BANK OF CANADA
BANQUE DU CANADA

Working Paper/Document de travail
2015-9

Securitization under Asymmetric Information over the Business Cycle

by Martin Kuncl

www.bank-banque-canada.ca

Bank of Canada Working Paper 2015-9

February 2015

Securitization under Asymmetric Information over the Business Cycle

by

Martin Kuncl

Canadian Economic Analysis Department
Bank of Canada
Ottawa, Ontario, Canada K1A 0G9
mkuncl@bankofcanada.ca

ISSN 1701-9397

Acknowledgements

For helpful comments and suggestions, I would like to thank Sergey Slobodyan, Markus Brunnermeier, Byeongu Jeong, Nobuhiro Kiyotaki, Filip Matějka, Benjamin Moll, Kalin Nikolov, Frederic Malherbe, Olena Senyuta, Alexander Ueberfeldt and seminar participants at Princeton University Student Macro Workshop, XXI MBF Conference, 6th RGS Doctoral Conference, 17th ICMAIF 2013, 5th IFABS Conference, 28th EEA Congress, Goethe University, European Central Bank, Econometric Society European Winter Meeting, Warwick Business School, Bank of England, Bank of Spain, CNMV, VU Amsterdam and Bank of Canada. Remaining errors are solely my own responsibility. Financial support of the Grant Agency of the Charles University (grant no. 638413), the Global Development Network (RRC 12+69) and the Czech Science Foundation Project No. P402/12/G097 DYME Dynamic Models in Economics is gratefully acknowledged.

Abstract

This paper studies the efficiency of financial intermediation through securitization in a model with heterogeneous investment projects and asymmetric information about the quality of securitized assets. I show that when retaining part of the risk, the issuer of securitized assets may credibly signal its quality. However, in the boom stage of the business cycle this practice is inefficient, information on asset quality remains private, and lower-quality assets accumulate on balance sheets of financial intermediaries. This prolongs and deepens a subsequent recession with an intensity proportional to the length of the preceding boom. In recessions, the model also produces amplification of adverse selection problems on resale markets for securitized assets. These are especially severe after a prolonged boom period and when securitized high-quality assets are no longer traded. The model also suggests that improperly designed regulation requiring higher explicit risk retention may become counterproductive due to a negative general-equilibrium effect; i.e., it may adversely affect both the quantity and the quality of investment in the economy.

JEL classification: E, E3, E32, E44, G, G0, G01, G2, G20
Bank classification: Business fluctuations and cycles; Economic models; Credit and credit aggregates; Financial markets; Financial stability; Financial system regulation and policies

Résumé

L’auteur étudie l’efficience de la titrisation comme moyen d’intermédiation financière à l’aide d’un modèle où sont formalisés des projets d’investissement hétérogènes et l’asymétrie de l’information sur la qualité des actifs titrisés. Il montre qu’en conservant une partie du risque, l’émetteur des actifs titrisés signale de manière crédible leur qualité. Toutefois, durant la phase d’expansion du cycle économique, cette pratique est inefficiente, l’information sur la qualité des actifs demeure privée, et des actifs de moindre qualité s’accumulent dans le bilan des intermédiaires financiers. Cela prolonge et accentue la récession qui s’installe ensuite avec une intensité proportionnelle à la longueur de la période d’expansion. Lors des récessions, le modèle produit une amplification des problèmes d’antisélection sur les marchés de la revente d’actifs titrisés. Ces problèmes sont particulièrement graves après une longue période d’expansion, et lorsque les actifs titrisés de grande qualité cessent de se négocier. Le modèle donne aussi à penser qu’une réglementation inadéquatement conçue, assortie d’exigences accrues en matière de conservation du risque, pourrait devenir contre-productive en raison d’un effet d’équilibre général néfaste; autrement dit, elle pourrait avoir des conséquences négatives sur les volumes et la qualité des investissements dans l’économie.

Classification JEL : E, E3, E32, E44, G, G0, G01, G2, G20
Classification de la Banque : Cycles et fluctuations économiques; Modèles économiques; Crédit et agrégats du crédit; Marchés financiers; Stabilité financière; Réglementation et politiques relatives au système financier

Non-Technical Summary

Due to its role in the financial crisis of the late 2000s, securitization has recently attracted a great deal of criticism. Most of the criticism points to various agency problems that stem from the asymmetry of information about the quality of securitized assets between the issuers and buyers. However, the design of securitized products contained risk-retention tools that were supposed to limit these problems. I address the question of whether the risk-retention tools worked efficiently in the period prior to the late-2000s financial crisis.

This paper suggests that risk retention contributes to a reduction in the asymmetry of information. However, in boom stages of the business cycle, including the period prior to the late-2000s financial crisis, this practice is inefficient as information about quality of securitized assets remains private and low-quality assets accumulate on balance sheets of financial intermediaries. The paper shows that this mechanism implies deeper and longer recessions proportional to the length of the preceding boom period. Those results are also relevant for the newly proposed regulations of securitization. Since self-regulation by risk retention is not sufficient, more standardization and transparency is necessary, especially in boom stages of the business cycle, to address the problem of asymmetry of information.

The mechanism presented in this paper can contribute to the understanding of the recent financial crisis, since it replicates some of the securitization market outcomes observed prior to and during the crisis. In the period preceding the crisis, many inefficient investments of unknown quality were undertaken. While this was not a problem as long as the economy was performing well, the large amount of low-quality loans in the economy contributed to the depth of the financial crisis. To the best of my knowledge existing models of securitization fail to produce those results in a rational-expectations framework.

1 Introduction

Securitization as well as the whole market-based system of financial intermediation grew significantly in importance in the decades preceding the financial crisis of the late 2000s (Adrian and Shin, 2009). However, due to its role in the financial crisis (e.g., Bernanke, 2010), securitization has recently attracted a great deal of criticism. New research focuses on the problematic aspects of securitization and is often very critical. Consider Shleifer and Vishny (2010), who argue that it creates systemic risks and inefficiencies in financial intermediation. Currently, the regulation of the financial sector is being redrafted and strengthened on national as well as international levels in many developed countries. The new regulation also addresses securitization practices.[1]

Most of the criticism points to various agency problems related to securitization that stem from the asymmetry of information about the quality of securitized assets between their issuers and buyers. However, these problems are not new, and various tools, such as explicit or implicit guarantees by issuers (in other words, risk retention by issuers), are used in the securitization process precisely to limit these problems. The question that is addressed in this paper is whether these tools work efficiently in the boom stage of the business cycle, i.e., in a period similar to the one preceding the late-2000s financial crisis.

In this paper, I model financial intermediation through securitization in a dynamic stochastic general-equilibrium model with heterogeneous investment opportunities and an asymmetry of information about the distribution of those investment opportunities. Following the empirical evidence in Bloom (2009) and Bloom et al. (2012), who find that the second moments of firms' total factor productivity (TFP) in the economy are countercyclical, the relative cross-sectional difference in the productivity (productivity dispersion) of investment projects in this model is also countercyclical. As a result, due to asymmetry of information, in periods of low productivity and high productivity dispersion, the economy is in a separating equilibrium, where only high-quality projects are being financed. However, in periods of high productivity and low productivity dispersion, the economy is in a pooling equilibrium, where both high- and low-quality projects are being financed and information about their qualities remains private.

I investigate whether the use of the aforementioned risk-retention tools can change the above result. In particular, I focus on the provision of the implicit guarantees by

[1] Pozsar et al. (2012) describe the role of securitization in shadow banking, and Adrian and Ashcraft (2012) review the proposals for new regulation.

issuers. An implicit guarantee (or recourse) is a non-contractual support provided by issuers of securitized products to holders of these assets.[2] This support is enforced in a reputation equilibrium, where failing to provide implicit support may be followed by punishment in the form of an inability to sell on the primary market for securitized assets. The reason why implicit guarantees were frequently preferred to higher explicit guarantees (such as tranche retention schemes) was mainly regulatory arbitrage. I show that reputation concerns can allow originators of securitized products to credibly signal the quality of loans by providing implicit recourse, and thus limit the problem of asymmetric information and improve the efficiency of financial intermediation. However, there are limits to the size of a credible implicit guarantee based on reputation. In the boom stage of the business cycle, where the difference in the productivities of projects is sufficiently small, the separating equilibrium would require levels of implicit recourse so high that they cannot be enforced through reputation.

Therefore, even though the provision of implicit guarantees increases the occurrence of separating equilibria, in the boom stage of the business cycle there are still only pooling equilibria, in which the information about the quality of loans remains private and the allocation of investment is inefficient. This has only very moderate effects as long as the economy stays in a boom, characterized by low productivity dispersion. However, the effect of an accumulated stock of low-quality assets becomes more pronounced in the subsequent downturn of the economy, which is thus deeper and longer. Also, the longer the boom, the larger the share of lower-quality assets on balance sheets and the deeper and longer will be the subsequent downturn.

In an extension of the model, I introduce asymmetric information between sellers and buyers of securitized assets on the resale market. The model then produces adverse selection, which is amplified in a recession. The negative impact of the adverse selection on the market price depends on the share of low-quality assets on the balance sheets of financial intermediaries. Therefore, adverse selection is especially severe in a recession following a prolonged boom period. When the price of securitized assets on the resale markets falls enough, even firms in need of liquidity find it unprofitable to sell high-quality assets for low market prices in order to finance new investment opportunities. Ultimately, securitized assets of high quality are no longer traded on the resale markets.

In another extension of the model, I analyze regulatory policy implications. This is a topical issue since, based on the already-mentioned critique of securitization, gov-

[2]For a review of empirical evidence on implicit recourse, a description of its types, and a discussion of its role in the securitization process, I refer the reader to section 2.

ernments and international organizations draft and pass new related regulations. An example is the requirement of higher explicit risk retention for the originators (issuers) of the securitized products in section 941 of the Dodd-Frank reform. In this paper, I do not explicitly model the frictions that would rationalize such regulations; nevertheless, the model points to some new general-equilibrium effects of the regulations, which suggests that the newly drafted regulations have limitations. In this model, when regulation limits the financial intermediation ability of the originator, the resulting lower supply of loans increases the prices of securitized assets and makes securitization more profitable. Also, the securitization of lower-quality loans is more profitable, which may result in a lower average quality of securitized loans in the economy. This effect is strongest when there is no tranching and the risk retention takes the form of a higher fraction of issued loans on issuers' balance sheets (equivalent to horizontal tranche retention). However, if tranching is allowed, and the risk-retention requirement takes the form of first-loss tranche retention by the originator, the positive signalling effect may outweigh the above-mentioned general-equilibrium effect. This is especially true when the related required capital retention for the originators is not substantially increased due to the first-loss tranche retention.

The mechanism presented in this paper can contribute to the understanding of the recent financial crisis, since it replicates some of the securitization market outcomes observed prior to and during the crisis. In the period preceding the crisis, many inefficient investments of unknown quality were undertaken. While this was not a problem as long as the economy was performing well, the large amount of low-quality loans in the economy contributed to the depth of the financial crisis. Also, during the crisis, the markets for securitized products were severely strained. The paper also points to some unexpected potential effects of the newly proposed regulation.

The paper is organized as follows. Section 2 reviews the related literature. Section 3 introduces the effects of the considered financial frictions in combination with the provision of implicit recourse. For analytical tractability, this section focuses on the steady state with only idiosyncratic risk and in which the aggregate variables are deterministic. Section 4 reports the results of the full-fledged model with aggregate risk obtained using global numerical methods and focuses on the switching between the separating and pooling equilibrium over the business cycle. Section 5 develops extensions of the model. In particular, I discuss some policy implications of the model and introduce adverse selection on resale markets. Section 6 offers some conclusions.

2 Literature review

This paper is related to several strands of literature. In this section, I focus on research related to securitization with implicit recourse and to financial intermediation imperfections, information frictions and business cycles.

2.1 Securitization and implicit recourse

Securitization is the process of selling cash flows related to the loans issued by the originator (often called the sponsor). The sale of loans is effectuated in a legally separate entity called a special purpose vehicle (SPV) or special purpose entity (SPE). The entity purchases the right to the cash flows with resources obtained by issuing securities in the capital market. The sponsor and the SPV are "bankruptcy remote," and the sale of loans is officially considered to be complete; i.e., the sponsor should transfer all risks to the buyers of the newly emitted securities. Loans are pooled in a portfolio that is then usually divided into several tranches ordered by seniority, which have a different exposure to risk. Before the recent financial crisis, securitization was perceived mainly as a means of dispersing credit risk and allocating it to less risk-averse investors, who would be compensated by higher returns, while highly risk-averse investors could invest in the most senior tranches with high ratings. Due to the role securitization played in the late-2000s financial crisis (e.g., Bernanke, 2010), it attracted a lot of criticism, and the attention of researchers turned more to the set of agency problems present at different stages of the securitization process (Shin, 2009). A detailed review of those agency problems can be found in Paligorova (2009).

Gorton and Pennacchi (1995) were among the first to point to moral hazard problems related to securitization and to address the issue of why securitization takes place despite them. Moral hazard problems stem from the fact that if the risk is transferred with a loan from the originator of the loan to the investor, the bank has a reduced incentive to monitor the loan quality. Gorton and Pennacchi (1995) argue that, before the 1980s, securitization was very limited. In the 1980s, several regulatory changes took place that effectively increased the cost of deposit funding. One key factor was the introduction of binding capital requirements for commercial banks.[3] Banks could avoid increased capital requirements by securitization, which moved some of the risky assets off their

[3] "In 1981 regulators announced explicit capital requirements for the first time in U.S. banking history: all banks and bank holding companies were required to hold primary capital of at least 5.5 percent of assets by June 1985" (Gorton and Metrick, 2010, p. 274).

balance sheets. This view that an important reason for securitization is regulatory arbitrage is shared by many economists (e.g., Gorton and Pennacchi, 1995; Gertler and Kiyotaki, 2010; Acharya et al., 2013; and Gorton and Metrick, 2010). Calomiris and Mason (2004) present some evidence suggesting that regulatory arbitrage is effectuated by securitizing banks to increase the efficiency of contracting where capital requirements are unreasonably high, rather than to abuse the safety net. The moral hazard and agency problems in general were then alleviated by the practice of keeping part of the loan on the balance sheet of the originator. Fender and Mitchell (2009) study different tranche retention designs and their effect on incentives. However, any loan sale, partial or complete, results in lower incentives to monitor the loan quality. One way to increase the confidence of buyers of securitized assets and the related sale price, while avoiding higher capital requirements, is the provision of implicit recourse.

Implicit recourse is a form of implicit support provided by the issuers of securitized products to the holders of these assets. They represent a guarantee of the quality of the loan and implicit risk retention by the originator. Guarantees are deliberately not explicit (i.e., non-contractual), to avoid the requirement of keeping additional capital on originators' balance sheets. This support materializes ex post (i.e., after the sale of securitized assets), typically in periods of lower asset performance, when the originator intervenes to increase the asset return. Much evidence suggests that implicit recourse was frequently used during the securitization process ("As the saying goes, the only securitization without recourse is the last" [Mason and Rosner, 2007, p. 38]). Gorton and Souleles (2006) show in a theoretical model that this mutually implicit collusion between buyers and originators of the securitized loans can be an equilibrium result in a repeated game due to the reputation concerns of originators, who want to pursue securitization in the future at favorable conditions. Several empirical studies documented concrete cases of implicit recourse or showed indirect evidence of its presence. Higgins and Mason (2004) study 17 discrete recourse events that were directed to an increase in the quality of receivables sponsored by 10 different credit card banks. The forms of the support provided were, for instance, adding higher-quality accounts to the pool of receivables, removing lower-quality accounts, increasing the discount on new receivables, increasing credit enhancement, and waiving servicing fees. Higgins and Mason (2004) argue that implicit recourse increases sponsors' stock prices in the short and the long run following the recourse. It also improves their long-run operating performance. They argue that recourse may help to signal to investors that shocks making recourse necessary are only transitory.

Another example showing that the risks were not fully transferred during securitization to the SPV is given by Brunnermeier (2009), who argues that when the SPV was subject to liquidity problems, which arise from a maturity mismatch between the SPV's assets and liabilities and a sudden reduced interest in the instruments emitted by the SPV, the sponsor would grant credit lines to it. Acharya et al. (2013) study explicit liquidity guarantees and argue that their large popularity was due to lower capital requirements compared to similar credit guarantees, strengthening the argument of the regulatory arbitrage motivation of originators.

In this paper, I concentrate on the relationship between investors and banks, where the latter have better information about the quality of loans, and I show that, due to reputation concerns, the bank has an incentive to signal this quality. This is in line with the suggestion by Higgins and Mason (2004) that implicit recourse is used as a signalling tool. However, the efficiency of signalling varies over the business cycle.

2.2 Financial intermediation imperfections, information frictions, and business cycles

This paper is related to the literature on financial frictions in macroeconomic models and the role of asymmetric information and reputation in financial intermediation.

In the recent financial crisis, we witnessed important disruptions of financial intermediation. It became clear that frictions in the financial sector are important and should not be omitted from macroeconomic models. The classic papers that endogenize financial frictions on the side of borrowers include Bernanke and Gertler (1989), Bernanke et al. (1999), and Kiyotaki and Moore (1997). These papers introduce an agency problem between borrowers and lenders, which gives rise to collateral requirements and credit rationing. The resulting endogenous amplification of the effects of the shocks in the economy is known as the "financial accelerator." Some of the recent macroeconomic models with financial frictions directly incorporate securitization. For example, Brunnermeier and Sannikov (2014) find that securitization enables the sharing of idiosyncratic risks, but may amplify systemic risk.

In this paper, I refer often to the Kiyotaki and Moore (2012) model of a monetary economy with differences in liquidity among different asset classes. Their model features borrowing and resaleability constraints and the stochastic uninsurable arrival of idiosyncratic investment shocks among market participants. I simplify this model, and in order to study the financial intermediation similar to securitization, I introduce asymmetric information and model signalling by the provision of reputation-based implicit recourse.

There is much literature on the adverse selection in lender-borrower relationships based on asymmetric information, expanding the original contribution of Akerlof (1970). In Parlour and Plantin (2008), the intensity of adverse selection on the markets for securitized assets (sold loans) depends on the proportion of liquidity sellers and informed sellers who want to sell low-quality loans. Kurlat (2013) models a similar adverse selection problem in an extension of Kiyotaki and Moore (2012) and shows that the proportion of sellers of high-quality assets is lower in a recession, which can lead to market shutdowns. Martin (2009) shows that the relationship between entrepreneurial wealth and aggregate investment, which is the basis of the already-mentioned "financial accelerator," may not be monotonic. In particular, in states with low entrepreneurial wealth, screening of borrowers using collateral requirements may be too costly, and therefore the economy is in a pooling equilibrium, in which good borrowers cross-subsidize bad borrowers.

Recent papers study the role of asymmetric information on the interbank market. Heider et al. (2009) show that asymmetric information about counterparty risk can produce market breakdowns. Boissay et al. (2013) explain, in a model with moral hazard and asymmetric information, why interbank market freezes are more likely after a credit boom. While in this paper I focus on securitization markets, I find similar results: the liquidity problems on the securitization markets are more severe in recessions especially after a prolonged boom period.

One of the major assumptions in the model is the existence of a productivity dispersion shock, which is inspired by the empirical evidence on countercyclical, cross-sectional variance in the TFP of U.S. firms in Bloom (2009) and Bloom et al. (2012). These authors also build models that assume a time-varying variance of idiosyncratic TFP shocks and show that a higher variance can cause a recession. Bigio (2013) uses a similar assumption and shows that a dispersion shock due to the existence of asymmetric information will worsen the adverse selection problem and lead to a recession. Compared to Bigio (2013), my model features reputation-based signalling, which is more effective when the dispersion is larger.

In this paper, the quality of investment allocation decreases during the boom stage of the business cycle. There are various papers that deal with the evolution of bank lending standards over the business cycle. In an empirical paper, Lown and Morgan (2006) document how bank lending standards in the United States deteriorated during the boom stage of the business cycle. In theoretical models with asymmetric information about the quality of borrowers and costly screening by banks, Dell'Ariccia and Marquez (2006) and Ruckes (2004) suggest the reasons for the countercyclical bank lending standards. In Dell'Ariccia and Marquez (2006), booms are periods with a lower share of low-quality borrowers; therefore, banks, due to competition, decide not to require collateral in those periods. In Ruckes (2004), boom periods are related to lower borrower default probabilities, which induce banks to screen less. This results in lower bank lending standards during booms, which is similar to the outcome of this paper. However, in my model, the asymmetric information exists among financial firms trading securitized loans, and the adverse selection can be alleviated by reputation-based signalling. Also unlike the mentioned models, my model is fully dynamic and better suited to study the time dimension of asymmetric information-related effects.

There are also several papers that study the importance of reputation in lender-borrower relationships. Nikolov (2012) introduces reputation to the model of Kiyotaki and Moore (1997) and shows that reputation represents intangible capital, which is more valuable in the boom stage of the business cycle, and therefore it further strengthens the collateral amplification mechanism. Ordoñez (2014) argues that unregulated banking disciplined only by reputation forces may be efficient due to the saving on regulatory and bankruptcy costs, but is more fragile. Chari et al. (2014) present a model where originators can differentiate themselves from others by keeping a larger fraction of their loan portfolio on the balance sheet. Pooling equilibria with adverse selection in which it is too costly for high-quality banks to separate are persistent and characterized by higher trade volumes.

My model is also related to research about the degree of asymmetric information over the business cycle. While some researchers, (e.g., Veldkamp, 2005) argue that booms are associated with a higher degree of trading and therefore more learning, others argue that information may be lost in boom periods of business cycles. Gorton and Ordoñez (2014) present a model where assets with an unknown value can serve as collateral for borrowing. In booms, none of the parties has the incentive to verify the value of an asset, and the economy saves on information acquisition costs and enjoys a "blissful ignorance" equilibrium, while in periods with low aggregate productivity lenders have incentives to verify the value of collateral, which leads to underinvestment. In my model, higher productivity will also be associated with less public information, but this would create inefficiencies.

