}$$

where $\tau_1 = \eta(x+s^l)\lambda\beta\frac{1}{1-(x+\rho)\lambda\beta}$ and $\tau_2 = (1-\eta)s^l\beta\frac{\lambda}{1-\rho\beta\lambda}$

$$\hat{w}_t = (1-\lambda)\hat{w}^*_t + \lambda(\hat{w}_{t-1} - \hat{\pi}_t)$$

$$\hat{p}^l_t = m\hat{c}_t + \hat{y}_t - \hat{l}_t$$

$$\hat{y}_t = \frac{c}{y}\hat{c}_t + \frac{i}{y}\hat{\imath}_t + \frac{g}{y}\hat{g}_t + \frac{\kappa x^2 n}{y}(\hat{x}_t + \frac{\hat{n}_t}{2})$$

$$\hat{y}_t = \hat{z}_t + \alpha\hat{k}_t + (1-\alpha)\hat{l}_t$$

$$\hat{r}^n_t = \rho_r\hat{r}^n_{t-1} + (1-\rho_r)(\rho_\pi\hat{\pi}_t + \rho_y\hat{y}_t) + \hat{\varepsilon}^r_t$$

7.5 Appendix E: Steady-state Calculations

$$\pi = 1$$

$$mc = \frac{\epsilon - 1}{\epsilon}$$

$$r^n = \frac{\pi}{\beta}$$

$$r^k = \frac{1}{\eta^e}$$

$$s = \frac{r^k}{r^n/\pi}$$

$$q = 1$$

$$i = \delta k$$

$$\frac{y}{k} = \frac{r^k - (1-\delta)}{\alpha mc}$$

$$\frac{l}{k} = (\frac{y}{k})^{-(1/1-\alpha)}$$

$$\frac{y}{l} = \frac{y}{k} / \frac{l}{k}$$

$$p^l = (1-\alpha)mc\frac{y}{l}$$

$$n = \frac{s^l}{1-\rho+s^l}$$

$$u = 1-n$$

$$x = s^l u/n$$

$$x(i) = x$$

$$m = s^l u$$

$$v = \frac{m}{q^l}$$

$$\sigma^m = \frac{m}{u^\sigma v^{1-\sigma}}$$

κ and w are solved from the following two steady-state conditions

$$\kappa x = \beta(p^l - w + \frac{\kappa}{2}x^2 + \rho\kappa x)$$

$$w = \eta(p^l + \frac{\kappa}{2}x^2 + s^l\kappa x) + (1-\eta)\bar{b}$$

where

$$\tilde{b} = \bar{b}/(p^l + \frac{\kappa}{2}x^2)$$

$$w^{flex} = w^{tar} = w^* = w$$

$$l = n$$

$$y = \frac{y}{l}l$$

$$k = l/(l/k)$$

$$N = k(N/k)$$

$$i = (i/k)k$$

$$c = y - i - (\frac{\kappa}{2})x^2 n$$

$$\lambda = 1/c$$

$$X = \frac{\lambda mcy}{1-\nu_p\beta\pi^\epsilon}$$

$$Y = \frac{\lambda y}{1-\nu_p\beta\pi^{\epsilon-1}}$$

$$p^* = (\frac{1 - \nu_p p^{\epsilon - 1}}{1 - \nu_p})^{1/(1-\epsilon)}$$

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