3 Model

To allow for maximum tractability, the set-up of the model is rather simple. The economy contains a continuum of financial firms that have stochastic heterogeneous investment opportunities. As a result, there is a need for financial intermediation in the model. Optimally, firms would want to transfer resources from firms without investment opportunities or with low-quality investment opportunities to firms with high-quality investment opportunities. The transfer of funds is possible through securitization, which is modelled as a sale of cash flows from the funded projects.[4] Financial intermediation is subject to frictions such as asymmetric information about distribution of investment opportunities, which makes it less efficient in the boom stage of the business cycle.

3.1 Model set-up

In this section I first describe the physical environment and specify the frictions in the model. Then I define the optimization problem of financial firms. Finally, I close the model by clearing both goods and asset markets.

3.1.1 Investment projects

There are three types of projects available to financial firms, and the allocation of firms to projects is stochastic through an i.i.d. shock:

- $(1-\pi)$ share of firms (subset $\mathcal{Z}_t$) do not have access to new investment projects;
- $\pi\mu$ share of firms (subset $\mathcal{H}_t$) have access to high-quality projects with high gross profit per unit of capital $r_t^h = A_t^h K_t^{\alpha-1}$; and
- $\pi(1-\mu)$ share of firms (subset $\mathcal{L}_t$) have access to low-quality projects with low gross profit per unit of capital $r_t^l = A_t^l K_t^{\alpha-1}$.

The realization of this shock cannot be insured by ex ante contracts.

[4] To keep the model simple, I do not model any alternative means of transferring funds such as debt. Elsewhere, I present an extension of this model, where different types of debt, such as deposits or interbank loans, are considered and replicate the main qualitative results of this paper (Kuncl, 2013).

Assumption 1: I assume that the relative difference in gross profits from high- and low-quality projects is countercyclical:

$$\frac{\partial}{\partial A_t} \frac{A_t^h - A_t^l}{A_t^l} < 0, \tag{3.1}$$

where A_t is the aggregate component of TFP. In other words, I assume that there is a dispersion shock perfectly negatively correlated with the productivity shock.

This assumption is inspired by the empirical evidence on countercyclical cross-sectional variance in the TFP of U.S. firms in Bloom (2009) and Bloom et al. (2012).[5] In this model, the TFP of the projects has an aggregate component, A_t, and a type-specific component, Δ_t^h and Δ_t^l, respectively: $A_t^h = A_t \Delta_t^h$ and $A_t^l = A_t \Delta_t^l$. To satisfy the assumption in (3.1), the ratio of type-specific TFP components has to be countercyclical, $\partial \left(\Delta_t^h / \Delta_t^l\right) / \partial A_t < 0$.

Some of the basic features of the model are inspired by Kiyotaki and Moore (2012). Similar to Kiyotaki and Moore (2012), financial firms are subject to an i.i.d. investment shock and face constant returns to scale, i.e., they take r_t^h and r_t^l as given; however, on the aggregate level there are decreasing returns to scale:

$$Y_t = r_t^h H_t + r_t^l L_t = \left(A_t^h \frac{H_t}{K_t} + A_t^l \frac{L_t}{K_t} \right) K_t^\alpha,$$

where $K_t = H_t + L_t$ and $H_t\,(L_t)$ are aggregate holdings of high(low)-quality capital.[6]

3.1.2 Frictions

Two **core frictions** are present in the model:

- Investing firms, which sell securitized loans, have to satisfy the ***"skin in the game"*** constraint (hereafter SGC), i.e., they have to keep at least a $(1-\theta)$-fraction of the investment on their balance sheet. This means they can sell at most a fraction θ of the current investment and the rest has to be financed from

[5]Motivated by the empirical evidence, Bloom (2009) and Bloom et al. (2012) construct models that assume time-varying second moments of idiosyncratic TFP shocks and show that a higher variance can cause a recession. This can be reinterpreted in a simpler setting as a negative correlation between productivity and dispersion shocks.

[6]Kiyotaki and Moore (2012) obtain this result by including labor in the production function and requiring a competitive wage to be paid to workers in order to run a project. Here, for simplicity, I omit the workers from the model, but I use the results of constant returns to scale on the individual level and decreasing returns to scale on the aggregate level by assumption.

their own resources. For simplicity, θ is taken throughout most of the paper as a parameter. However, in section 5 this friction is endogenized by the existence of a moral hazard problem.

- There is an ***asymmetry of information*** about the above-described allocation of investment opportunities among firms. Each firm knows the type of project it is assigned to in the current period, but it is not aware of the allocation of projects among other firms.

The second friction is motivated by a certain opacity of securitized assets and by the aforementioned criticism of securitization, which takes the asymmetric information as the source of most of the related agency problems (for details see the literature review). The first friction can also be observed in reality. But the main reason for having it in this otherwise simple model is that despite the competition among financial firms, a binding SGC increases equilibrium prices above the costs of investment and, therefore, makes the securitization process profitable. Only when securitization is profitable does a reputation equilibrium with implicit recourse exist. As I explain later, when a firm defaults on the implicit recourse, it suffers a punishment by other firms, which will stop buying securitized assets from it.[7] And this represents a loss for the firm only when securitization is profitable.

3.1.3 Firms' problem

Each financial firm (indexed by i) chooses the control variables $\{c_{i,t+s},\, x_{i,t+s},\, \{a_{i,j,t+s+1}\}_j,\, h^S_{i,t+s+1},\, l^S_{i,t+s+1},\, r^G_{i,t+s+1},\, \chi_{i,t+s}\}_{s=0}^{\infty}$ to maximize the expected discounted utility from its future consumption stream:

$$\sum_{s=0}^{\infty} \beta^s u\left(c_{i,t+s}\right),$$

where $u\left(c_{i,t+s}\right) = \log\left(c_{i,t+s}\right)$. The budget constraint for all firms is

$$\begin{aligned} c_{i,t} + x_{i,t}\left(1 - q^G_{i,t}\right) + \sum_{j \in \mathcal{I}_t} a_{i,j,t+1} q^G_{j,t} + h^S_{i,t+1} q^h_t + l^S_{i,t+1} q^l_t \quad &+ \quad \chi_{i,t} cir_{i,t} \\ = \sum_{j \in \mathcal{I}_{t-1}} a_{i,j,t}\left(r^{\hat{G}}_{j,t} + \lambda q_{j,t}\right) \quad &+ \quad h^S_{i,t}\left(r^h_t + \lambda q^h_t\right) + l^S_{i,t}\left(r^l_t + \lambda q^l_t\right) \; \forall i, \forall t. \end{aligned}$$

[7] I assume that it is possible to commit to not buy securitized assets from a particular firm and show that such a commitment can be credible; i.e., even ex post it would be optimal to stick to the punishment rule. In other words, the punishment is renegotiation-proof under certain conditions. However, I assume that it is not possible to prevent a particular firm from buying securitized assets from others; i.e., a threat of complete autarky is not possible. I believe that also, in reality, it is easier to commit not to buy assets from particular issuers rather than forbid them to buy securitized assets.

All firms face the SGC and information asymmetry constraint and firms with no investment opportunity face an additional constraint $x_{i,t} = 0 \; \forall i \in \mathcal{Z}_t$.

This constrained maximization problem describes the following options of firms. The resources of firms consist of stochastic gross profits from projects financed in the past and the market value of a non-depreciated part λ of those projects. They consume the $c_{i,t}$ part of those resources. If they have an investment opportunity, they can originate new projects at unitary costs $x_{i,t}$.[8] I denote the subset of firms that decide to invest, i.e., originate new projects (issue new loans), as $\mathcal{I}_t$. Firms can also buy securitized cash flows from projects, newly originated by other firms, on the primary market $\{a_{i,j,t+1}\}_j$ for prices $\{q_{j,t}\}_j$, or buy securitized cash flows from older projects of known high(low) quality on the secondary (resale) market $h^S_{i,t+1} \left(l^S_{i,t+1}\right)$ for price $q^h_t \left(q^l_t\right)$. The $j \in \mathcal{I}_t$ subscript denotes originating firms, and superscripts h, l denote the known quality of the traded asset. Originating firms can securitize and sell cash flows from the new projects. If they sell a part of their investment,[9] they can provide implicit recourse to buyers of these newly securitized assets in the form of a promise for minimum gross profit per unit of capital next-period $r^G_{i,t+1}$. An asset with implicit recourse is traded for a market price $q^G_{i,t}$, which depends on the information structure in the equilibrium, i.e., on the beliefs of buyers about the type of sold asset. The following period, after the realization of the aggregate shock, each firm can decide whether to default on the implicit recourse from the previous period, which is represented by χ_{t+1}.[10] If a firm honors the implicit recourse, it has to spend part of its resources covering related costs $cir_{i,t}$. Details on the cost of implicit recourse and the choice of default are discussed in section 3.2.4. The timing of shocks and the choice of controls by firms within each period are shown in Figure 3.1.

Note that since profits (cash flows) are observed and Δ^h, Δ^l, and A_t are public information, the uncertainty about the quality of financed projects is resolved at the latest in the period following investment in the project. Therefore, depending on the particular equilibrium, the quality of assets traded on the primary market may be either

[8] Gertler and Kiyotaki (2010), in their study of the interbank market, based on the same modelling approach as Kiyotaki and Moore (2012), refer to investments in projects as loans to entrepreneurs who run those projects. Entrepreneurs are able to offer perfectly state-contingent debt, and since financial firms (banks) have all the bargaining power, they can extract the entire profit from entrepreneurs. Following this approach, I will sometimes refer to the investment in projects as loans, too, and later calibrate this model using the performance of mortgage-backed securities.

[9] The amount of new loans kept on the balance sheet is the difference between investment x_t and the next-period holdings of assets of firm i issued by firm i: $a_{i,i,t+1}$, while $x_t - a_{i,i,t+1} \geqq 0$.

[10] χ_{t+1} takes the value of 1 in the case of no default and 0 in the case of default.

public or private information, and when these assets are traded in the next period on the resale market their quality is already public information. Therefore, we can collapse all assets issued in past periods into two categories of high- and low-quality assets: h^S, l^S.[11] Laws of motion for high- and low-quality assets traded on resale markets are

$$\begin{aligned} H^S_{t+1} &= \sum_i h^S_{i,t+1} = \sum_i \sum_{j \in \mathcal{H}_{t-1}} \lambda a_{i,j,t} + \sum_i \lambda h^S_{i,t}, \\ L^S_{t+1} &= \sum_i l^S_{i,t+1} = \sum_i \sum_{j \in \mathcal{L}_{t-1}} \lambda a_{i,j,t} + \sum_i \lambda l^S_{i,t}. \end{aligned}$$

Since the uncertainty about qualities of projects lasts only for one period, for simplicity and tractability I also restrict the guarantee on the loan performance to one period after the issuance.[12]

Since utility is logarithmic and budget constraints are linear in individual holdings of assets, the policy functions will also be linear in the individual holdings of wealth. Due to logarithmic utility, all firms will always consume a constant fraction of their current wealth (for derivations see Appendix B):

$$c_{i,t} = (1-\beta)\left(\sum_{j \in \mathcal{I}_{t-1}} a_{i,j,t}\left(r^{\hat{G}}_{j,t} + \lambda q_{j,t}\right) + h^S_{i,t}\left(r^h_t + \lambda q^h_t\right) + l^S_{i,t}\left(r^l_t + \lambda q^l_t\right)\right) \ \forall i, \forall t.$$

Linear policy functions and i.i.d. investment opportunities enable a straightforward aggregation. An application of the law of large numbers implies that the aggregate quantities and prices do not depend on the distribution of wealth across individual firms (Σ); therefore, we do not have to keep track of it.

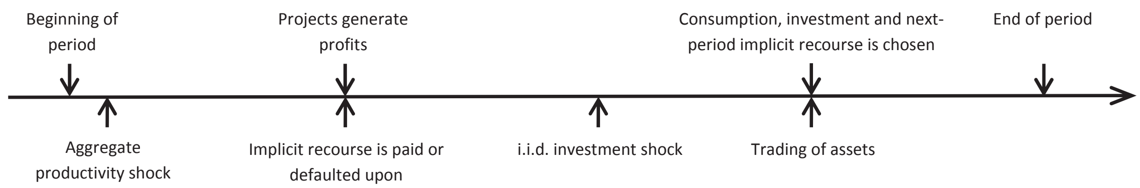
3.1.4 Goods and asset markets

The model features a market for consumption goods and for capital goods (securitized cash flows from projects). In every period all projects generate gross profits in the form of consumption goods. Consumption goods must be either consumed or converted into capital goods by investing in new projects. The goods market clears when all current

[11] Later, in an extension of this model, I relax this assumption and introduce asymmetric information on resale markets.

[12] In a related paper, Kuncl (2014) allows for infinite-horizon implicit guarantees, with the result that public information about the quality of assets traded on the secondary market is persistently unavailable. As a result, Kuncl (2014) presents a richer dynamics of adverse selection on resale markets, though the model is also more complex.

Figure 3.1. Timing of shocks and the choice of a firm's controls within each period



output Y_t is consumed or invested: $Y_t = C_t + X_t$.

Capital goods are traded on asset markets. There is a secondary market on which assets of known quality are traded, and a primary market for newly issued assets whose quality is either known or does not depend on the type of equilibrium. As derived in Appendix B, the conditions for the clearing of asset markets come from the first-order conditions (FOC) of firms, which buy on asset markets (subset $\mathcal{S}_t$), and which I call saving firms $i \in \mathcal{S}_t$. These conditions imply that the discounted return of all assets traded on markets have to be equal to 1, and that, in equilibrium, saving firms will be indifferent between holding different assets.

Asset markets clear when prices satisfy the FOC of the saving firms with respect to portfolio allocation:

$$\begin{aligned}
E_t\left[\beta \frac{c_{i,t}}{c_{i,t+1}} \frac{r^{\hat{G}}_{t+1} + \lambda q_{j,t+1}}{q^G_{j,t}}\right] &= 1 \; \forall i \in \mathcal{S}_t, \forall j \in \mathcal{I}_t, \\
E_t\left[\beta \frac{c_{i,t}}{c_{i,t+1}} \frac{r^h_{t+1} + \lambda q^h_{t+1}}{q^h_t}\right] &= 1 \; \forall i \in \mathcal{S}_t, \\
E_t\left[\beta \frac{c_{i,t}}{c_{i,t+1}} \frac{r^l_{t+1} + \lambda q^l_{t+1}}{q^l_t}\right] &= 1 \; \forall i \in \mathcal{S}_t.
\end{aligned}$$

Recall that all assets depreciate over time, so the law of motion for capital (stock of projects) is $K_{t+1} = \lambda K_t + X_t$.[13]

3.2 Model solution in special cases

To provide intuition for the results of this paper, this section explicitly derives a solution of the model analytically in the steady state, i.e., when aggregate productivity is fixed

[13] Similar laws hold for both types of capital (low quality and high quality): $H_{t+1} = \lambda H_t + X^h_t$, $L_{t+1} = \lambda L_t + X^l_t$. As in Kiyotaki and Moore (2012), I assume that the subjective discount factor exceeds the share of capital left after depreciation: $\beta > \lambda$.

$(A_t = A)$, while idiosyncratic shocks still take place. In section 4, I report numerical results from the fully stochastic case.

To demonstrate the effect of the core frictions in the model, I first briefly show the behavior and solution of the model without frictions, and then I successively introduce a binding "skin in the game" constraint and the asymmetric information. I show that when the SGC is binding, a reputation equilibrium exists, where implicit recourse can be provided. Consequently, I show the solution of the model in the case of interest, i.e., where both frictions hold and the provided implicit recourse can signal the quality of securitized assets.

3.2.1 Case with no financial frictions: first-best

If neither of the two frictions is present—i.e., project allocation is public information and the SGC is not binding—in equilibrium only firms with high-quality investment opportunities will invest, securitize loans, and sell them to firms with low or unproductive investment opportunities (see Figure 3.2). Since there is no asymmetric information and only high-quality projects are being financed, there is only one type of asset traded in the economy. When I omit the variables that turn out to be zero in equilibrium, the budget constraints of individual firms with different investment opportunities are

$$\begin{aligned} c_{i,t} + x_{i,t} + (h_{i,t+1} - x_{i,t})\, q_t^h &= h_{i,t}(r_t^h + \lambda q_t^h)\ \forall i \in \mathcal{H}_t, \\ c_{i,t} + h_{i,t+1} q_t^h &= h_{i,t}(r_t^h + \lambda q_t^h)\ \forall i \in \mathcal{L}_t, \\ c_{i,t} + h_{i,t+1} q_t^h &= h_{i,t}(r_t^h + \lambda q_t^h)\ \forall i \in \mathcal{Z}_t. \end{aligned}$$

Because of competition among firms with high-quality investment opportunities, the price of loans is equal to the unit costs of financing the project (issuing the loan), $q^h = 1$.

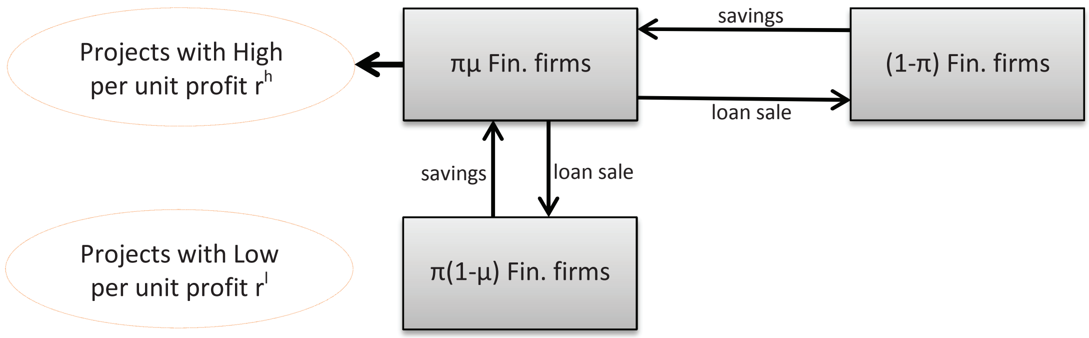
Combining the aggregate consumption function, the goods market clearing condition, and the law of motion for capital, we obtain[14]:

$$r^h + \lambda = \frac{1}{\beta}. \tag{3.2}$$

The current period gross profit per unit of invested capital plus the value of non-depreciated assets is equal to the time preference rate; therefore, the amount of invest-

[14] For details see Appendix A.1.

Figure 3.2. Case without frictions: First-best case



Note: In the first-best case, only firms with access to projects with high profit per unit of capital invest, and they sell some of these projects to the remaining firms.

ment is indeed first-best.

3.2.2 Introducing the "skin in the game" constraint (SGC)

In this section, I show that a binding SGC (θ fraction of new loans that at most can be sold) increases the equilibrium prices above the replacement rate, which makes securitization profitable. As noted above, only when securitization is profitable can a reputation equilibrium exist. The SGC is also a common practice observed in securitization contracts in the form of tranche retention schemes.[15] This constraint can be motivated and endogenized by a moral hazard problem, which is derived in section 5. Section 5 also discusses some potential policy implications when making θ a policy parameter. In this section, I assume for simplicity a constant θ.

By lowering θ, we limit the capacity of firms with access to high-quality projects to issue new investments. When this capacity is lower than the demand for new investments at the zero-profit price $q^h = 1$, the SGC becomes binding, and the price has to increase above the unit costs of investment to clear the market. Securitization becomes profitable.

If the SGC is binding for firms with access to high-quality projects, i.e., their holdings of newly issued assets represent a $(1 - \theta)$ share of their investment $h_{i,t+1} = a_{i,i,t+1} =$

[15]For simplicity, I do not model the existence of different tranches. The "skin in the game" constraint is analogous to keeping a "vertical slice" of all tranches.

$(1-\theta)\, x_{i,t}\ \forall i \in \mathcal{H}_t$[16], we can rewrite their budget constraint as

$$c_{i,t} + \frac{\left(1-\theta q_t^h\right)}{(1-\theta)} h_{i,t+1} = h_{i,t}(r_t^h + \lambda q_t^h) + l_{i,t}(r_t^l + \lambda q_t^l)\ \forall i \in \mathcal{H}_t. \tag{3.3}$$

Combining these two equations and the consumption function, we can find the level of investment of the constrained firm with access to high-quality projects:

$$x_{i,t}^h = \frac{\beta\left(h_{i,t}(r_t^h + \lambda q_t^h) + l_{i,t}(r_t^l + \lambda q_t^l)\right)}{\left(1-\theta q_t^h\right)}\ \forall i \in \mathcal{H}_t. \tag{3.4}$$

All policy functions are again linear, and therefore can be easily aggregated and, as Appendix A.2 shows, we can obtain the following proposition.

Proposition 1. *If "skin in the game" is sufficiently large to be binding, i.e., θ is sufficiently low to satisfy*

$$1-\theta > \frac{\pi\mu}{1-\lambda},$$

then in the deterministic steady state:

(i) the price of high-quality assets q^h exceeds 1;

(ii) the steady-state level of output and capital is lower than in the first-best case.

The above proposition is analogous to Claim 1 in Kiyotaki and Moore (2012), but for a complete characterization of the model's steady state, we also need the following proposition.

Proposition 2. *Suppose the condition from Proposition 1 holds, then depending on parameter values, the deterministic steady state is characterized by one of the following cases:*

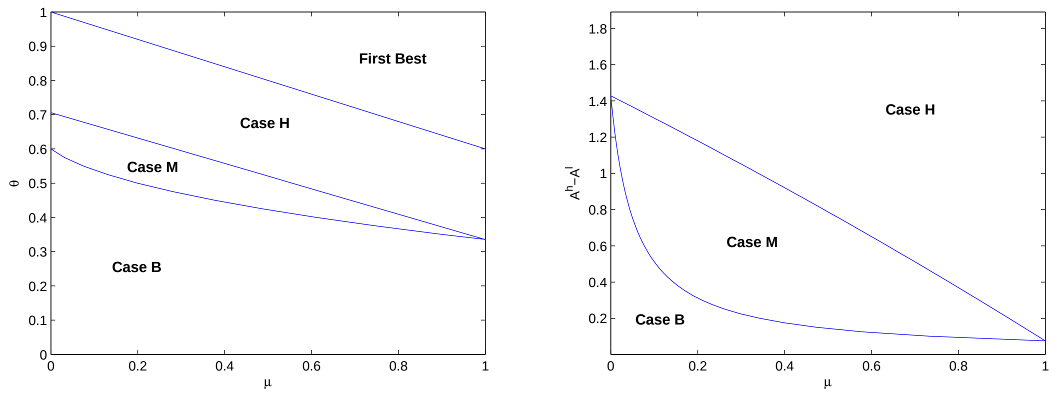
Case H: Only firms with access to high-quality projects issue credit and securitize ($q^l < 1$);

Case M: Firms with access to low-quality loans use a mixed strategy and issue credit with probability ψ, ($q^l = 1$);

Case B: All firms with access to high- and low-quality projects issue credit and securitize ($q^l > 1$).

[16] I show below that for a subset of parameters, firms with access to low-quality projects will also optimally choose to invest and securitize loans in equilibrium. They may also face the binding "skin in the game" constraint, i.e., $l_{i,t+1}^l = a_{i,i,t+1} = (1-\theta)\, x_{i,t}^l\ \forall i \in \mathcal{L}_t$.

Figure 3.3. Type of deterministic steady state depending on selected parameter values



The above cases are ranked from the least restricted ($q^l < 1$), where output and capital levels are relatively the closest to the first-best case, to the most restricted ($q^l > 1$), where output and capital are the lowest:

$$\begin{aligned} Y_{FB} &> Y_H > Y_M > Y_B, \\ K_{FB} &> K_H > K_M > K_B. \end{aligned}$$

Here subscript FB denotes the first-best case, and subscript H, M and B denote the above described cases.

Proofs of the above propositions are in the appendices (A.2 and A.3).

Figure 3.3 shows the effect of selected parameter values on the type of steady state. In the left panel we can see that lowering θ or μ moves the steady state from an unrestricted first-best case to more restricted cases. The right panel shows that lowering the difference in the productivity of the two types makes it more likely that low-quality projects are financed in the steady state.

3.2.3 Introducing asymmetric information

In this section, I describe the consequences of introducing asymmetric information about the allocation of investment opportunities among firms on the model solution. I focus on the effect of asymmetric information between issuers of securitized assets and their first buyers.[17]

[17] In this benchmark version of the model I assume that past projects are not anonymous; therefore, the quality of all existing projects becomes public information in the period following their securitization. Consequently, there is no adverse selection on resale markets. In an extension of this model

Unless the difference in qualities is large enough, firms with access to low-quality projects mimic firms with access to high-quality projects. Since it is not possible to distinguish between the projects, saving firms, which want to diversify their portfolio, buy both high- and low-quality securitized assets at the rate corresponding to the probabilities of their arrival. This means that, in equilibrium, a μ fraction of investment is allocated to high-quality and a $1-\mu$ fraction to low-quality projects.

Proposition 3. *Compared to the public information case, the allocation of capital is generally less efficient (more in favor of low-quality projects); therefore, capital is less productive and the steady-state amount of capital and output is lower.*

For a proof see Appendix A.4.

The public information case will be equal to the private information case only if the difference in quality is large enough. The firm with low-quality investment opportunities will avoid mimicking firms with high-quality investment opportunities as long as the return from buying high-quality assets exceeds the return from mimicking:

$$R \mid buying\, high\, loans \quad > \quad R \mid mimicking.$$

As shown in Appendix A.5, in the steady state this condition implies

$$\frac{A^h}{A^l} > \frac{(1-\theta)\,q^h}{1-\theta q^h} = \frac{(1-\pi\mu)\,(1-\lambda)\,(1-\theta)}{\pi\mu\lambda + (1-\lambda)\,\theta\pi\mu}. \tag{3.5}$$

If the ratio of high and low productivity does not satisfy (3.5), the resulting pooling equilibrium will be less efficient than the public information case. The separation condition can be rewritten as

$$q^l < \frac{1-\theta q^h}{1-\theta}. \tag{3.6}$$

Since, by Proposition 1, $q^h > 1$, (3.6) implies that a necessary condition for the existence of a separating equilibrium is that the equilibrium price of low-quality assets is lower than the costs of investing $q^l < 1$.

Note also that tightening the SGC, i.e., lowering θ, will only increase the lower bound for the ratio of productivities in condition 3.5 and, therefore, make mimicking

(section 5.3), I relax this assumption and show that asymmetric information on resale markets can lead to market shutdowns similar to Kurlat (2013). Kuncl (2014) focuses on the interaction of adverse selection on the resale markets and the provision and default on infinite-horizon implicit recourse.

more likely. This result is driven by the general-equilibrium effect. A lower θ increases the prices in the economy and, therefore, makes mimicking more profitable.

Proposition 4. *Under private information, increasing the "skin in the game," i.e., lowering θ, makes a pooling equilibrium in which firms with low-quality investment opportunities mimic firms with high-quality investment opportunities more likely.*

3.2.4 Introducing implicit recourse and the reputation equilibrium case

Proposition 3 implies that the outcome of the private information case is generally inefficient compared to the public information case. Firms with high-quality investment opportunities have incentives to distinguish themselves from low-quality investment firms. However, under Proposition 4, we can see that retaining higher "skin in the game" does not lead to a separating equilibrium.

Signalling would be more efficient if originators could keep more risk from the investment on their balance sheets without restricting their investment capacity. This could be achieved either by tranching and first-loss tranche retention by the originator (explicit risk retention) or by the provision of implicit recourse by the originator (implicit risk retention). In the real world, explicit risk retention would typically lead to higher capital requirements and would thus restrict the investment potential of originators, while implicit risk retention was popular precisely for the potential to arbitrage regulation. While in this simple model I do not model capital requirements regulation or the reasons for it, I focus my analysis on the efficiency of implicit recourse.

It turns out that by providing **implicit recourse**, a firm with high-quality investment opportunities can distinguish itself without restricting its investment potential. Under this strategy, the issuing firm promises a minimum gross profit per unit of invested capital r_t^G to the buyers of securitized loans. Should the actual gross profits in the following period fall below this minimum, the issuing firm would reimburse the difference. This promise is not enforced by any explicit contract; rather, it is the result of collusion between issuers of loans and their buyers. Implicit recourse can be credible in a reputation equilibrium, where securitizing firms aim to keep their reputation of sticking to the promise, and firms buying securitized projects enforce this promise by punishing the issuing firms in case of default on the implicit recourse. I assume a trigger punishment strategy that prevents a firm without a reputation of honoring implicit recourse from selling securitized assets on the market. The punishment has to be credible; therefore, in this reputation equilibrium, buyers of securitized products with implicit support aim to keep the reputation of being "tough investors," i.e., a reputation of always punishing firms that did not fulfill their promise.

At this point, it is convenient to write the problem recursively:

$$
\begin{aligned}
V^{ND}\left(\bar{s}, w-cir; \bar{S}\right) &= \pi\left(\mu V^{ND,h}\left(\bar{s}, w-cir; \bar{S}\right) + (1-\mu) V^{ND,l}\left(\bar{s}, w-cir; \bar{S}\right)\right) && (3.7)\\
&\quad + (1-\pi) V^{ND,z}\left(\bar{s}, w-cir; \bar{S}\right),\\
V^{D}\left(\bar{s}, w; \bar{S}\right) &= \pi\left(\mu V^{D,h}\left(\bar{s}, w; \bar{S}\right) + (1-\mu) V^{D,l}\left(\bar{s}, w; \bar{S}\right)\right) + (1-\pi) V^{D,z}\left(\bar{s}, w; \bar{S}\right), && (3.8)\\
V^{ND,k}\left(\bar{s}, w; \bar{S}\right) &= \max_{c,x,\left\{a'_j\right\}_j, h^{S\prime}, l^{S\prime}, r^{G\prime}} [\log(c) && (3.9)\\
&\quad + \beta E\left[\max\left(V^{ND}\left(\bar{s}', w'-cir'; \bar{S}'\right), V^{D}\left(\bar{s}', w'; \bar{S}'\right)\right)\right]],\\
V^{D,k}\left(\bar{s}, w; \bar{S}\right) &= \max_{c,x,\left\{a'_j\right\}_j, h^{S\prime}, l^{S\prime}} \left[\log(c) + \beta E V^{D}\left(\bar{s}', w'; \bar{S}'\right)\right], && (3.10)
\end{aligned}
$$

where V^{ND} $\left(V^{D}\right)$ are the value functions for the firm that never defaulted (has already defaulted) on implicit recourse. w is individual wealth before deducting the costs of implicit recourse cir, $\bar{s} = \left\{\{a_j\}_j, h^S, l^S\right\}$ is a vector of individual state variables, $\bar{S} = \{K, \omega, A\}$ is a vector of aggregate state variables, and superscript k, which can take values $\{h, l, z\}$, represents the type of investment opportunity that the firm faces in the current period.

Equations (3.7) and (3.8) show the investment shock that takes place after the realization of the aggregate productivity shock and the decision on (non-)default on implicit recourse from the previous period. After the investment shock, firms optimally choose the level of consumption, the quantity of securitized loans they buy on the primary and secondary market, and, if they have an investment opportunity, they choose the optimal level of investment in new projects, the securitization of their cash flows, the fraction of the new investment that is sold, and the implicit recourse they provide.[18] This problem is described by equations (3.9) and (3.10) for firms with a reputation for having never defaulted on implicit recourse and for firms without this reputation, respectively.

The above problem is constrained by budget constraints that take the following form for investing firms for which the "skin in the game" constraint is binding (e.g., in the case where firms have high-quality investment opportunities):

$$
c_{i,t} + \frac{(1-\theta q^G_{i,t})}{(1-\theta)} h_{i,t+1} + cir_{i,t} = \sum_{j \in \mathcal{I}_{t-1}} a_{i,j,t}\left(r^{\hat{G}}_{j,t} + \lambda q_{j,t}\right) + h^S_{i,t}(r^h_t + \lambda q^h_t) + l^S_{i,t}(r^l_t + \lambda q^l_t)\ \forall i \in \mathcal{H}_t,
$$

[18]Recall that the timing of shocks and the choice of controls by firms within each period are shown in Figure 3.1.

where the price of securitized loans issued by firm j, $q^G_{j,t}$, depends on the information structure, i.e., on the beliefs of buyers about the type of sold asset $\varphi_{j,t} \mid r^G_{j,t}$. When the "skin in the game" is binding, the costs of implicit recourse are given by

$$cir_{i,t+1} = \theta x_{i,t} \left(r^G_{i,t} - r^k_t \right) \; \forall i \notin \mathcal{S}_t, k \in \{h, l\}.$$

The incentive-compatible constraints (ICCs), which have to be satisfied in equilibrium for the existence of reputation-based implicit recourse, are

$$V^{ND}\left(\bar{s}', w' - cir'; \bar{S}'\right) \geq V^{D}\left(\bar{s}', w'; \bar{S}'\right), \tag{3.11}$$

for the current originators of securitized assets (subset $\mathcal{I}_t$), and

$$V^{P}\left(\bar{s}'; \bar{S}'\right) \geq V^{NP}\left(\bar{s}'; \bar{S}'\right), \tag{3.12}$$

for the current buyers of securitized assets (subset $\mathcal{S}_t$). V^P, V^{NP} are the value functions for the firm that has always punished for default on implicit recourse, and for the firm that failed to punish for default and suffers the negative consequences, respectively.

Condition 3.11 determines the level of implicit recourse that can be credibly provided; i.e., it is not defaulted upon, given the trigger strategy punishment rule. The trigger punishment strategy has to be credible. Therefore, the saving firm that observes default on implicit recourse has to be ex post better off punishing the investing firm that defaulted rather than not punishing it. This corresponds to condition 3.12.[19]

Definition 1. A recursive competitive equilibrium consists of prices $\{q^h\left(\bar{S}\right),\ q^l\left(\bar{S}\right),$ $\left\{q^G_j\left(\bar{S}\right)\right\}_j\}$ and gross profits per unit of capital $\left\{r^h\left(\bar{S}\right), r^l\left(\bar{S}\right)\right\}$, individual decision rules $\{c\left(\bar{s};\bar{S}\right), x\left(\bar{s};\bar{S}\right), h^{S\prime}\left(\bar{s};\bar{S}\right), l^{S\prime}\left(\bar{s};\bar{S}\right), r^{G\prime}\left(\bar{s};\bar{S}\right), \left\{a'_j\left(\bar{s}, r^G_j, \varphi_j \mid r^G_j; \bar{S}\right)\right\}_j, \chi\left(\bar{s}, r^G; \bar{S}\right)\}$, value functions $\{V^{ND}\left(\bar{s};\bar{S}\right), V^{ND,k}\left(\bar{s};\bar{S}\right), V^{D}\left(\bar{s};\bar{S}\right), V^{D,k}\left(\bar{s};\bar{S}\right), V^{NP}\left(\bar{s};\bar{S}\right), V^{P}\left(\bar{s};\bar{S}\right)\}$, and the law of motion for $\bar{S} = \{K, \omega, A, \Sigma\}$ such that: (i) individual decision rules and value functions solve each firm's problems taking prices, gross profits per unit of capital, and law of motion for $\bar{S} = \{K, \omega, A, \Sigma\}$ as given; (ii) both asset and goods markets clear; and (iii) the law of motion for $\bar{S} = \{K, \omega, A, \Sigma\}$ is consistent with the individual firm's decisions.

[19] I show that this condition holds in Appendix A.6.

3.2.5 Public information case with implicit recourse

Although one might think that the public information case is uninteresting, it is an important benchmark. If originating firms could coordinate, they would not be providing implicit recourse in the case, when it does not serve as a tool that would distinguish the firm type. However, due to competition, firms tend to outbet each other.

Should promises always be credible, the optimal level of implicit recourse would be determined by the following FOC (note that the individual firm ignores the effects of this choice on aggregate variables):

$$\frac{\partial V^{ND}}{\partial r^G} = \frac{\partial V^{ND'}}{\partial \left(w' - cir'\right)} \frac{\partial \left(w' - cir'\right)}{\partial r^G} = 0.$$

I show in Appendix A.7 that this condition implies that $q^j = 1$, which means that as far as there are positive profits from securitization, the competition will drive the level of implicit recourse so high that profits from securitization are zero. However, when profits from securitization are zero, the punishment has zero costs, and the original non-defaulting ICC (3.11) is not satisfied. This leads us to the following conclusion.

Proposition 5. *As long as the implicit recourse is credible, firms find it optimal to increase it up to the level where $q^j = 1$. So the level of implicit recourse is defined by the maximum recourse, which can be sustained by the no-default condition (3.11).*

For details on the derivation, see Appendix A.7. The steady state, in this case, is characterized by the following propositions.

Proposition 6. *Suppose that the condition from Proposition 1 holds, then depending on parameter values, a deterministic steady state is characterized by one of the following cases:*

Case 1: Only firms with access to high-quality projects issue credit, securitize loans, and provide implicit recourse $r^G_{h,cred}$ ($q^h > 1$, $q^l < 1, G^h_{cred} \geq r^h$);

Case 2: Firms with access to high-quality projects issue credit, securitize loans, and provide implicit recourse $r^G_{h,cred}$, and firms with access to low-quality projects use a mixed strategy and issue credit with probability ψ and provide implicit recourse $r^G_{l,cred}$ ($q^h > 1$, $q^l = 1$, $r^G_{h,cred} \geq r^h$, $r^G_{l,cred} = r^l$);

Case 3: All firms with access to high- and low-quality projects issue credit, securitize, and provide implicit recourse $r^G_{h,cred}$ and $r^G_{l,cred}$, respectively ($q^h > 1$, $q^l > 1$, $r^G_{h,cred} \geq r^h$, $r^G_{l,cred} \geq r^l$).

Note that $r^G_{k,cred}$ is the maximum implicit recourse that can be credibly provided by firms with a $k \in \{h, l\}$ type of investment opportunity.

Proposition 7. *Compared to the public information case without implicit recourse, the amount of capital and output are higher, the allocation of capital is more in favor of high-quality projects and wealth is less concentrated inside firms with investment opportunities. This holds unless the provided implicit recourse has no value ($r^G_{h,cred} = r^h$), and therefore the result is identical to the case without implicit recourse.*

3.3 Case of interest: Implicit recourse as a signal of loan quality

In this section, I analyze the case of interest, where the SGC is binding, there is asymmetric information about the distribution of firms to investment opportunities, and implicit recourse can signal the type of investment opportunity.

As proven in section 3.2.4, implicit recourse can be credibly provided in a reputation equilibrium. Under asymmetric information, implicit recourse can be interpreted as a signal of the loan quality. Investing firms (subset $\mathcal{I}_t$) sell securitized cash flows from newly financed projects and provide implicit recourse $r^G_{j,t+1} \in (0, \infty)$. The fact that a particular firm sells securitized cash flows and provides $r^G_{j,t+1}$ is the message that this firm is sending to potential buyers of its securitized cash flows. Saving firms (subset $\mathcal{S}_t$), observing any message sent with positive probability, use Bayes' rule to compute the posterior assessment that the message comes from each type. Without restriction on out-of-equilibrium beliefs (beliefs about the types conditioned on observing messages that are not sent in equilibrium), there is a multiplicity of perfect Bayesian equilibria (PBE), generally both pooling and separating. I use the intuitive criterion (Cho and Kreps, 1987) as a refinement to eliminate the dominated equilibria with unreasonable out-of-equilibrium beliefs. As a result, when at least one separating PBE exists, after the application of intuitive criterion, we eliminate all except one unique separating equilibrium. When no separating PBE exists, the economy is in a pooling equilibrium. So in the dynamic version, when state variables change, the economy may be switching between a pooling and a separating equilibrium.

Pooling equilibria: In pooling equilibria, both firms with access to high- and low-quality investment opportunities choose to provide the same level of implicit recourse given the beliefs of investors (see Figure 3.8). They both provide r^{G*} with probability 1. Saving firms observe this message and use Bayes' rule to compute the posterior assessment that messages are sent by each type:

$$\varphi\left(j \in \mathcal{H}_t \mid r_j^G = r^{G*}\right) = \frac{\varphi\left(j \in \mathcal{H}_t\right) \cdot 1}{\varphi\left(j \in \mathcal{H}_t\right) \cdot 1 + \varphi\left(j \in \mathcal{L}_t\right) \cdot 1 + \varphi\left(j \in Z_t\right) \cdot 0} = \frac{\mu\pi}{\mu\pi + (1-\mu)\pi} = \mu.$$

Absent aggregate risk, there are several candidates for the pooling PBE:

Case 1: Firms with access to both high- and low-quality projects select with probability 1: $r^{G*} = r_{l,cred,p}^G$, where $r_{l,cred,p}^G$ is the maximum implicit recourse that can be provided by firms with low-quality assets under pooling. Saving firms' out-of-equilibrium beliefs that sustain this equilibrium can be the following: $\varphi\left(j \in \mathcal{H}_t \mid r_{l,cred,p}^G < r_j^G < r_{h,cred,s}^G\right) = 0$ and unrestricted for intervals $0 < r_j^G < r_{l,cred,p}^G$, and $r_j^G > r_{h,cred,s}^G$.[20] In this equilibrium, no firm defaults on implicit guarantees. None of the firms have the incentive to unilaterally decrease the implicit recourse or increase it.

Note that choosing $r_j^G < r_{l,cred,p}^G$ is not an equilibrium, since both types will have incentives to increase implicit recourse to $r_j^G = r_{l,cred,p}^G$ due to competition, no matter what the beliefs of investors are, since both types would fulfill the implicit recourse in this interval.

Case 2: Firms with access to both high and low-quality projects select $r_j^G = r^{G*}$ such that:

$$r_{lb,p}^G \leq r^{G*} \leq \min\left(r_{minsep}^G, r_{h,cred,p}^G\right).$$

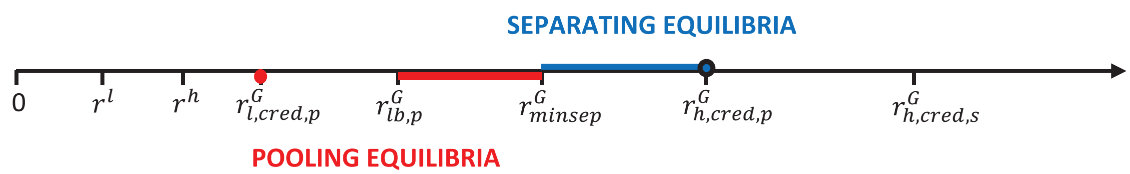
Saving firms' out-of-equilibrium beliefs that sustain this equilibrium can be the following: $\varphi\left(j \in \mathcal{H}_t \mid r^{G*} < r_j^G < r_{h,cred,s}^G\right) = 0$, and $\varphi\left(j \in \mathcal{H}_t \mid 0 < r_j^G < r^{G*}\right) \leq \mu$ and unrestricted for the interval $r_j^G > r_{h,cred,s}^G$.

r_{minsep}^G is the minimum level of implicit recourse, which the low types would not mimic under any beliefs (see derivation in Appendix A.9). $r_{lb,p}^G$ is the lower bound on r^G, where firms with high-quality investments do not have incentives to deviate to $r_{l,cred,p}^G$. For r^G such that $r_{l,cred,p}^G < r^G < r_{lb,p}^G$, both types have incentives to decrease implicit recourse to $r_j^G = r_{l,cred,p}^G$, since equilibrium defaults on the implicit recourse of firms with low investment, which bring investors lower utility than when $r^G = r_{l,cred,p}^G$. This negative effect on price, together with potentially higher costs of higher implicit recourse (when $r^G > r^h$), outweighs the positive effect of higher implicit recourse on the price.

Separating equilibria: There is potentially a continuum of separating equilibria, where firms with access to low-quality projects save and buy securitized assets from

[20] $r_{h,cred,s}^G$ is the maximum level of implicit recourse that can be promised credibly by a firm with high-quality investment opportunities in a separating equilibrium (see below).

Figure 3.4. The case where the intuitive criterion selects a unique separating equilibrium



firms with access to high-quality projects (see Figure 3.7). Firms with access to high-quality projects invest, securitize, and provide implicit recourse $r^{G*} \in \left(r^G_{minsep}, r^G_{h,cred,s}\right)$ with probability 1, where r^G_{minsep} is the minimum implicit recourse that prevents mimicking by firms with low investment opportunities. Saving firms observe this message and use Bayes' rule to compute the posterior probability that the message is sent by each type:

$$\varphi\left(j \in \mathcal{H}_t \mid G_j = G^*\right) = \frac{\varphi\left(j \in \mathcal{H}_t\right) \cdot 1}{\varphi\left(j \in \mathcal{H}_t\right) \cdot 1 + \varphi\left(j \in \mathcal{L}_t\right) \cdot 0 + \varphi\left(j \in Z_t\right) \cdot 0} = \frac{\mu\pi}{\mu\pi} = 1.$$

Saving firms' out-of-equilibrium beliefs that sustain this equilibrium can be the following: $\varphi\left(j \in \mathcal{H}_t \mid r^{G*} < r^G_j < r^G_{h,cred}\right) = 0$ and unrestricted for intervals $0 < r^G_j < r^{G*}$ and $r^G_j > r^G_{h,cred,s}$.

Application of intuitive criterion: If a separating equilibrium exists, then all pooling equilibria are dominated, and therefore fail the intuitive criterion (Cho and Kreps, 1987). In particular, due to competition among firms with access to high-quality investments, the intuitive criterion selects only one separating equilibrium, where firms with access to high-quality investments invest, securitize, and provide the maximum credible implicit recourse $r^{G*} = r^G_{h,cred,s}$.[21] When there are multiple separating PBE, after applying the intuitive criterion, we eliminate all dominated equilibria with unreasonable out-of-equilibrium beliefs and obtain a unique separating equilibrium. In a case where no separating PBE exists, application of the intuitive criterion does not affect the potential multiplicity pooling equilibria. Below, I derive conditions for the uniqueness of pooling equilibrium.

The condition for the existence of a separating equilibrium:

Thanks to Proposition 5, we know that firms have incentives to unilaterally increase the provided implicit recourse up to the maximum credible level. But then, if low-quality firms are already at the maximum credible level, where the costs of default-

[21]This case is shown in Figure 3.4.

Figure 3.5. The case where there is no separating equilibrium

Figure 3.6. The case with unique pooling equilibrium

ing and keeping the implicit recourse are equalized, they are better off if they increase the implicit recourse without increasing the costs further, but potentially benefit from being mistaken for a firm with access to high-quality projects. Therefore, no separating equilibrium can exist in which firms with low-quality investment would provide a different level of implicit recourse. Firms with low-quality investments always prefer to mimic firms with high-quality investments rather than provide a lower implicit recourse and disclose their quality.

Therefore, separation can take place only when the costs of mimicking become so large that investing in high-quality assets is preferred. Under the deterministic case, this condition can be expressed analytically. The implicit recourse r^G has to be high enough to satisfy:

$$V^l \mid mimicking < V^l \mid buying\ high\ loans. \tag{3.13}$$

This brings us to one of the main findings in this paper.

Proposition 8. *Under asymmetric information, a separating equilibrium is possible in the deterministic steady state if and only if*

$$\frac{A^h}{A^l} > \frac{(1-\theta B)\, q^h}{1-\theta B q^h}, \tag{3.14}$$

where $B \equiv \frac{q^G}{q^h} = \frac{r^G+\lambda q^h}{r^h+\lambda q^h}$ *is the price premium for the equilibrium implicit guarantee. This implies that separating equilibrium:*

(i) exists if and only if the level of aggregate productivity does not exceed the threshold level $\tilde{A}$;

(ii) exists if and only if $q^l < 1$; and

(iii) is more likely in the presence of reputation-based implicit recourse.

In a separating equilibrium, firms with low-quality investment projects save and buy securitized assets from firms with high investment opportunities.

Sketch of proof: The derivation of (3.14) comes directly from the no-mimicking condition 3.13.[22] Point (i) comes directly from Assumption 1 about the countercyclical relative difference of cash flows from projects of different quality. Since the ratio of TFP on the left-hand side (LHS) of (3.14) increases with aggregate TFP A, the mentioned threshold is defined as $\Delta^h\left(\tilde{A}\right)/\Delta^l\left(\tilde{A}\right) = (1-\theta B)\,q^h/\left(1-\theta B q^h\right)$.

Crucially, as I show in Appendix A.8, in a separating steady state, both q^h and B and, therefore, the whole right-hand side (RHS) of (3.14) are independent of the steady-state level of aggregate productivity A and are uniquely determined by the intensity of frictions and the punishment for default on implicit recourse.

After a substitution of the share of TFP by the ratio of prices from the asset market clearing condition, condition 3.14 can be rewritten as

$$q^l < \frac{1-\theta B q^h}{1-\theta B},$$

which implies that in a separating equilibrium $q^l < 1$, since, by Proposition 1, $q^h > 1$.

Finally, when comparing the lower bound on the TFP ratio, consistent with the separating equilibrium in cases without implicit recourse (eq. 3.5) and in cases with implicit recourse (eq. 3.14), we can show that the latter is lower. This implies that in the case with implicit recourse, the separation condition (eq. 3.14) is more likely to be satisfied.[23]

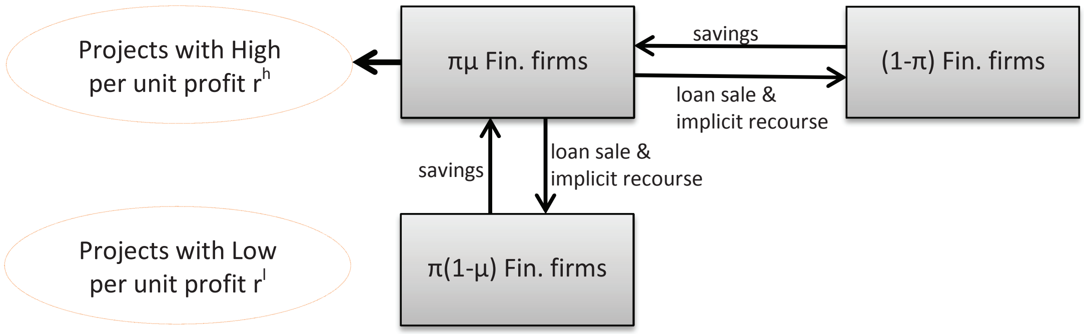
Uniqueness of pooling equilibrium:

When a separating equilibrium does not exist, there is generally a continuum of pooling equilibria (see Figure 3.5). However, it turns out that for a large subset of parameter space, there is only one pooling equilibrium with $r^{G*} = r^G_{l,cred,p}$, independent of a specific type of out-of-equilibrium beliefs (see Figure 3.6). I calibrate the model to have only one pooling equilibrium. The advantage of this calibration is that punishment

[22] See Appendix A.8 for the derivation.

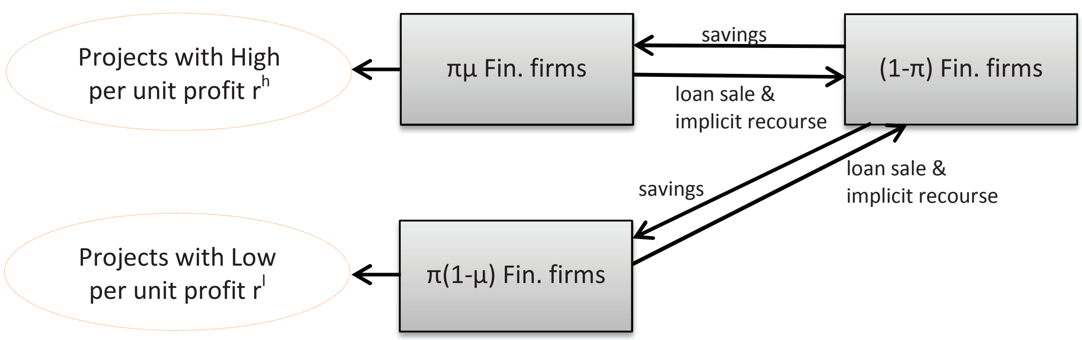
[23] The proof is in Appendix A.8.

Figure 3.7. A private information case with implicit recourse: separating equilibrium



Note: In the separating equilibrium, the implicit recourse provided by the firms with access to high-quality projects is high enough, so that it is not profitable for firms with access to low-quality projects to mimic them. They are better off buying high-quality projects.

Figure 3.8. A private information case with implicit recourse: pooling equilibrium



Note: In the pooling equilibrium, both firms with access to high-quality projects and firms with access to low-quality projects provide the same level of implicit recourse. They are indistinguishable, and, therefore, both types of firm invest in projects and sell them to firms with no investment opportunities.

is never triggered in equilibrium. It still provides the disciplining role, but the dynamic results are not influenced by the exercise of a particular punishment rule.[24]

To obtain such an equilibrium, I have to find parameter values such that $r^G_{lb,p} > r^G_{h,cred,p}$; i.e., the minimum level of implicit recourse for which it pays off to provide recourse higher than $r^G_{l,cred,p}$ is not credible in equilibrium, since it exceeds $r^G_{h,cred,p}$.

It turns out that this condition is satisfied for a low enough share of high-quality investment opportunities, μ, and a high enough difference in type-specific TFP in a pooling equilibrium:

$$\mu < \frac{1-\theta q^l}{q^h - \theta q^l}.$$

For details see Appendix A.9.

4 Dynamics and numerical examples

In this section, I show a solution of the fully stochastic version of the model with asymmetric information, binding SGC and implicit recourse. The allocation of projects to firms is still driven by an i.i.d. shock. The aggregate productivity for simplicity follows a two-state Markov chain $A_t \in \left(A^H, A^L\right)$[25] with a transition matrix $P = [p, 1-p; 1-p, p]$.[26]

When analyzing the dynamic properties of the model, I focus on the switching between the separating and the pooling equilibrium over the business cycle. Even though in the steady state there is a separating equilibrium, when the aggregate productivity increases and the economy is in the boom $A_t = A^H$, a separating equilibrium is no longer sustainable. The economy is in the pooling equilibrium, where both types of firms provide the same level of implicit support and both invest in new projects. This follows directly from Proposition 8. The intuition behind the result is the following. As the aggregate productivity increases, the relative difference in productivity of the two non-zero profit project types is reduced. Therefore, a higher implicit recourse is needed to satisfy the separation condition (3.13). Intuitively, following Proposition 8, the condition says that $q^l < \left(1-\theta B q^h\right) / (1-\theta B) < 1$ is necessary for separation, but

[24] Kuncl (2014) generalizes this model and contains economy-wide defaults on the implicit recourse in a deep recession. But even in this case the punishment is not triggered, since it stops being renegotiation-proof.

[25] Note that the capital superscripts H, L refer to the aggregate state of the economy and not to the type of investment opportunity.

[26] The case when A_t follows a Markov chain is easier to calibrate but is not crucial for the results. An earlier version of this paper works with an AR(1) process for the aggregate TFP.

in a boom even the quality of low type projects is relatively high, and therefore one has to provide high implicit recourse to drive the prices of low-quality projects low enough. At some point, the level of implicit recourse required to achieve separation exceeds the maximum level that can be credibly provided, and the economy switches to the pooling equilibrium.

Calibration of parameters: Since I extend the model of Kiyotaki and Moore (2012), I use the same parameters for $\alpha = 0.4$, $\beta = 0.99$, and $\pi = 0.05$. The persistence parameter for the productivity process is $p = 0.86$.[27] Parameters A^H, A^L are chosen to match the annual standard deviation of GDP in the United States, which is 2.8%.[28] The remaining parameters are chosen to replicate the performance (delinquency rates) of securitized assets, which has been at the core of recent debates over the efficiency of securitization—subprime residential mortgage-backed securities issued (RMBS) in the United States: $\mu = 0.63$, $\Delta^l\left(A^H\right)/\Delta^h\left(A^H\right) = 0.94$ and $\Delta^l\left(A^L\right)/\Delta^h\left(A^L\right) = 0.71$.[29] The annual depreciation $\lambda = 0.78$ is chosen to replicate the weighted average life for RMBS of 54.5 months (Centorelli and Peristiani, 2012). And finally the fraction of loans that can be sold is set to $\theta = 0.75$ to allow for the switching between pooling and separating equilibrium over the business cycle.

Solution method: The fully stochastic model is solved using a global numerical approximation method.[30] In particular, I find the price and the value functions by iterating them on the grid of state variables until convergence.[31]

Impulse responses: Figure 4.1 shows how the economy behaves in a particular episode of three periods in a state with high aggregate TFP, followed by three periods in a state with low aggregate TFP. Then the productivity shocks are switched off and the economy converges to the steady state.[32] The point of this exercise is to show the switch from separating equilibrium to pooling and back and its effects on output.

[27]This corresponds to an autocorrelation of TFP shocks at the quarterly frequency of 0.95.

[28]A similar approach is used in Nikolov (2012).

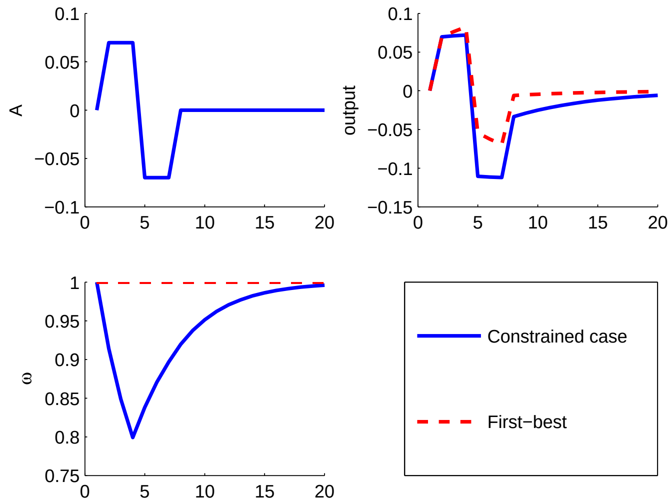
[29]For details see Appendix C.

[30]See, e.g., Judd (1998) for the description of global numerical methods and their distinction from local numerical methods.

[31]Details are in Appendix D.

[32]In this case with a Markov chain for aggregate productivity, the steady-state productivity is defined as the mean of the ergodic distribution across $\left(A^H, A^L\right)$, and in this zero-probability steady state, the expectations about the occurrence of either state are set to 50%.

Figure 4.1. Impulse responses



For comparison on the graph, I report impulse responses[33] of the constrained model under private information, with binding "skin in the game" and with an implicit recourse provision, as well as an unconstrained and efficient first-best case. Note that the graph depicts deviations from each model's steady state. Only the share of high-quality assets on the balance sheets (ω) is shown in absolute value. So even though, on the graph, both the first-best and the constrained cases start at the same point, the first-best case is characterized by higher absolute levels of steady-state output and capital.

The figure shows that, as the constrained economy moves to the boom stage of the business cycle, the equilibrium type changes from separating to pooling; i.e., the share of high-quality projects (ω) decreases, while ω remains constant in the first-best case at 100%. The lower share of high-quality projects in the constrained case slows the growth of output slightly, as well as the accumulation of capital already in the boom, but the effect is small, since in the boom stage the difference in the two qualities is rather small. However, the inefficiency in the allocation of capital continues to accumulate. As the economy exogenously moves to a recession with a higher difference in qualities, one can see that the accumulated inefficiency in the allocation of capital is more pronounced. Therefore, booms have almost the same relative size in a constrained and first-best case, but busts that follow a boom stage are much deeper in a constrained case.

[33] The impulse responses start from a steady state to which they converge after a long period of zero-productivity shocks; i.e., aggregate productivity stays at the steady-state productivity. Then, I introduce the described sequence of productivity shocks after which the shocks are zero again.

Figure 4.2. The longer the boom stage, the deeper the subsequent recession

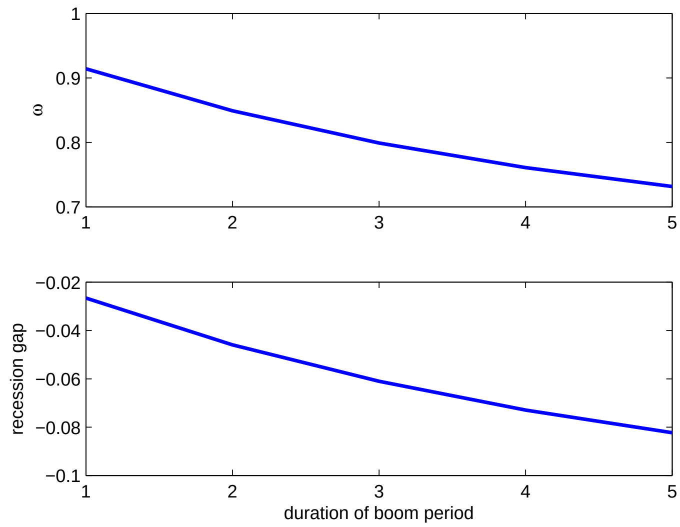


Figure 4.2 shows that the longer the boom period is preceding the recession, the larger the fraction is of low-quality assets accumulated in the pooling equilibrium, and the larger the difference in the depth of a recession is compared to the first-best case (a recession gap). This result follows directly from the above described property of the model, i.e., the existence of pooling equilibrium in the boom.

5 Extensions

5.1 Endogenizing the "skin in the game"

So far, the "skin in the game" (or, equivalently, the share of loans that can be sold, θ) has been taken as an exogenous parameter. In this section, I will sketch a simple moral hazard problem, which rationalizes the existence of this constraint.

Consider that firms can divert funds from the sale of current period loans needed to cover the unit investment costs. This cannot be immediately verified. To eliminate this problem, investors require the issuing firms to retain sufficiently large "skin in the game" $(1-\theta)$, i.e., to finance a fraction $1-\theta$ of funds in the project from their own resources. The ICC then determines a sufficiently high θ that would prevent this moral hazard problem[34]:

$$V^{D}\left(w\beta R' \mid diverting\, funds\right) \leq V^{ND}\left(w\beta R' \mid investing\, properly\right),$$

where the return from diverting funds is $R' \mid diverting\, funds = \left(\frac{\theta q^{G}}{(1-\theta)}\right)^{x}$, with x being the number of times the individual recycles the returns from this operation to issue and sell new "castles-in-the-air" projects. Since I do not restrict the practice of the sequential issuance of loans, which is technically needed even under proper investing, the ICC will always fail unless $\theta q^{G} \leq (1-\theta)$, which translates to

$$\theta \leq \frac{1}{q^{G}+1}. \tag{5.1}$$

Thus, the higher the sale price of loans q^{G}, the more "skin in the game" $(1-\theta)$ is required to prevent the stated moral hazard problem.

Note that, in this version of the model, I have two sources of asymmetric information. The first is the potential diversion of resources needed to invest properly, which cannot be immediately observed. The "skin in the game" is found to be an efficient tool to prevent this behavior, while the loss of reputation and subsequent punishment are not efficient. The second source of information asymmetry is the unobserved allocation of investment opportunities among firms. According to Proposition 4, the "skin in the game" is not an efficient tool to eliminate this asymmetry, while reputation-based implicit recourse has higher potential to overcome the related inefficiencies.

Even with an endogenous "skin in the game," the main qualitative result of the paper, which is the endogenous switching between the pooling and separating equilibrium, remains unchanged.[35]

5.2 "Skin in the game" as a policy parameter

The "skin in the game" can be considered as a potential policy parameter. For instance, Section 941 of the Dodd-Frank reform requires a minimum explicit risk retention of 5%.

[34] It is intuitive to assume that if a firm would divert funds, other firms will use at least the same punishment tools as for the case of implicit recourse default.

[35] For the proof see Appendix A.10. Also note that the assumption of the moral hazard problem is absolutely essential, since without it the solution would be first-best even under asymmetric information. Under first-best, securitization is not profitable; therefore, firms with access to low-quality investment do not have any incentives to mimic firms with high-quality investments. Therefore, neither reputation equilibria nor implicit recourse would take place.

Note that the "skin in the game" represents retention of a fraction of assets. Compared to a risk retention of the first-loss tranche, the "skin in the game," which would be equivalent to horizontal tranche retention, has lower signalling potential. The reason is the domination of a negative general-equilibrium effect discussed below. Such an effect would be limited if the risk retention requirement is satisfied by first-loss tranche retention without a substantial increase in potential capital requirements.

If, as in the extension of the model from section 5.1, the "skin in the game" is determined endogenously by a moral hazard problem, and securitization is the only means of financial intermediation, a policy that tries to increase the "skin in the game" beyond the endogenously determined value would not improve the efficiency of financial intermediation. The reasons are twofold.

First, a larger "skin in the game" increases the profits from securitization and lowers the aggregate quantity of investment (this follows from Propositions 1 and 2). Second, higher profits also make the issuance and sale of loans profitable, even for firms with lower-quality projects, which would otherwise be buyers of high-quality projects. This holds both in the symmetric information case from Proposition 2 and under asymmetric information, since the pooling equilibrium is more likely (see Proposition 4 and Proposition 8). Therefore, both the quantity and quality of investment are lower with a larger "skin in the game" than with the level of this constraint determined by the market.

In contrast to some other models of securitization, such as Gorton and Pennacchi (1995), my model does not feature continuous monitoring or effort. I have an option of fund diversion, which is observed only with a time lag. At a high level of abstraction, this can be understood as the analogy to costly monitoring in Gorton and Pennacchi (1995), where the level of monitoring would take only two values (no monitoring or full monitoring). This moral hazard problem indeed determines the optimum level of "skin in the game." Given that everyone is rational, not only is there no reason to increase the "skin in the game" above the level determined by the equilibrium, but increasing it would have negative effects on the economy as described above.

One could possibly introduce additional frictions, which would create benefits of the newly proposed regulation. However, those possible benefits can be outweighed by the adverse general-equilibrium effect, especially when the regulation is too excessive.

5.3 Adverse selection on resale markets

So far, we have considered the asymmetry of information between the originators of securitized assets and buyers of these assets. In this section, I extend the asymmetry of information to the resale market. In particular, I assume that the holder of the asset can learn the quality of the underlying asset, while the buyer cannot. This leads to a typical adverse selection on the resale market.

The new result in this paper comes from the interaction of the adverse selection on resale markets with the switching between pooling and separating equilibria.[36] The severity of the adverse selection on the secondary markets depends both on the difference in qualities and on the share of low-quality assets on the balance sheets. Therefore, intuitively the adverse selection is more important in a recession than in a boom. But, also, the longer the boom period is, the larger the share is of low-quality loans on the market, and the more acute the adverse selection issue becomes in the subsequent recession. If adverse selection is strong enough, securitized loans of high quality stop being traded on the resale markets altogether, which further deepens the recession.

The motivations for including this section are the problems witnessed on the securitization markets during the late-2000s financial crisis.

After outlining the main results of this extension, let me explain their derivation from the new assumption in more detail. The assumption of asymmetric information on resale markets has the following impact on the model behavior. First, when an asset is resold, there is a unique price that is independent of the quality of this asset q_t^s. If an asset is not resold, the owner who knows its quality will value the high-quality asset q_t^h and the low-quality asset q_t^l, but this is not the market price. Second, prices depend on the share of high-quality assets on the resale market.[37] In every period, firms find out the quality of assets on their balance sheets and sell all low-quality assets. Unlike original issuers in the period when the investment was made, they no longer have the technology to provide implicit recourse. High-quality assets are sold on the market only by firms with investment opportunities who need liquidity.

Therefore, the share of high-quality assets on the resale market is

$$f_t^h = \frac{\pi\mu\omega_t}{\pi\mu + (1-\pi\mu)(1-\omega_t)}$$

[36] Kuncl (2014) extends the issue of adverse selection on the resale markets further in a model with infinite-horizon implicit recourse and equilibrium defaults on such recourse.

[37] See Appendix A.11 for details.

in the case of a separating equilibrium and

$$f_t^h = \frac{\pi \omega_t}{\pi + (1-\pi)(1-\omega_t)}$$

in the case of a pooling equilibrium.

If, due to the adverse selection, the price of assets on the resale market drops low enough, even firms that sell assets for liquidity reasons will stop selling high-quality assets. The price is so low that the return from taking advantage of the investment opportunity would not compensate for the cost of selling a valuable asset at a low market price. In a steady state, this situation takes place if

$$R^h > q^s \frac{R^h - \theta R^G}{1 - \theta q^{IR}},$$

where $R^h = r^h + \lambda \pi \mu q^s + \lambda(1-\pi\mu) q^h$ and $R^G = r^G + \lambda \pi \mu q^s + \lambda(1-\pi\mu) q^h$. As shown in Appendix A.11, this condition implies that the share of high-quality assets traded on the resale market has to be low enough to satisfy:

$$f^h < 1 - \frac{q^h - 1}{(q^h - q^l)(1 - \theta B)}.$$

If this condition is satisfied, there will not be complete market shutdowns, since low-quality assets would still be sold at a fair price, but the volume of sales would greatly diminish by the absence of high-quality assets, and the level of overall investment in the economy would also be significantly lower.

6 Conclusion

In this paper, I show that, in general, reputation concerns allow originators of securitized products to signal the quality of securitized loans by providing implicit recourse and thus they limit the problem of private information typical for securitization. However, there are limits to the efficiency of these particular reputation-based tools, which become more pronounced in boom stages of the business cycles. The minimum level of implicit recourse that would prevent mimicking by firms with investment projects of lower quality exceeds the level that can be credibly promised. In the resulting pooling equilibrium, the information about the quality of loans is lost, and the investment allocation becomes more inefficient. Due to this mechanism, large inefficiencies in the allocation of capital can be accumulated during the boom period of the business cycle. The accumulated inefficiencies can then amplify a subsequent downturn of the economy. Additionally, the longer the duration of the boom stage of the business cycle the deeper will be the fall of output in a subsequent recession.

In an extension of the model, I introduce asymmetric information on the resale market for securitized loans. The model predicts adverse selection on resale markets, which lowers the market price and consequently depresses further the investment and output in the economy. This adverse selection is the most amplified in a recession, particularly in the case when the recession is preceded by a prolonged boom period. If the adverse selection is severe enough, high-quality securitized loans are no longer traded at all.

The results of this paper also have implications for macroprudential policy that requires higher explicit risk retention. If the risk retention limits excessively the ability of firms with investment opportunities to do financial intermediation, then such regulation may be counterproductive. In particular, when explicit risk retention has the form of a fraction of investment (horizontal tranche retention), such a requirement restricts the supply of loans and, through the general-equilibrium effect, makes securitization more profitable. As a result, this regulation may lower both the quantity and the quality (higher likelihood of pooling equilibria) of investment in the economy. This effect is less pronounced in the case when originators retain the first-loss tranche and higher risk retention does not reduce their investment substantially, e.g. through higher capital requirements regulation.

The mechanism presented in this paper can contribute to the understanding of the recent financial crisis, since it describes the experience of securitization markets prior to and during the recent financial crisis. In the period preceding the crisis, many inefficient investments of unknown quality were undertaken. While this was not problematic as long as the economy was performing well, the large amount of low-quality loans in the economy ultimately contributed to the depth of the financial crisis. The paper also points to some unexpected negative effects of the newly proposed regulation.

References

Acharya, V. V., Schnabl, P., Suarez, G., 2013. Securitization without risk transfer. Journal of Financial Economics 107 (3), 515–536.

Adrian, T., Ashcraft, A., 2012. Shadow banking regulation. Annual Review of Financial Economics 4 (1), 99–140.

Adrian, T., Shin, H., 2009. Money, liquidity, and monetary policy. American Economic Review: Papers and Proceedings 99 (2), 600–605.

Akerlof, G., 1970. The market for "lemons": Quality uncertainty and the market mechanism. The Quarterly Journal of Economics 84 (3), 488–500.

Bernanke, B., 2010. Economic challenges: Past, present and future. A speech at the Dallas Regional Chamber, Dallas, Texas, April 7, 2010.
URL `http://www.federalreserve.gov/newsevents/speech/20100407a.htm`

Bernanke, B., Gertler, M., 1989. Agency costs, net worth, and business fluctuations. American Economic Review 79 (1), 14–31.

Bernanke, B., Gertler, M., Gilchrist, S., 1999. The financial accelerator in a quantitative business cycle framework, 1st Edition. Vol. 1. In: Taylor, J. and Woodford, M. (Eds.), Handbook of Macroeconomics. Elsevier, Ch. 21, pp. 1341–1393.

Bigio, S., 2013. Endogenous liquidity and the business cycle. Mimeo. Columbia Business School.

Bloom, N., 2009. The impact of uncertainty shocks. Econometrica 77 (3), 623–685.

Bloom, N., Floetotto, M., Jaimovich, N., Sapora-Eksten, I., Terry, S., 2012. Really uncertain business cycles. NBER Working Papers (18245).

Boissay, F., Collard, F., Smets, F., 2013. Booms and systemic banking crises. Working Paper Series 1514, European Central Bank.

Brunnermeier, M., 2009. Deciphering the liquidity and credit crunch 2007-2008. Journal of Economic Perspectives 23 (1), 77–100.

Brunnermeier, M., Sannikov, Y., 2014. A macroeconomic model with a financial sector. American Economic Review (forthcoming).

Calomiris, C., Mason, J., 2004. Credit card securitization and regulatory arbitrage. Journal of Financial Services Research 26 (1), 5–27.

Centorelli, N., Peristiani, S., 2012. The role of banks in asset securitization. FRBNY Economic Policy Review 18 (2), 47–63.

Chari, V. V., Shourideh, A., Zetlin-Jones, A., 2014. Reputation and persistence of adverse selection in secondary loan markets. American Economic Review forthcoming.

Cho, I.-K., Kreps, D. M., 1987. Signaling games and stable equilibria. The Quarterly Journal of Economics 102 (2), 179–222.

Dell'Ariccia, G., Marquez, R., 2006. Lending booms and lending standards. The Journal of Finance 61 (5), 2511–2546.

Demyanyk, Y., Van Hemert, O., 2011. Understanding the subprime mortgage crisis. Review of Financial Studies 24 (6), 1848–1880.

Fender, I., Mitchell, J., 2009. Incentives and tranche retention in securitization: A screening model. BIS Working Paper 289, 1–41.

Gertler, M., Kiyotaki, N., 2010. Financial intermediation and credit policy in business cycle analysis, 1st Edition. Vol. 3. In: Friedman, B. and Woodford, M. (Eds.), Handbook of Monetary Economics. Elsevier, Ch. 11, pp. 547–599.

Gorton, G., Metrick, A., 2010. Regulating the shadow banking system. Brookings Papers on Economic Activity 41 (2), 261–312.

Gorton, G., Ordoñez, G., 2014. Collateral crises. American Economic Review 104 (2), 343–378.

Gorton, G., Pennacchi, G., 1995. Banks and loan sales: Marketing nonmarketable assets. Journal of Monetary Economics 35 (3), 389–411.

Gorton, G., Souleles, N., 2006. Special purpose vehicles and securitization. In: Carey, M. and Stulz, R. (Eds.), The risks of financial institutions. University of Chicago Press, Chicago, pp. 549–602.

Heider, F., Hoerova, M., Holthausen, C., 2009. Liquidity hoarding and interbank market spreads: The role of counterparty risk. Working Paper Series 1126, European Central Bank.

Higgins, E., Mason, J., 2004. What is the value of recourse to asset-backed securities? A clinical study of credit card banks. Journal of Banking and Finance 28 (4), 875–899.

Judd, K. L., 1998. Numerical methods in economics. MIT press.

Kiyotaki, N., Moore, J., 1997. Credit cycles. Journal of Political Economy 105 (2), 211–248.

Kiyotaki, N., Moore, J., 2012. Liquidity, business cycles and monetary policy. NBER Working Papers (17934).

Kuncl, M., 2013. Securitization under asymmetric information and macroprudential regulation. Mimeo. European Central Bank.

Kuncl, M., 2014. Adverse selection on re-sale markets for securitized assets. Mimeo. CERGE-EI. Dissertation chapter.

Kurlat, P., 2013. Lemons markets and the transmission of aggregate shocks. American Economic Review 103 (4), 1463–1489.

Lown, C., Morgan, D., 2006. The credit cycle and the business cycle: New findings using the loan officer opinion surveys. Journal of Money, Credit and Banking 38 (6), 1575–1597.

Martin, A., 2009. A model of collateral, investment and adverse selection. Journal of Economic Theory 144 (4), 1572–1588.

Mason, J., Rosner, J., 2007. Where did the risk go? How misapplied bond ratings cause mortgage backed securities and Collateralized Debt Obligation market disruptions. SSRN Working Paper series, 1–87.

Nikolov, K., 2012. A model of borrower reputation as intangible collateral. Working Paper Series 1490, European Central Bank.

Ordoñez, G., 2014. Confidence banking and strategic default. Mimeo. University of Pennsylvania and NBER.

Paligorova, T., 2009. Agency conflicts in the process of securitization. Bank of Canada Review (Autumn), 33–47.

Parlour, C., Plantin, G., 2008. Loan sales and relationship banking. The Journal of Finance 63 (3), 1291–1314.

Pozsar, Z., Adrian, T., Ashcraft, A., Boesky, H., 2012. Shadow banking. Federal Reserve Bank of New York Staff Reports (458).

Ruckes, M., 2004. Bank competition and credit standards. The Review of Financial Studies 17 (4), 1073–1102.

Shin, H., 2009. Securitization and financial stability. The Economic Journal 119 (536), 309–332.

Shleifer, A., Vishny, R., 2010. Unstable banking. Journal of Financial Economics 97 (3), 306–318.

Veldkamp, L., 2005. Slow boom, sudden crash. Journal of Economic Theory 124 (2), 230–257.

Appendices

A Proofs

A.1 First-best case

Due to logarithmic utility, firms always consume $1-\beta$ fraction of their wealth: $c = (1-\beta)\, h\left(r^h+\lambda\right)$. This policy function is linear, so it is trivial to aggregate it across the continuum of firms to obtain the equation describing the evolution of aggregate variables: $C = (1-\beta)\, H\left(r^h+\lambda\right)$.

From the market clearing condition, we know that $X = Y - C = Hr^h - C$. And from the law of motion for capital, we know that in the steady state $X = (1-\lambda)\, H$. Combining these two conditions, we obtain:

$$Hr^h - C = (1-\lambda)\, H.$$

Substituting for aggregate consumption we get:

$$\begin{aligned} Hr^h - (1-\beta)\, H\left(r^h+\lambda\right) &= (1-\lambda)\, H, \\ r^h+\lambda &= \frac{1}{\beta}. \end{aligned}$$

A.2 Proof of Proposition 1

In the first-best equilibrium, $q^h = 1$. Should the SGC be binding, then by Proposition 1, $q^h > 1$. Let us consider the least restricted case where still only firms with access to high-quality loans are originating new projects and securitize their cash flows, and the SGC is not sufficiently binding to allow a firm with access to low-quality investment opportunities to profitably issue loans $q^l < 1$.

Under the binding SGC, the aggregate investment into high-quality projects will be (obtained as an aggregation of eq. 3.4):

$$X_t^H = \pi\mu \frac{\beta\left(H_t\left(\left(A_t+\Delta^h\right)K_t^{\alpha-1}+\lambda q_t^h\right)+L_t\left(\left(A_t+\Delta^l\right)K_t^{\alpha-1}+\lambda q_t^l\right)\right)}{\left(1-\theta q_t^h\right)}. \tag{6.1}$$

Prices of particular assets are determined from the Euler equations of saving firms. In equilibrium, these firms are indifferent between investing in high- or low-quality projects:

$$E_t\left[\frac{\frac{r_{t+1}^h+\lambda q_{t+1}^h}{q_t^h}}{\left(\omega_{t+1}\frac{r_{t+1}^h+\lambda q_{t+1}^h}{q_t^h}+\left(1-\omega_{t+1}\right)\frac{r_{t+1}^l+\lambda q_{t+1}^l}{q_t^l}\right)}\right]=1 \tag{6.2}$$

$$E_t\left[\frac{\frac{r^l_{t+1}+\lambda q^l_{t+1}}{q^l_t}}{\left(\omega_{t+1}\frac{r^h_{t+1}+\lambda q^h_{t+1}}{q^h_t}+(1-\omega_{t+1})\,\frac{r^l_{t+1}+\lambda q^l_{t+1}}{q^l_t}\right)}\right]=1, \tag{6.3}$$

where ω_t is the share of high-quality projects in the overall assets in the economy: $\omega_t = {}^{H_t}/_{K_t}$. The derivation of these conditions can be found in Appendix B.

Finally, the goods market clearing condition has to hold, too:

$$Y_t = C_t + X_t. \tag{6.4}$$

Case 1: Only firms with access to high-quality projects originate new projects and securitize their cash flows:

Condition 6.1 and a combination of 6.2 and 6.3, 6.4 can be rewritten when in steady state:

$$\begin{aligned}
(1-\lambda)\left(1-\theta q^h\right) &= \pi\mu\beta\left(r^h+\lambda q^h\right),\\
\frac{A^h}{q^h} &= \frac{A^l}{q^l},\\
r^h &= (1-\lambda)+(1-\beta)\left(r^h+\lambda q^h\right).
\end{aligned}$$

Combining these equations, we can obtain

$$\begin{aligned}
q^h_H &= \frac{(1-\lambda)(1-\pi\mu)}{(1-\lambda)\,\theta+\pi\mu\lambda}, \qquad (6.5)\\
K_H &= \left[\frac{(1-\lambda)+\frac{(1-\beta)\lambda(1-\lambda)(1-\pi\mu)}{(1-\lambda)\theta+\pi\mu\lambda}}{\beta A^h}\right]^{\frac{1}{\alpha-1}}.
\end{aligned}$$

As long as $q^h = 1$, we find $K_H = \left[\frac{1}{A^h}\left(\frac{1}{\beta}-\lambda\right)\right]^{\frac{1}{\alpha-1}}$, which is the first-best level of capital. If $(1-\lambda)(1-\pi\mu) > (1-\lambda)\,\theta+\pi\mu\lambda$, then $q^h > 1$. The deterministic steady-state level of capital is then lower than in the first-best case:

$$K_H = \left[\frac{(1-\lambda)+(1-\beta)\,\lambda q^h_H}{\beta A^h}\right]^{\frac{1}{\alpha-1}} < \left[\frac{(1-\lambda)+(1-\beta)\,\lambda}{\beta A^h}\right]^{\frac{1}{\alpha-1}} = K_{FB}.$$

A.3 Proof of Proposition 2

Proposition 2 claims that the steady-state equilibrium can be of three types depending on the parameter values. In the proof of Proposition 1 above, I already described the least restricted case, where only a firm with access to high-quality projects will be issuing and securitizing loans. By continuing to tighten the SGC, we will increase the price of the low-quality asset to 1 ($q^l = 1$). At this point, the firms with access to low-quality loans will be indifferent between buying high-quality securitized assets or issuing and securitizing their own loans. Origination of more low-quality projects counterweights the effect of tightening the SGC, and therefore the prices stay at the same levels ($q^l = 1$, $q^h = A^h/A^l$). For an interval of θ, there will be a steady state in which firms with access to low-quality investment will play a mixed strategy when giving credit with probability ψ. As θ decreases ("skin in the game" rises), ψ increases all the way up to 1, where a third type of steady state takes place. In this case, firms with access to both high- and low-quality projects will all be issuing credit and always securitizing.

Case 2: Firms with access to low-quality projects originate new projects with probability ψ:

Steady-state conditions are the following:

$$(1-\lambda)\left(1-\theta q^h\right)\omega = \pi\mu\beta\left(\omega\left(r^h+\lambda q^h\right)+(1-\omega)\left(r^l+\lambda q^l\right)\right), \tag{6.6}$$

$$(1-\lambda)\left(1-\theta q^l\right)(1-\omega) = \pi(1-\mu)\psi\beta\left(\omega\left(r^h+\lambda q^h\right)+(1-\omega)\left(r^l+\lambda q^l\right)\right), \tag{6.7}$$

$$\frac{A^h}{q^h} = \frac{A^l}{q^l}, \tag{6.8}$$

$$q^l = 1, \tag{6.9}$$

$$\omega r^h + (1-\omega)\,r^l = (1-\lambda) + (1-\beta)\left(\omega\left(r^h+\lambda q^h\right)+(1-\omega)\left(r^l+\lambda q^l\right)\right). \tag{6.10}$$

Let us define

$$q \equiv \frac{q^h}{A^h} = \frac{q^l}{A^l}, \tag{6.11}$$

and

$$D \equiv \omega A^h + (1-\omega)\,A^l. \tag{6.12}$$

Using (6.11) and (6.12), and combining equations (6.6), (6.7), and (6.8), we obtain:

$$(1-\lambda)(1-\theta q D) = \pi\left(\mu+\varphi(1-\mu)\right)\beta D\left(K^{\alpha-1}+\lambda q\right),$$

$$(1-\lambda) - \pi\left(\mu+\psi(1-\mu)\right)\beta D K^{\alpha-1} = qD\left[(1-\lambda)\,\theta + \pi\left(\mu+\psi(1-\mu)\right)\beta\lambda\right]. \tag{6.13}$$

We can also rewrite (6.10):

$$\beta D K^{\alpha-1} = 1-\lambda+(1-\beta)\,D\lambda q. \tag{6.14}$$

Combining (6.13) and (6.14), we obtain:

$$q_M = \frac{(1-\lambda)(1-\pi(\mu+\psi(1-\mu)))}{(1-\lambda)\theta+\pi(\mu+\psi(1-\mu))\lambda}\frac{1}{D}. \tag{6.15}$$

Substituting (6.15) back into (6.14), we obtain:

$$K_M = \left[\frac{(1-\lambda)+\frac{(1-\beta)\lambda(1-\lambda)(1-\pi(\mu+\psi(1-\mu)))}{(1-\lambda)\theta+\pi(\mu+\psi(1-\mu))\lambda}}{\beta D}\right]^{\frac{1}{\alpha-1}}. \tag{6.16}$$

Case 3: Firms with access to both high- and low-quality projects are always originating new projects:

The deterministic steady state is defined by

$$(1-\lambda)\left(1-\theta q^h\right)\omega = \pi\mu\beta\left(\omega\left(r^h+\lambda q^h\right)+(1-\omega)\left(r^l+\lambda q^l\right)\right), \tag{6.17}$$

$$(1-\lambda)\left(1-\theta q^l\right)(1-\omega) = \pi(1-\mu)\beta\left(\omega\left(r^h+\lambda q^h\right)+(1-\omega)\left(r^l+\lambda q^l\right)\right), \tag{6.18}$$

$$\frac{A^h}{q^h} = \frac{A^l}{q^l}, \tag{6.19}$$

$$\omega r^h+(1-\omega)r^l = (1-\lambda)+(1-\beta)\left(\omega\left(r^h+\lambda q^h\right)+(1-\omega)\left(r^l+\lambda q^l\right)\right). \tag{6.20}$$

Using (6.11) and (6.12), and combining equations (6.17), (6.18), and (6.19), we obtain:

$$(1-\lambda)(1-\theta q D) = \pi\beta D\left(K^{\alpha-1}+\lambda q\right),$$

$$(1-\lambda)-\pi\beta D K^{\alpha-1} = qD\left[(1-\lambda)\theta+\pi\beta\lambda\right]. \tag{6.21}$$

We can also rewrite (6.20):

$$\beta D K^{\alpha-1} = 1-\lambda+(1-\beta)D\lambda q. \tag{6.22}$$

Combining (6.21) and (6.22), we get:

$$q_B = \frac{(1-\lambda)(1-\pi)}{(1-\lambda)\theta+\pi\lambda}\frac{1}{D}. \tag{6.23}$$

Substituting (6.23) back into (6.22), we get:

$$K_B = \left[\frac{(1-\lambda) + \frac{(1-\beta)\lambda(1-\lambda)(1-\pi)}{(1-\lambda)\theta+\pi\lambda}}{\beta D} \right]^{\frac{1}{\alpha-1}}. \tag{6.24}$$

The second part of the proposition claims that $K_H > K_M > K_B$. To show this, let us first focus on the part of the formula within brackets for capital: since in Case 1 $q_H^l < 1$, then $q_H^h < \frac{A^h}{A^l}$. And since $q_M^l = 1$, then $\frac{(1-\lambda)(1-\pi(\mu+\varphi(1-\mu)))}{(1-\lambda)\theta+\pi(\mu+\varphi(1-\mu))\lambda} = \frac{D_M}{A^l}$. The following inequality then holds:

$$\frac{(1-\lambda)+(1-\beta)\,\lambda q_H^h}{\beta A^h} < \frac{(1-\lambda)}{\beta A^h} + (1-\beta)\,\lambda \frac{1}{\beta A^l} < \frac{(1-\lambda)}{\beta D_M} + (1-\beta)\,\lambda \frac{1}{\beta A^l} = \frac{(1-\lambda) + \frac{(1-\beta)\lambda(1-\lambda)(1-\pi(\mu+\psi(1-\mu)))}{(1-\lambda)\theta+\pi(\mu+\psi(1-\mu))\lambda}}{\beta D_M}.$$

This implies that

$$K_H = \left[\frac{(1-\lambda)+(1-\beta)\,\lambda q_H^h}{\beta A^h} \right]^{\frac{1}{\alpha-1}} > \left[\frac{(1-\lambda) + \frac{(1-\beta)\lambda(1-\lambda)(1-\pi(\mu+\psi(1-\mu)))}{(1-\lambda)\theta+\pi(\mu+\psi(1-\mu))\lambda}}{\beta D_M} \right]^{\frac{1}{\alpha-1}} = K_M.$$

Similarly, we can show that $K_M > K_B$. Since $w_B < w_M$, then $D_B < D_M$. And since $q_B^l > 1$, then $\frac{(1-\lambda)(1-\pi)}{(1-\lambda)\theta+\pi\lambda} > \frac{D_B}{A^l}$. This implies that

$$\frac{(1-\lambda) + \frac{(1-\beta)\lambda(1-\lambda)(1-\pi(\mu+\psi(1-\mu)))}{(1-\lambda)\theta+\pi(\mu+\psi(1-\mu))\lambda}}{\beta D_M} = \frac{(1-\lambda)}{\beta D_M} + (1-\beta)\,\lambda \frac{1}{\beta A^l} < \frac{(1-\lambda)}{\beta D_B} + (1-\beta)\,\lambda \frac{1}{\beta A^l} < \frac{(1-\lambda) + \frac{(1-\beta)\lambda(1-\lambda)(1-\pi)}{(1-\lambda)\theta+\pi\lambda}}{\beta D_B},$$

$$K_M = \left[\frac{(1-\lambda) + \frac{(1-\beta)\lambda(1-\lambda)(1-\pi(\mu+\psi(1-\mu)))}{(1-\lambda)\theta+\pi(\mu+\psi(1-\mu))\lambda}}{\beta D_M} \right]^{\frac{1}{\alpha-1}} > \left[\frac{(1-\lambda) + \frac{(1-\beta)\lambda(1-\lambda)(1-\pi)}{(1-\lambda)\theta+\pi\lambda}}{\beta D_B} \right]^{\frac{1}{\alpha-1}} = K_B.$$

A.4 Proof of Proposition 3

When we introduce asymmetric information about the allocation of investment opportunities and condition 3.5 is satisfied, capital and output levels are lower than in the first-best case due to the inefficient allocation of capital. This is true even when the SGC is not binding enough to influence prices.

When the SGC is not binding, the average gross profit from one unit of invested capital in the economy equals

$$\bar{r} = \mu r^h + (1-\mu) r^l = \frac{1}{\beta} - \lambda.$$

The level of capital in this private information case $K_{private}$ is determined by

$$K_{private} = \left[\frac{1}{\mu A^h + (1-\mu)A^l}\left(\frac{1}{\beta} - \lambda\right)\right]^{\frac{1}{\alpha-1}} < \left[\frac{1}{A^h}\left(\frac{1}{\beta} - \lambda\right)\right]^{\frac{1}{\alpha-1}} = K_{FB}.$$

Suppose now that $(1-\pi)(1-\lambda) > \pi\lambda + (1-\lambda)\theta$, in which case the SGC starts to bind. The steady-state conditions then collapse into the two following equations in $(K_{private}, q)$:

$$(1-\lambda)(1-\theta q) = \pi\beta\left(\mu r^h + (1-\mu)r^l + \lambda q\right),$$

$$\mu r^h + (1-\mu)r^l = (1-\lambda) + (1-\beta)\left(\mu r^h + (1-\mu)r^l + \lambda q\right),$$

where $q = \mu q^h + (1-\mu)q^l$. From this we can easily derive:

$$\begin{aligned} q &= \frac{(1-\pi)(1-\lambda)}{\pi\lambda + (1-\lambda)\theta}, \\ K_{private} &= \left[\frac{(1-\lambda) + (1-\beta)\lambda q}{\beta\left(\mu A^h + (1-\mu)A^l\right)}\right]^{\frac{1}{\alpha-1}}. \end{aligned} \tag{6.25}$$

In the proof of Propositions 1 and 2, we already proved that $K_{FB} > K_H > K_M > K_B$. To prove Proposition 3, it suffices to prove that $K_B > K_{private}$, where $K_{private}$ is the level of capital under private information about the allocation of investment opportunities. To obtain $K_B > K_{private}$, we need:

$$K_B^{\alpha-1} < K_{private}^{\alpha-1},$$

$$\frac{(1-\lambda) + \frac{(1-\beta)\lambda(1-\lambda)(1-\pi)}{(1-\lambda)\theta+\pi\lambda}}{\beta\left(\omega A^h + (1-\omega)A^l\right)} < \frac{(1-\lambda) + \frac{(1-\beta)\lambda(1-\lambda)(1-\pi)}{(1-\lambda)\theta+\pi\lambda}}{\beta\left(\mu\Delta A^h + (1-\mu)A^l\right)},$$

$$\omega > \mu.$$

Writing equations (6.17) and (6.18) into a ratio, we obtain:

$$\frac{(1-\lambda)\left(1-\theta q^h\right)\omega}{(1-\lambda)\left(1-\theta q^l\right)(1-\omega)} = \frac{\pi\mu\beta\left(\omega\left(r^h + \lambda q^h\right) + (1-\omega)\left(r^l + \lambda q^l\right)\right)}{\pi(1-\mu)\beta\left(\omega\left(r^h + \lambda q^h\right) + (1-\omega)\left(r^l + \lambda q^l\right)\right)}.$$

Since $q^h > q^l$, we obtain:

$$\frac{\omega}{(1-\omega)} = \frac{\left(1-\theta q^l\right)}{\left(1-\theta q^h\right)}\frac{\mu}{(1-\mu)} > \frac{\mu}{(1-\mu)},$$

and this implies that $\omega > \mu$.

A.5 Proof of Proposition 4

Under the private information case, firms with low-quality investment opportunities prefer to buy high-quality loans rather than to mimic firms with high-quality investment opportunities if

$$\begin{aligned}
R \mid mimicking &< R \mid buying\, high\, loans, \\
\frac{r^l + \lambda q^l}{\frac{1-\theta q^h}{1-\theta}} &< \frac{r^h + \lambda q^h}{q^h}, \\
\frac{(1-\theta)\, q^h}{1-\theta q^h} &< \frac{r^h + \lambda q^h}{r^l + \lambda q^l} = \frac{q^h}{q^l}, \\
q^l &< \frac{1-\theta q^h}{1-\theta}.
\end{aligned}$$

Substituting for q from (6.5) and using $\frac{A^h}{q^h} = \frac{A^l}{q^l}$, we get:

$$\frac{A^h}{A^l} > \frac{(1-\pi\mu)(1-\lambda)(1-\theta)}{\pi\mu\lambda + (1-\lambda)\,\theta\pi\mu}.$$

A.6 Credibility of the trigger punishment strategy

A necessary condition for the existence of the reputation equilibrium in which implicit recourse is being provided is the credibility of the punishment rule. The saving firm, which observes default on the implicit recourse, has to prefer punishing the defaulting firm rather than non-punishing it, even ex post. This is expressed in condition (3.12). Here, I derive analytically both elements of that inequality in the case of the pooling steady state, where the level of aggregate TFP is constant. In the fully stochastic version, this can be solved numerically. Following the same steps as in Appendix A.9, we can find that the value function of the firm that always punished, and therefore has a reputation of being a "tough investor," is

$$V^P(w) = \frac{\log\left[(1-\beta)\, w\right]}{1-\beta} + \frac{\beta \log(\beta)}{(1-\beta)^2} + \frac{\beta}{(1-\beta)^2}\left(\pi\mu \log\left(R^{h,IR}\right) + \pi(1-\mu)\log\left(R^{l,IR}\right) + (1-\pi)\log\left(R^s\right)\right),$$

and the value function of the firm that failed to punish and therefore lost its reputation of being a "tough investor" is

$$V^{NP}(w) = \frac{\log\left[(1-\beta)\, w\right]}{1-\beta} + \frac{\beta \log(\beta)}{(1-\beta)^2} + \frac{\beta}{(1-\beta)^2}\left(\pi \log\left(R^{h,IR}\right) + \pi(1-\mu)\log\left(R^{l,IR}\right) + (1-\pi)\log\left(R^{s,NP}\right)\right).$$

If a firm loses its reputation of being a "tough investor," other firms will expect that this firm will never punish in the future, and, as a consequence, they will never again provide implicit support to this firm. So when a firm without the reputation of being a "tough investor" buys assets with implicit support issued in the primary market, its return is $R^{s,NP} = \frac{\mu\left(r^h+\lambda q^h\right)+(1-\mu)\left(r^l+\lambda q^l\right)}{q^G}$. Firms with a "tough investors" reputation have a return of $R^s = \frac{\hat{r^G}+\mu\lambda q^h+(1-\mu)\lambda q^l}{q^G}$. Note that firms without a reputation cannot obtain a lower price from originators of securitized assets without implicit recourse, since granting a lower and fair price would reveal the identity of low-quality assets of the issuing firm. Similarly, if firms without a "tough investor" reputation buy assets without implicit recourse on the secondary (resale) markets, they are also in a disadvantageous position. When firms with a "tough investor" reputation sell high-quality assets to firms with a reputation, they charge a market price q^h. However, if firms without a "tough investor" reputation have the outside option of only buying on the primary market, they will be willing to buy a high-quality asset even for the price q^G. The price for which a high-quality asset is sold on the secondary market to the firms without a reputation is somewhere in the interval $q^{h,NP} \in \left(q^h, q^G\right)$, depending on the bargaining power of sellers and buyers. Unless all bargaining power is on the side of firms without reputation, then $q^{h,NP} > q^h$. This implies that $R^{s,NP} < R^s$, and therefore saving firms are better off punishing, and inequality (3.12) would be satisfied.

It is well known that trigger strategies are often not renegotiation-proof. While in this paper I do not address this problem in detail and rule out renegotiation by assumption, it can be shown that for a large set of parameter space and relative bargaining power of different agents in the economy, renegotiation is not optimal. Therefore, a trigger strategy will be robust even in the case when renegotiation is allowed.

Suppose one firm decides to default on the implicit support (which is the case when the ICC for non-defaulting, eq. 3.11, does not hold). Other firms decide whether to punish this firm and face lower returns in the future $R^{s,NP}$, as shown above, or whether not to punish and negotiate for better terms with the defaulted firms, i.e., buy the assets from them for a lower price $q^{h,RN} < q^h$, giving it a return $R^{s,RN} > R^s$. However, those benefits from renegotiation are limited by the fact that the defaulted firm would be selling the assets only with probability $\pi\mu$, and the quantity of assets the firm can sell is limited and proportional to its equity. Even if the quantity of the assets sold by the defaulted firm is large enough, renegotiation would not be optimal as long as

$$R^s > \pi\mu R^{s,RN} + (1-\pi\mu)\, R^{s,NP}.$$

This depends on prices $q^h, q^{h,NP}, q^{h,RN}$, which themselves depend upon the relative bargaining power of different agents in the economy.

A.7 Proof of Proposition 5

I claimed that if the implicit recourse were to be credible, the optimal level of promise would mean $q^j = 1$, and therefore zero profit for securitizing firms. The relevant FOC can be transformed in the following way.

Let us consider FOC for firms with high-quality investment opportunities. I guess and verify that the remaining firms would not invest at all:

$$\begin{aligned}
\frac{\partial V^{ND}}{\partial r^{G}}=\frac{\partial V^{ND'}}{\partial\left(w'-cir'\right)}\frac{\partial\left(w'-cir'\right)}{\partial r^{G}} &= 0,\\
\frac{\partial V^{ND'}}{\partial\left(w'-cir'\right)}\frac{\partial}{\partial r^{G}}\frac{\left(1-\theta\right)\beta w\left(r^{j'}+\lambda q^{j}\right)-\theta\beta w\left(r^{G}-r^{j}\right)}{1-\theta q^{G}} &= 0,\\
\frac{\partial V^{ND'}}{\partial\left(w'-cir'\right)}\frac{\partial}{\partial r^{G}}\frac{\beta w\left(r^{j'}+\lambda q^{j}-\theta\left(r^{G'}+\lambda q^{j}\right)\right)}{1-\theta q^{G}} &= 0.
\end{aligned}$$

After substituting in this case with constant aggregate productivity $q^{G,j}=\frac{r^{G'}+\lambda q^{j}}{r^{j'}+\lambda q^{j}}q^{j}$, this condition implies that

$$\frac{\partial V^{ND'}}{\partial\left(w'-cir'\right)}\frac{\partial}{\partial r^{G}}\frac{\beta w\left(r^{j'}+\lambda q^{j}\right)\left(1-\theta\frac{q^{G,j}}{q^{j}}\right)}{1-\theta q^{G,j}}=0,$$

and since $\frac{\partial V^{ND'}}{\partial(w'-cir')}>0$, and as long as higher r^{G} credibly brings value to buyers $\frac{\partial q^{G,j}}{\partial r^{G}}>0$, the above condition simplifies to

$$\frac{\partial}{\partial q^{G,j}}\frac{\left(1-\theta\frac{q^{G,j}}{q^{j}}\right)}{1-\theta q^{G,j}}=\frac{\theta\left(q^{j}-1\right)}{q^{j}\left(1-\theta q^{G,j}\right)^{2}}=0.$$

This implies $q^{j}=1$ conditional that the level of r^{G} is credible, i.e. is not defaulted upon at least in some states of the world.

Note that for when the level of r^{G} satisfies this condition, the return from investing and securitizing is equal to the return from investing but not securitizing, i.e., securitization does not increase the return:

$$\begin{aligned}
R\mid investing\,\&\,securitizing &= R\mid investing\\
\frac{\left(r^{j}+\lambda q^{j}-\theta\left(r^{G}+\lambda q^{j}\right)\right)}{1-\theta\frac{r^{G}+\lambda q^{j}}{r^{j}+\lambda q^{j}}q^{j}} &= \frac{r^{j}+\lambda q^{j}}{1}.
\end{aligned}$$

When you substitute in the above condition $q^{j}=1$, the condition is exactly satisfied for all parameter values.

A.8 Proof of Proposition 8

To complete the proof of Proposition 8 sketched in the main text, I first need to derive from (3.13) the (3.14) and show that the RHS of equation (3.14) is independent of the level of aggregate productivity A. This means that variables B and q^h should be independent of the level of aggregate productivity A.

Under separation, steady-state conditions are as follows:

$$\begin{aligned}
(1-\lambda)\left(1-\theta q^G\right) &= \pi\mu\beta\left(r^h+\lambda q^h\right), && (6.26)\\
r^h &= (1-\lambda)+(1-\beta)\left(r^h+\lambda q^h\right), && (6.27)\\
\frac{r^G+\lambda q^h}{q^G} &= \frac{\left(A+\triangle^h\right)K^{\alpha-1}+\lambda q^h}{q^h}, && (6.28)\\
V^{ND}\left(w'-cir'\right) &= V^D\left(w'\right). && (6.29)
\end{aligned}$$

Using the following property given by the logarithmic utility function:

$$\begin{aligned}
V(w) &= \log\left((1-\beta)w\right)+\beta\log\left((1-\beta)\beta Rw\right)+\beta^2\log\left((1-\beta)\beta^2R^2w\right)+\beta^3\log\left((1-\beta)\beta^3R^3w\right)\ldots\\
&= \frac{1}{1-\beta}\log(w)+\log\left((1-\beta)\right)+\beta\log\left((1-\beta)\beta R\right)+\beta^2\log\left((1-\beta)\beta^2R^2\right)+\beta^3\log\left((1-\beta)\beta^3R^3\right)\ldots\\
&= \frac{1}{1-\beta}\log(w)+V(1),
\end{aligned}$$

we can transform the no-default condition expressed in (6.29) in the following way:

$$\begin{aligned}
V^D\left(w'\right) &= V^D\left(w\beta\frac{(1-\theta)\left(r^h+\lambda q^h\right)}{\left(1-\theta q^G\right)}\right)=V^D(w)+\frac{1}{1-\beta}\log\left(\beta\frac{(1-\theta)\left(r^h+\lambda q^h\right)}{\left(1-\theta q^G\right)}\right)\\
V^{ND}\left(w'-cir'\right) &= V^{ND}\left(w\beta\frac{(1-\theta)\left(r^h+\lambda q^h-\frac{\theta}{1-\theta}\left(r^G-r^h\right)\right)}{\left(1-\theta q^G\right)}\right)\\
&= V^{ND}(w)+\frac{1}{1-\beta}\log\left(\beta\frac{(1-\theta)\left(r^h+\lambda q^h-\frac{\theta}{1-\theta}\left(r^G-r^h\right)\right)}{\left(1-\theta q^G\right)}\right).
\end{aligned}$$

For simplicity, let us express the value functions separately from individual wealth in the following way, which is easy to do given the log utility: $V(w)=V(1)+\frac{1}{1-\beta}\log(w)$. We can also find solutions for value functions with wealth normalized to unity, which we can denote simply as $V=V(1)$:

$$\begin{aligned}
V^{ND} &= \log(1-\beta)+\beta\left(\pi\mu V^{ND}\left(\beta R^{h,IR}\right)+\pi(1-\mu)V^{ND}\left(\beta R^l\right)+(1-\pi)V^{ND}\left(\beta R^z\right)\right)\\
&= \log(1-\beta)+\beta\left(\frac{\pi\mu\log\left(\beta R^{h,IR}\right)}{1-\beta}+\pi(1-\mu)\frac{\log\left(\beta R^l\right)}{1-\beta}+(1-\pi)\frac{\log\left(\beta R^z\right)}{1-\beta}+V^{ND}\right)\\
&= \frac{\log(1-\beta)}{1-\beta}+\frac{\beta\log(\beta)}{(1-\beta)^2}+\frac{\beta}{(1-\beta)^2}\left(\pi\mu\log\left(R^{h,IR}\right)+\pi(1-\mu)\log\left(R^l\right)+(1-\pi)\log\left(R^z\right)\right).
\end{aligned}$$

$$
\begin{aligned}
V^D &= \log(1-\beta)+\beta\left(\pi\mu V^D\left(\beta R^{h,D}\right)+\pi(1-\mu)V^D\left(\beta R^l\right)+(1-\pi)V^D\left(\beta R^z\right)\right)\\
&= \log(1-\beta)+\beta\left(\frac{\pi\mu\log\left(\beta R^{h,D}\right)}{1-\beta}+\pi(1-\mu)\frac{\log\left(\beta R^l\right)}{1-\beta}+(1-\pi)\frac{\log\left(\beta R^z\right)}{1-\beta}+V^D\right)\\
&= \frac{\log(1-\beta)}{1-\beta}+\frac{\beta\log(\beta)}{(1-\beta)^2}+\frac{\beta}{(1-\beta)^2}\left(\pi\mu\log\left(R^{h,D}\right)+\pi(1-\mu)\log\left(R^l\right)+(1-\pi)\log\left(R^z\right)\right).
\end{aligned}
$$

Substituting the above derived conditions into the no-default condition (6.29) and canceling the terms equal for both value functions, we obtain:

$$
\log\left(\beta(1-\theta)\left(r^h+\lambda q^h-\frac{\theta}{1-\theta}\left(r^G-r^h\right)\right)\right)+\frac{\beta\pi\mu}{1-\beta}\log\left(R^{h,IR}\right)=\log\left(\beta(1-\theta)\left(r^h+\lambda q^h\right)\right)+\frac{\beta\pi\mu}{1-\beta}\log\left(R^{h,D}\right),
$$

where the LHS shows the utility from consumption when wealth is reduced by repayment of implicit recourse and from the future discounted benefit of having a good reputation. The RHS, then, shows higher immediate utility from savings on implicit recourse, but the future utility is lower, since the firm can no longer issue and sell new loans. This equation can further be simplified using (6.28) and substituting for the returns:

$$
\begin{aligned}
\log\left(\frac{r^h+\lambda q^h-\theta\left(r^G+\lambda q^h\right)}{(1-\theta)\left(r^h+\lambda q^h\right)}\right) &= -\frac{\beta\pi\mu}{1-\beta}\log\left(\frac{R^{h,IR}}{R^{h,D}}\right)\\
&= -\frac{\beta\pi\mu}{1-\beta}\log\left(\frac{(1-\theta)\left(r^h+\lambda q^h-\frac{\theta}{1-\theta}\left(r^G-r^h\right)\right)}{\left(1-\theta q^G\right)}\frac{1}{\left(r^h+\lambda q^h\right)}\right)\\
&= -\frac{\beta\pi\mu}{1-\beta}\log\left(\frac{r^h+\lambda q^h-\theta\left(r^G+\lambda q^h\right)}{r^h+\lambda q^h-\theta q^h\left(r^G+\lambda q^h\right)}\right).
\end{aligned}
$$

Now let us denote the price premium for the equilibrium implicit guarantee $B\equiv\frac{q^G}{q^h}=\frac{r^G+\lambda q^h}{r^h+\lambda q^h}$. Then we can express the above equation as follows:

$$
\log\left(\frac{1-\theta B}{1-\theta}\right) = \frac{\beta\pi\mu}{1-\beta}\log\left(\frac{1-\theta Bq^h}{1-\theta B}\right), \tag{6.30}
$$

which is an equation in two unknown endogenous variables $\left(B,q^h\right)$ depending on time preference parameters β and parameters defining the strength of the financing frictions (π,μ,θ).

We can express a second steady-state condition in two endogenous variables $\left(B,q^h\right)$ combining two remaining conditions for the steady state (6.26, 6.27):

$$
(1-\lambda)\left(1-\theta Bq^h\right)=\pi\mu\left(1-\lambda+\lambda q^h\right). \tag{6.31}
$$

Combining the two equations (6.30, 6.31), we can obtain the solution to both the price of the high-quality asset q^h and the price premium for the equilibrium implicit guarantee B. Crucially, the solution does not depend on the level of aggregate productivity A, which is one step we needed to show to complete the proof of Proposition 8.

The second step is to derive (3.14) from (3.13). Note that in the separating equilibrium, selected by the intuitive criterion, mimicking firms with access to low-quality projects would find it optimal to default on implicit recourse, since in a separating equilibrium, $r^{G*} > r^{G}_{l,cred,s}$.

As with condition 6.29, we can transform the following condition for separation (3.13):

$$\begin{aligned} V^{l}\,(mimicking\,\&\,default) &< V^{l}\,(buying\;high\;loans) \\ \log\left(\frac{\beta\,(1-\theta)\left(r^{l}+\lambda q^{l}\right)}{(1-\theta q^{G})}\right)+\frac{\beta\pi\mu}{1-\beta}\log\left(R^{h,D}\right) &< \log\left(\beta\frac{\left(r^{h}+\lambda q^{h}\right)}{q^{h}}\right)+\beta\pi\mu\log\left(R^{h,IR}\right) \\ -\frac{\beta\pi\mu}{1-\beta}\log\left(\frac{R^{h,IR}}{R^{h,D}}\right) &< \log\left(\frac{(1-\theta q^{G})}{(r^{l}+\lambda q^{l})\,(1-\theta)}\frac{\left(r^{h}+\lambda q^{h}\right)}{q^{h}}\right) \\ \frac{\beta\pi\mu}{1-\beta}\log\left(\frac{1-\theta B q^{h}}{1-\theta B}\right) &< \log\left(\frac{\left(1-\theta B q^{h}\right)}{(1-\theta)}\frac{1}{q^{l}}\right). \end{aligned}$$

Using (6.30) and the preceding transformations, we can replace the LHS to get:

$$\begin{aligned} \log\left(\frac{1-\theta B}{1-\theta}\right) &< \log\left(\frac{\left(1-\theta B q^{h}\right)}{(1-\theta)}\frac{1}{q^{l}}\right) \\ q^{l} &< \frac{1-\theta B q^{h}}{1-\theta B}. \end{aligned} \tag{6.32}$$

If we divide (6.32) by q^h and substitute the ratio of prices by the steady-state asset market clearing condition $^{A^h}/_{q^h} = {}^{A^l}/_{q^l}$, then we obtain:

$$\frac{A^{h}}{A^{l}} > \frac{(1-\theta B)\,q^{h}}{1-\theta B q^{h}}.$$

Proposition 8 (iii) also claims that the inequality in (3.5) is less likely to be satisfied than in (3.14). To prove that, let us first rewrite the denominator of (3.5) using (6.5), which says:

$$\left(1-\theta q^{h}\right)(1-\lambda) = \pi\mu\left(1-\lambda+\lambda q^{h}\right),$$

to obtain:

$$\frac{A^{h}}{A^{l}} > \frac{(1-\theta)\,(1-\lambda)}{\pi\mu\left(\frac{1-\lambda}{q^{h}}+\lambda\right)}.$$

Similarly, let us rewrite the denominator of (3.14) using (6.31) to obtain:

$$\frac{A^h}{A^l} > \frac{(1-\theta B)(1-\lambda)}{\pi\mu\left(\frac{1-\lambda}{q^h}+\lambda\right)}.$$

We can show that

$$\frac{1-\lambda}{\pi\mu} = \frac{(1-\theta)(1-\lambda)}{\pi\mu\left(\frac{1-\lambda}{q^h}+\lambda\right)} \mid no\, implicit\, recourse > \frac{(1-\theta B)(1-\lambda)}{\pi\mu\left(\frac{1-\lambda}{q^h}+\lambda\right)} \mid implicit\, recourse,$$

because the price premium for implicit recourse B is, by definition, higher than one, and $q^h \mid no\, implicit\, recourse > q^h \mid implicit\, recourse$. The latter comes directly from comparing (6.5) and (6.31), which when combined give:

$$\frac{1-\lambda+\lambda q^h}{1-\theta q^h} \mid no\, implicit\, recourse = \frac{1-\lambda+\lambda q^h}{1-\theta B q^h} \mid implicit\, recourse.$$

Further, this can be satisfied only if $q^h \mid no\, implicit\, recourse > q^h \mid implicit\, recourse.$

A.9 Other derivations from section 3.3

Conditions for the minimum level of implicit recourse needed for separation G_{minsep}:

At G_{minsep}, firms with low-quality investments are indifferent between mimicking and separating:

$$\begin{aligned}
V^l \mid mimicking\,\&\, default &= V^l \mid buying\ high\ loans \\
\log\left(\frac{\beta(1-\theta)\left(r^l+\lambda q^l\right)}{(1-\theta q^G)}\right) + \beta\pi\mu\log\left(R^{h,D}\right) &= \log\left(\beta\frac{\left(r^h+\lambda q^h\right)}{q^h}\right) + \beta\pi\mu\log\left(R^{h,IR}\right) \\
-\beta\pi\mu\log\left(\frac{1-\theta B_{min}}{1-\theta}\right) &= \log\left(\frac{\left(1-\theta B_{min} q^h\right)}{(1-\theta)} q^l\right).
\end{aligned} \tag{6.33}$$

Combining (6.33) with the following equilibrium investment condition

$$(1-\lambda)\left(1-\theta B_{min} q^h\right) = \pi\mu\left(1-\lambda+\lambda q^h\right), \tag{6.34}$$

where $B_{min} \equiv \frac{q^G}{q^h} = \frac{(A+G_{minsep})K^{\alpha-1}+\lambda q^h}{r^h+\lambda q^h}$, gives $\left\{G_{minsep}, q^h, B_{min}\right\}$.

Conditions for a unique pooling equilibrium:

A necessary condition for firms to have incentives to increase G above $G^l_{cred,p}$ is that it must be considered as profitable to, at least, individually deviate above $G^l_{cred,p}$. The following condition should, therefore, be satisfied:

$$\frac{\partial V^{ND}}{\partial r^G} = \frac{\partial V^{ND}}{\partial R^{h,IR}} \frac{\partial R^{h,IR}}{\partial r^G} > 0.$$

Since $\frac{\partial V^{ND}}{\partial R^{h,IR}} > 0$, this becomes:

$$\frac{\partial R^{h,IR}}{\partial r^G} = \frac{\partial}{\partial r^G} \frac{\left(\left(r^h - \frac{\theta}{1-\theta}\left(r^G - r^h\right)\right) + \lambda q^h\right)(1-\theta)}{1 - \theta \frac{\left(\mu r^G + (1-\mu) r^l\right) + \lambda\left(\mu q^h + (1-\mu) q^l\right)}{r^h + \lambda q^h} q^h} > 0.$$

In taking the derivative, we obtain:

$$\begin{aligned}
-\theta K^{\alpha-1} \left(1 - \theta \frac{\left(\mu r^G + (1-\mu) r^l\right) + \lambda\left(\mu q^h + (1-\mu) q^l\right)}{r^h + \lambda q^h} q^h\right) & \\
+ \frac{\theta \mu q^h K^{\alpha-1}}{r^h + \lambda q^h} \left(r^h - \frac{\theta}{1-\theta}\left(r^G - r^h\right) + \lambda q^h\right)(1-\theta) & > 0,
\end{aligned}$$

$$\begin{aligned}
\left(r^h - \frac{\theta}{1-\theta}\left(r^G - r^h\right) + \lambda q^h\right)(1-\theta)\mu q^h &> r^h + \lambda q^h - \theta\left(\mu r^G + (1-\mu) r^l\right) \\
&\quad + \lambda\left(\mu q^h + (1-\mu) q^l\right) q^h \\
\left(\mu q^h - 1\right)\left(r^h + \lambda q^h\right) &> \theta q^h (\mu - 1)\left(r^l + \lambda q^l\right). \qquad (6.35)
\end{aligned}$$

As long as $\left(\mu q^h - 1\right) > 0$, the condition (6.35) always holds, since $\mu < 1$. When $\left(\mu q^h - 1\right) < 0$, then we get:

$$\left(r^h + \lambda q^h\right) < \theta \frac{q^h (1-\mu)}{(1 - \mu q^h)} \left(r^l + \lambda q^l\right),$$

which is not satisfied if

$$\frac{A^h}{A^l} > \theta \frac{q^h (1-\mu)}{(1 - \mu q^h)},$$

or when rewritten:

$$\mu < \frac{1 - \theta q^l}{q^h - \theta q^l}.$$

This implies that the share of high-quality assets has to be low enough, or, in a pooling equilibrium, the relative difference in TFP has to be large enough.

A.10 Endogenizing the "skin in the game"

If we endogenize the SGC with the moral hazard problem described in section 5, we obtain the incentive-compatible constraint (5.1). In this section, I would like to show briefly that the main results concerning the provision of implicit recourse and the endogenous switching between the pooling equilibrium and the separating equilibrium hold.

First, we have to check whether firms have the incentive to provide implicit support. The check is equivalent to the proof of Proposition 5, as discussed in section A.7, which boils down to show that

$$\frac{\partial}{\partial q^{G,j}} \frac{\left(1-\theta \frac{q^{G,j}}{q^j}\right)}{1-\theta q^{G,j}} = \frac{\left(q^j-1\right)}{q^j\left(1-\theta q^{G,j}\right)^2} \frac{\partial \theta q^{G,j}}{\partial q^{G,j}} \geq 0.$$

Since $\frac{\partial \theta q^{G,j}}{\partial q^{G,j}} = \frac{\partial}{\partial q^{G,j}} \frac{q^{G,j}}{q^{G,j}+1} = \frac{1}{\left(q^{G,j}+1\right)^2} > 0$, the above condition corresponds again to $q^j \geq 1$. This means that, in equilibrium, implicit recourse will be provided.

Given (5.1), the separating equilibrium in the deterministic steady state is defined by

$$\begin{aligned}
(1-\lambda)\left(1-\theta B q^h\right) &= \pi\mu\left(1-\lambda+\lambda q^h\right) \\
\log\left(\frac{1-\theta B}{1-\theta}\right) &= \frac{\beta\pi\mu}{1-\beta}\log\left(\frac{1-\theta q^h B}{1-\theta B}\right) \\
\theta &= \frac{1}{Bq^h+1}.
\end{aligned}$$

Which simplifies into two equations, which are independent on the level of TFP A:

$$\begin{aligned}
(1-\lambda)\left(\frac{1}{Bq^h+1}\right) &= \pi\mu\left(1-\lambda+\lambda q^h\right) \\
\log\left(\frac{B\left(q^h-1\right)+1}{Bq^h}\right) &= \frac{\beta\pi\mu}{1-\beta}\log\left(\frac{1}{B\left(q^h-1\right)+1}\right).
\end{aligned}$$

The conditions for the existence of a separating equilibrium (3.14) become:

$$\frac{A^h}{A^l} > q^h\left(B\left(q^h-1\right)+1\right).$$

A.11 Adverse selection on resale markets

We derive the pricing conditions from the FOC of saving firms. In the case of a separating equilibrium, they are the following. The value of a high-quality asset q_t^h reflects the expected gross profit next period and the value of the asset next period, which is q_{t+1}^h if the firm has no investment opportunities and keeps the asset on the balance sheet, or q^s_{t+1} if the firm has an investment opportunity and sells the asset:

$$E_t\left[\frac{1}{\Xi_{t+1}}\frac{r^h_{t+1}+\lambda\pi\mu q^s_{t+1}+\lambda\left(1-\pi\mu\right)q^h_{t+1}}{q^h_t}\right]=1.$$

The value of the low-quality asset reflects the expected next-period gross profits and the expected next-period resale price, since low assets are always sold on the resale market:

$$E_t\left[\frac{1}{\Xi_{t+1}}\frac{r^l_{t+1}+\lambda q^s_{t+1}}{q^l_t}\right]=1.$$

The price of the newly issued asset with implicit support in a separating equilibrium and the price of an asset sold on the resale market satisfy the following:

$$E_t\left[\frac{1}{\Xi_{t+1}}\frac{r^G_{t+1}+\lambda f^h_t\left(\pi\mu q^s_{t+1}+\lambda\left(1-\pi\mu\right)q^h_{t+1}\right)}{q^G_t}\right]=1,$$

$$E_t\left[\frac{1}{\Xi_{t+1}}\frac{f^h_t r^h_{t+1}+\left(1-f^h_t\right)r^l_{t+1}+\lambda f^h_t\left(\pi\mu q^s_{t+1}+\lambda\left(1-\pi\mu\right)q^h_{t+1}\right)+\lambda\left(1-f^h_t\right)q^s_{t+1}}{q^s_t}\right]=1,$$

where

$$\begin{aligned}\Xi_{t+1} \quad = \quad & X_t\frac{r^G_{t+1}+\lambda q^s_{t+1}}{q^G_t}+\lambda K_t[(\pi\mu+(1-\pi\mu)(1-\omega_t))\,\frac{f^h_t r^h_{t+1}+\left(1-f^h_t\right)r^l_{t+1}+\lambda q^s_{t+1}}{q^s_t}\\ & +(1-\pi\mu)\,\omega_t\frac{r^h_{t+1}+\lambda\pi\mu q^s_{t+1}+\lambda\left(1-\pi\mu\right)q^h_{t+1}}{q^h_t}].\end{aligned}$$

Also note that $q^s_t=f^h_t q^h_t+\left(1-f^h_t\right)q^l_t$.

Conditions for no trade of high-quality assets: For investing firms preferring to keep their high-quality loans rather than selling them and investing such obtained liquidity, the following condition has to be satisfied in the deterministic steady state:

$$R^h>q^s\frac{R^h-\theta R^G}{1-\theta q^G},$$

where $R^h=r^h_{t+1}+\lambda\pi\mu q^s_{t+1}+\lambda\left(1-\pi\mu\right)q^h_{t+1}$, and $R^h=r^h_{t+1}+\lambda\pi\mu q^s_{t+1}+\lambda\left(1-\pi\mu\right)q^h_{t+1}$.

This can be transformed as follows:

$$\begin{aligned}R^h-\theta q^h R^G &> q^s R^h-\theta q^s R^G\\ R^h\left(1-q^s\right) &> \theta R^G\left(q^h-q^s\right).\end{aligned}$$

Substituting $q^s = f^h q^h + \left(1 - f^h\right) q^l$, and $B = {}^{R^G}/_{R^h}$, we get:

$$\begin{aligned}
1 - f^h q^h - \left(1 - f^h\right) q^l &> \theta B \left(1 - f^h\right)\left(q^h - q^l\right) \\
\frac{1 - f^h q^h}{1 - f^h} &> \theta B q^h + \left(1 - \theta B\right) q^l \\
f^h \left(q^l - q^h\right)\left(1 - \theta B\right) &> \theta B q^h - 1 + \left(1 - \theta B\right) q^l \\
f^h &< 1 - \frac{q^h - 1}{\left(q^h - q^l\right)\left(1 - \theta B\right)}.
\end{aligned}$$

B Derivation of firms' policy functions

In this section, I derive the policy functions of firms in the most general case. It is convenient to rewrite the firm's problem characterized in section 3.1.3 in a recursive formulation:

$$\begin{aligned}
V^{ND}\left(\bar{s}, w - cir; \bar{S}\right) &= \pi\left(\mu V^{ND,h}\left(\bar{s}, w - cir; \bar{S}\right) + \left(1-\mu\right) V^{ND,l}\left(\bar{s}, w - cir; \bar{S}\right)\right) \\
&\quad + \left(1-\pi\right) V^{ND,z}\left(\bar{s}, w - cir; \bar{S}\right), \\
V^{D}\left(\bar{s}, w; \bar{S}\right) &= \pi\left(\mu V^{D,h}\left(\bar{s}, w; \bar{S}\right) + \left(1-\mu\right) V^{D,l}\left(\bar{s}, w; \bar{S}\right)\right) + \left(1-\pi\right) V^{D,z}\left(\bar{s}, w; \bar{S}\right), \\
V^{ND,k}\left(\bar{s}, w; \bar{S}\right) &= \max_{c, x, \left\{a'_j\right\}_j, h^{S\prime}, l^{S\prime}, r^{G\prime}} \left[\log\left(c\right) + \beta E\left[\max\left(V^{ND}\left(\bar{s}', w' - cir'; \bar{S}'\right), V^{D}\left(\bar{s}', w'; \bar{S}'\right)\right)\right]\right], \\
V^{D,k}\left(\bar{s}, w; \bar{S}\right) &= \max_{c, x, h', l'} \left[\log\left(c\right) + \beta E V^{D}\left(\bar{s}', w'; \bar{S}'\right)\right],
\end{aligned}$$

subject to the budget constraints that take the following form for investing firms for which the SGC is binding:

$$c_{i,t} + \frac{\left(1 - \theta q^G_{i,t}\right)}{\left(1 - \theta\right)} h_{i,t+1} + cir_{i,t} = \sum_{j \in \mathcal{I}_{t-1}} a_{i,j,t}\left(r^{\hat{G}}_{j,t} + \lambda q_{j,t}\right) + h^S_{i,t}(r^h_t + \lambda q^h_t) + l^S_{i,t}(r^l_t + \lambda q^l_t)\ \forall i \in \mathcal{H}_t \cap \mathcal{I}_t,$$

$$c_{i,t} + \frac{\left(1 - \theta q^G_{i,t}\right)}{\left(1 - \theta\right)} l_{i,t+1} + cir_{i,t} = \sum_{j \in \mathcal{I}_{t-1}} a_{i,j,t}\left(r^{\hat{G}}_{j,t} + \lambda q_{j,t}\right) + h^S_{i,t}(r^h_t + \lambda q^h_t) + l^S_{i,t}(r^l_t + \lambda q^l_t)\ \forall i \in \mathcal{L}_t \cap \mathcal{I}_t.$$

The ICCs, which have to be satisfied in equilibrium for reputation-based implicit recourse to exist, are the following:

$$\begin{aligned}
V^{ND}\left(\bar{s}, w - cir; \bar{S}\right) &\geq V^{D}\left(\bar{s}, w; \bar{S}\right), \\
V^{P}\left(\bar{s}; \bar{S}\right) &\geq V^{NP}\left(\bar{s}; \bar{S}\right),
\end{aligned}$$

where V^{ND}, V^D, V^P and V^{NP} are the value functions if the firm never defaulted, defaulted, always punished a default on implicit recourse or failed to punish, respectively.

From FOC, we can obtain the following Euler equations in cases where the SGC is binding for all investing firms:

$$E_t\left[\beta\frac{c_{i,t}}{c_{i,t+1}}\frac{r^{\hat{G}}_{t+1}+\lambda q_{j,t+1}}{q^G_{j,t}}\right] = 1\ \forall i\in\mathcal{S}_t, \forall j\in\mathcal{I}_t, \tag{6.36}$$

$$E_t\left[\beta\frac{c_{i,t}}{c_{i,t+1}}\frac{r^h_{t+1}+\lambda q^h_{t+1}}{q^h_t}\right] = 1\ \forall i\in\mathcal{S}_t, \tag{6.37}$$

$$E_t\left[\beta\frac{c_{i,t}}{c_{i,t+1}}\frac{r^l_{t+1}+\lambda q^l_{t+1}}{q^l_t}\right] = 1\ \forall i\in\mathcal{S}_t, \tag{6.38}$$

$$E_t\left[\beta\frac{c_{i,t}}{c_{i,t+1}}\frac{r^h_{t+1}+\lambda q^h_{t+1}}{\frac{\left(1-\theta q^G_t\right)}{(1-\theta)}}\right] = 1\ \forall i\in\mathcal{H}_t\cap\mathcal{I}_t, \tag{6.39}$$

$$E_t\left[\beta\frac{c^l_t}{c^l_{t+1}}\frac{r^l_{t+1}+\lambda q^l_{t+1}}{\frac{\left(1-\theta q^{G,l}_t\right)}{(1-\theta)}}\right] = 1\ \forall i\in\mathcal{L}_t\cap\mathcal{I}_t. \tag{6.40}$$

I guess and verify that all investing firms provide the same level of implicit support $r^G_{j,t+1}=r^G_{t+1}\ \forall j\in\mathcal{I}_t$ (see the discussion in section 3.3 for details). Then, I guess and verify that policy functions have the following form.

Due to the logarithmic utility function, all firms consume a $(1-\beta)$ fraction of their wealth:

$$c_{i,t}=(1-\beta)\left(\sum_{j\in\mathcal{I}_{t-1}}a_{i,j,t}\left(r^{\hat{G}}_{j,t}+\lambda q_{j,t}\right)+h^S_{i,t}\left(r^h_t+\lambda q^h_t\right)+l^S_{i,t}\left(r^l_t+\lambda q^l_t\right)\right)\ \forall i.$$

Under binding SGC, firms with access to high-quality investment opportunities $\mathcal{H}_t$ invest all of the unconsumed part of their wealth into new projects and sell the maximum fraction of investment θ to saving firms:

$$h_{i,t+1}=a_{i,i,t+1}=\frac{\beta\left(\sum_{j\in\mathcal{I}_{t-1}}a_{i,j,t}\left(r^{\hat{G}}_{j,t}+\lambda q_{j,t}\right)+h^S_{i,t}\left(r^h_t+\lambda q^h_t\right)+l^S_{i,t}\left(r^l_t+\lambda q^l_t\right)\right)}{\frac{\left(1-\theta q^G_{i,t}\right)}{(1-\theta)}}\ \forall i\in\mathcal{H}_t\cap\mathcal{I}_t,$$

$$l_{i,t+1}=0\ \forall i\in\mathcal{H}_t\cap\mathcal{I}_t.$$

In the pooling equilibrium, firms with access to low-quality investment opportunities $\mathcal{L}_t$ also invest all of the unconsumed part of their wealth into new projects, and if the

SGC is binding, they sell the maximum fraction of the investment θ to saving firms:

$$l_{i,t+1} = a_{i,i,t+1} = \frac{\beta\left(\sum_{j\in\mathcal{I}_{t-1}} a_{i,j,t}\left(r_{j,t}^{\hat{G}} + \lambda q_{j,t}\right) + h_{i,t}^{S}\left(r_t^h + \lambda q_t^h\right) + l_{i,t}^{S}\left(r_t^l + \lambda q_t^l\right)\right)}{\frac{\left(1-\theta q_{i,t}^{G}\right)}{(1-\theta)}} 1 \ \forall i \in \mathcal{L}_t \cap \mathcal{I}_t,$$

$$h_{i,t+1} = 0 \ \forall i \in \mathcal{L}_t \cap \mathcal{I}_t.$$

If the economy is in a separating equilibrium, the intersection $\mathcal{L}_t \cap \mathcal{I}_t = \mathrm{O}$ is an empty set, and firms with access to low-quality investment opportunities $\mathcal{L}_t$ are not investing in new projects, but rather are buying securitized assets from other firms $\mathcal{L}_t \subset \mathcal{S}_t$.

Saving firms $\mathcal{S}_t$ are, in equilibrium, indifferent between investing in different types of assets. All of them try to diversify their investment, so I guess and verify that, in equilibrium, all will allocate the same fraction of wealth into different types of assets:

$$h_{i,t+1}^{S} = \frac{\zeta^{hS}\beta\left(\sum_{j\in\mathcal{I}_{t-1}} a_{i,j,t}\left(r_{j,t}^{\hat{G}} + \lambda q_{j,t}\right) + h_{i,t}^{S}\left(r_t^h + \lambda q_t^h\right) + l_{i,t}^{S}\left(r_t^l + \lambda q_t^l\right)\right)}{q_t^h} \ \forall i \in \mathcal{S}_t,$$

$$l_{i,t+1}^{S} = \frac{\zeta^{lS}\beta\left(\sum_{j\in\mathcal{I}_{t-1}} a_{i,j,t}\left(r_{j,t}^{\hat{G}} + \lambda q_{j,t}\right) + h_{i,t}^{S}\left(r_t^h + \lambda q_t^h\right) + l_{i,t}^{S}\left(r_t^l + \lambda q_t^l\right)\right)}{q_t^t} \ \forall i \in \mathcal{S}_t,$$

$$\begin{aligned} h_{i,t+1}^{P} &= \sum_{j\in\mathcal{H}_t\cap\mathcal{I}_t} a_{i,j,t+1} \\ &= \frac{\zeta^{hP}\beta\left(\sum_{j\in\mathcal{I}_{t-1}} a_{i,j,t}\left(r_{j,t}^{\hat{G}} + \lambda q_{j,t}\right) + h_{i,t}^{S}\left(r_t^h + \lambda q_t^h\right) + l_{i,t}^{S}\left(r_t^l + \lambda q_t^l\right)\right)}{q_t^G} \ \forall i \in \mathcal{S}_t, \end{aligned}$$

$$\begin{aligned} l_{t+1}^{P} &= \sum_{j\in\mathcal{L}_t\cap\mathcal{I}_t} a_{i,j,t+1} \\ &= \frac{\zeta^{lP}\beta\left(\sum_{j\in\mathcal{I}_{t-1}} a_{i,j,t}\left(r_{j,t}^{\hat{G}} + \lambda q_{j,t}\right) + h_{i,t}^{S}\left(r_t^h + \lambda q_t^h\right) + l_{i,t}^{S}\left(r_t^l + \lambda q_t^l\right)\right)}{q_t^G} \ \forall i \in \mathcal{S}_t, \end{aligned}$$

where $\zeta^{hS} + \zeta^{lS} + \zeta^{hP} + \zeta^{lP} = 1$.

The consumption of the firms in the following period depends on the return from their investment:

$$
\begin{aligned}
c_{i,t+1} &= (1-\beta)\,[h^S_{i,t+1}\left(r^h_{t+1}+\lambda q^h_{t+1}\right)+l^S_{i,t+1}\left(r^l_{t+1}+\lambda q^l_{t+1}\right) \\
&\quad +h^P_{i,t+1}\left(r^{\hat{G},h}_{t+1}+\lambda q^h_{t+1}\right)+l^P_{t+1}\left(r^{\hat{G},l}_{t+1}+\lambda q^l_{t+1}\right)]\ \forall i\in\mathcal{S}_t, \\
c_{i,t+1} &= (1-\beta)\left(h_{i,t+1}\left(r^h_{t+1}+\lambda q^h_{t+1}\right)\right)\ \forall i\in\mathcal{H}_t\cap\mathcal{I}_t, \\
c_{i,t+1} &= (1-\beta)\left(l_{i,t+1}\left(r^l_{t+1}+\lambda q^l_{t+1}\right)\right)\ \forall i\in\mathcal{L}_t\cap\mathcal{I}_t.
\end{aligned}
$$

Using these guesses and substituting in (6.39) and (6.40), we can see that these conditions always hold.

The remaining Euler equations (6.37), (6.38) and (6.36), after substitutions, can be rewritten into:

$$E_t\left[\frac{\frac{r^h_{t+1}+\lambda q^h_{t+1}}{q^h_t}}{\Xi_{t+1}}\right]=1,$$

$$E_t\left[\frac{\frac{r^l_{t+1}+\lambda q^l_{t+1}}{q^l_t}}{\Xi_{t+1}}\right]=1,$$

$$E_t\left[\frac{\frac{r^{\hat{G}}_{t+1}+\lambda q_{t+1}}{q^G}}{\Xi_{t+1}}\right]=1,$$

where $\Xi_{t+1}=\zeta^{hS}\frac{r^h_{t+1}+\lambda q^h_{t+1}}{q^h_t}+\zeta^{lS}\frac{r^l_{t+1}+\lambda q^l_{t+1}}{q^l_t}+\zeta^{hP}\frac{r^{\hat{G},h}_{t+1}+\lambda q^h_{t+1}}{q^G_t}+\zeta^{lP}\frac{r^{\hat{G},l}_{t+1}+\lambda q^l_{t+1}}{q^G_t}$.

The allocation of saving firms (those with zero-profit projects) between high and low investment projects have to satisfy the market clearing conditions on both primary and secondary markets for high and low projects:

$$
\begin{aligned}
\lambda H_t &= \zeta^{hS}\beta\sum_{i\in\mathcal{S}_t}\left(\sum_{j\in\mathcal{I}_{t-1}}a_{i,j,t}\left(r^{\hat{G}}_{j,t}+\lambda q_{j,t}\right)+h^S_{i,t}\left(r^h_t+\lambda q^h_t\right)+l^S_{i,t}\left(r^l_t+\lambda q^l_t\right)\right), \\
\lambda L_t &= \zeta^{lS}\beta\sum_{i\in\mathcal{S}_t}\left(\sum_{j\in\mathcal{I}_{t-1}}a_{i,j,t}\left(r^{\hat{G}}_{j,t}+\lambda q_{j,t}\right)+h^S_{i,t}\left(r^h_t+\lambda q^h_t\right)+l^S_{i,t}\left(r^l_t+\lambda q^l_t\right)\right),
\end{aligned}
$$

$$
\begin{aligned}
\theta\quad & \frac{\beta\sum_{i\in\mathcal{H}_t\cap\mathcal{I}_t}\left(\sum_{j\in\mathcal{I}_{t-1}}a_{i,j,t}\left(r^{\hat{G}}_{j,t}+\lambda q_{j,t}\right)+h^S_{i,t}\left(r^h_t+\lambda q^h_t\right)+l^S_{i,t}\left(r^l_t+\lambda q^l_t\right)\right)}{(1-\theta q^G_t)} \\
&= \frac{\zeta^{hP}\sum_{i\in\mathcal{S}_t}\left(\sum_{j\in\mathcal{I}_{t-1}}a_{i,j,t}\left(r^{\hat{G}}_{j,t}+\lambda q_{j,t}\right)+h^S_{i,t}\left(r^h_t+\lambda q^h_t\right)+l^S_{i,t}\left(r^l_t+\lambda q^l_t\right)\right)}{q^G_t},
\end{aligned}
$$

$$\theta \quad \frac{\beta \sum_{i\in\mathcal{L}_t\cap\mathcal{I}_t}\left(\sum_{j\in\mathcal{I}_{t-1}} a_{i,j,t}\left(r_{j,t}^{\hat{G}}+\lambda q_{j,t}\right)+h_{i,t}^{S}\left(r_t^h+\lambda q_t^h\right)+l_{i,t}^{S}\left(r_t^l+\lambda q_t^l\right)\right)}{\left(1-\theta q_t^G\right)}$$
$$=\frac{\zeta^{lP}\sum_{i\in\mathcal{S}_t}\left(\sum_{j\in\mathcal{I}_{t-1}} a_{i,j,t}\left(r_{j,t}^{\hat{G}}+\lambda q_{j,t}\right)+h_{i,t}^{S}\left(r_t^h+\lambda q_t^h\right)+l_{i,t}^{S}\left(r_t^l+\lambda q_t^l\right)\right)}{q_t^G}.$$

And the goods market clears, too: $Y_t = C_t + X_t$.

C Calibration of the parameters used in section 4

In section 4, I explain the choice of most of the model parameters. Here I would like to specifically comment on the choice of the share of high-quality investment opportunities μ and the dispersion of the type-specific component of high- and low-quality projects in the two Markov states $\Delta^l\left(A^H\right)/\Delta^h\left(A^H\right)$, $\Delta^l\left(A^L\right)/\Delta^h\left(A^L\right)$.

I choose these parameters to replicate the performance (delinquency rates) of securitized assets, which has been at the core of recent debates over the efficiency of securitization—subprime RMBS. Demyanyk and Van Hemert (2011) study the delinquency rates of subprime mortgage loans. In Figure 6.1, which is taken from Demyanyk and Van Hemert (2011), they report the actual delinquency rates of these loans in the left panel, and in the right panel they report the delinquency rates adjusted by the effect of various observable characteristics of the loans and the economy. They conclude that the quality of the loans measured by the adjusted delinquency rates has deteriorated significantly since 2004. This finding is consistent with the switching mechanism presented in this paper. As can be seen in the left panel of Figure 6.2, the United States emerged from a recession in 2003, and in 2004, the output again reached its potential. The model predicts that, as the economy moves to the boom stage of a business cycle, the equilibrium in the signalling game becomes pooling, and as a consequence, low-quality loans start to be financed. As shown in the right panel of Figure 6.1, the boom period of 2004-2007 is associated with lower-quality loans, and the economic downturn of 2001-2003 is associated with higher-quality loans.

I used the reported delinquency rates by Demyanyk and Van Hemert (2011) to calibrate the model parameters.[38] I particularly want to match the delinquency rate of high-quality loans after 12 months in the low state to the delinquency of the 2001 vintage, which is 12.5%; the delinquency rate of high-quality loans after 12 months in

[38] The model presented in this paper does not model loan repayments explicitly. If I assume that a delinquent fraction of loans/projects do not generate cash flows in the current period, then I can compute the ratio of gross profits in the two types of projects.

the high state to the average of the delinquency of the 2002 and 2003 vintage, which is approximately 7%; the delinquency rate of a mix of high- and low-quality loans after 12 months in the high state to the delinquency of the 2005 vintage, which is 9.5%; and the delinquency rate of the mix of high- and low-quality loans after 12 months in the low state to the delinquency of the 2007 vintage, which is 22.5%. This gives the following: $\Delta^l\left(A^H\right)/\Delta^h\left(A^H\right) = 0.94$ and $\Delta^l\left(A^L\right)/\Delta^h\left(A^L\right) = 0.71$.

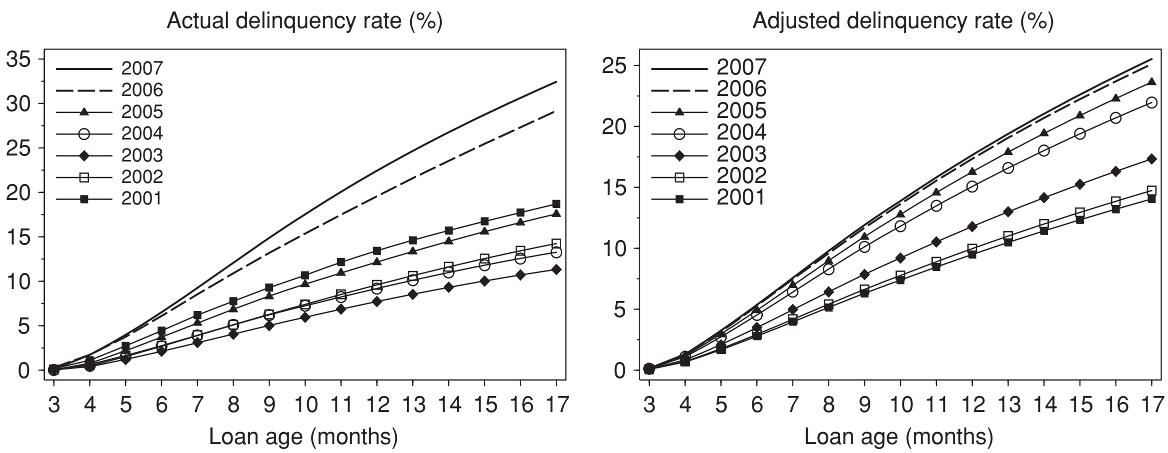
Calibration of the share of high-quality investment opportunities μ is more complicated, since I do not have disaggregated data for the United States. However, assuming that the growth in the volume of subprime mortgage loans between 2003 and 2004 was driven mainly by the entry of firms with access to low-quality loans into the market, we would obtain $\mu = 0.6$. Since this estimate is rather rough, I use loan level data from Moody's Performance Data Services (PDS) database for the United Kingdom. When we compare the delinquency rates of the collateral of the RMBS in the period with the lowest output gap, i.e., in the period 2009Q3, on the one hand for loans issued in previous boom stages of the business cycle, i.e., in 2005Q3-2008Q1 (left panel of Figure 6.3), and on the other hand for loans issued in previous recessions, i.e., in periods 2001Q3-2003Q2 and 2004Q3-2005Q2 (right panel of Figure 6.3), we find a significant difference. In particular, it seems that we can distinguish two relatively clear-cut groups in the subset of RMBS issued in the boom period. One has very low delinquency rates (below 4%) and the other has, at times, much higher delinquency rates. When I use the threshold delinquency rate of 4% to identify high- and low-quality assets and combine the reported frequency with volumes, I find the share of high-quality investment opportunities $\mu = 0.63$. This is approximately consistent with my initial guess for the subprime mortgage loans in the United States, so I use this parameter level.

D Numerical solutions of the fully stochastic dynamic model

Since, depending on the state variables, the economy is switching between a separating and a pooling equilibrium, I use global numerical approximation methods. Perturbation methods used frequently in the macroeconomic literature, which obtain linear approximations for the equilibrium conditions around a steady state and find solutions to the resulting linear system of equations, would fail to capture the switching between different types of equilibrium in this model. I find numerical approximation on a grid of state variables A, K, and ω by iteration. This solution captures different behavior of the model in different states of the economy.

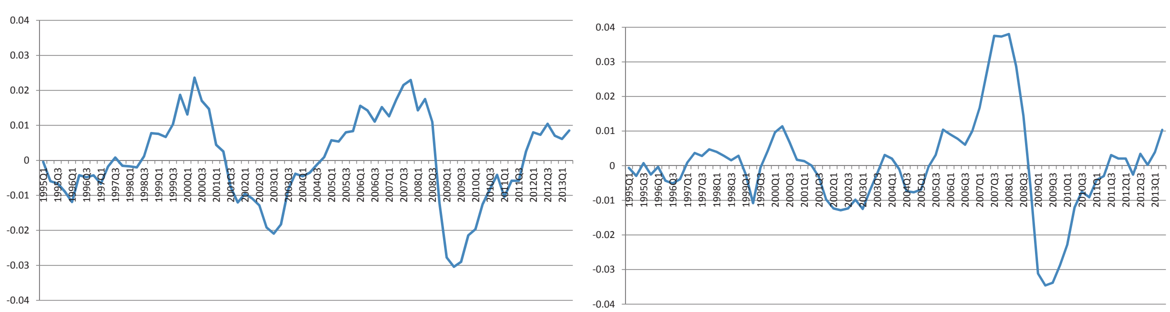
The model solution can be reduced to finding the solution for the set of three non-

Figure 6.1. Actual and adjusted delinquency rates for subprime mortgages by Demyanyk and Van Hemert (2011).



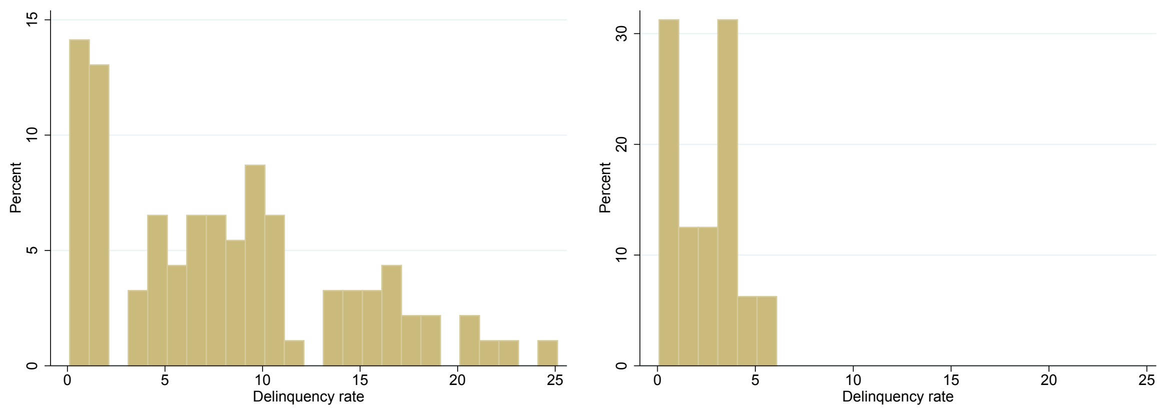
Note: On p.1, Demyanyk and Van Hemert (2011) describe their figure: "The figure shows the age pattern in the actual (left panel) and adjusted (right panel) delinquency rate for the different vintage years. The delinquency rate is defined as the cumulative fraction of loans that were past due 60 or more days, in foreclosure, real-estate owned, or defaulted, at or before a given age. The adjusted delinquency rate is obtained by adjusting the actual rate for year-by-year variation in FICO scores, loan-to-value ratios, debt-to-income ratios, missing debt-to-income ratio dummies, cash-out refinancing dummies, owner-occupation dummies, documentation levels, percentage of loans with prepayment penalties, mortgage rates, margins, composition of mortgage contract types, origination amounts, MSA house price appreciation since origination, change in state unemployment rate since origination, and neighborhood median income."

Figure 6.2. Log of the output gap in the United States (left panel) and the United Kingdom (right panel)



Note: Data are from Eurostat for the United Kingdom and from FRED (St. Louis FED) for the United States. I construct the output gap using the Hodrick-Prescott filter with the smoothing parameter 1600.

Figure 6.3. Histograms of delinquency rates for collateral of the RMBS issued in the United Kingdom in 2009Q3 for loans issued in the boom (left panel) and for loans issued in the bust (right panel)



Note: The figure shows histograms of the delinquency rates of the collateral for the RMBS, which are defined as the amount of receivables that are 90 or more days past due divided by the original collateral balance (in %). The source of the data is Moody's PDS database. The left panel shows the delinquency rate for the subset of RMBS issued in the boom periods 2005Q3-2008Q1, and the right panel shows the RMBS issued in recessions in the periods 2001Q3-2003Q2 and 2004Q3-2005Q2.

predetermined endogenous variables $q^h\left(\bar{S}\right)$, $q^l\left(\bar{S}\right)$ and $V_{diff}\left(\bar{S}\right) \equiv V^{ND}\left(\bar{S}\right) - V^{D}\left(\bar{S}\right)$, which I will denote as $\bar{\Gamma}\left(\bar{S}\right) = \left\{q^h, q^l, V_{diff}\right\} \mid \bar{S}$. Expectations about their next-period values determine the current level of all endogenous variables. Once I know the value of $\bar{\Gamma}\left(\bar{S}\right)$, I can find the remaining endogenous variables, including the law of motion for the endogenous state variables K and ω. Therefore, all equilibrium conditions can be rewritten as

$$E\left(\bar{\Gamma}, \bar{\Gamma}', \bar{S}'\left(\bar{\Gamma}\right)\right) \mid \bar{S} = 0.$$

I use the following algorithm to find the numerical approximation to the model solution.

Initiation: I construct a grid for the three aggregate states $\bar{S} = (A, K, \omega)$. The aggregate productivity takes only two values, so I have a vector $\bar{A} = \left\{A^H, A^L\right\}$. I choose n values for K with equal distances among them and the median being the steady-state value of K, and obtain a vector $\bar{K}$ of size n. Finally, I choose n values of ω from the interval of possible values that ω can take, i.e., from $(\mu, 1)$, and obtain a vector $\bar{\omega}$ of size n. Then I construct a grid $\mathcal{S}$ of state variables as all possible combinations of $\bar{A}, \bar{K}$ and $\bar{\omega}$.

I make an initial guess for non-predetermined endogenous variables of interest $\bar{\Gamma}$ on the grid: $\bar{\Gamma}_0\left(\mathcal{S}\right)$. I choose the stopping criterion $\varepsilon > 0$ and set the iteration counter to zero, $l = 0$.

Step 1: For all combinations of state variables on the grid $\bar{S} \in \mathcal{S}$, I compute $\bar{\Gamma}_{l+1}$, which satisfies[39]

$$E\left(\bar{\Gamma}_{l+1}, \bar{\Gamma}'_l, \bar{S}'\left(\bar{\Gamma}_l\right)\right) \mid \bar{S} = 0.$$

Note that to compute the next-period values of $\bar{\Gamma}'_l$, I need to know the next-period values of state variables $\bar{S}'\left(\bar{\Gamma}_l\right)$. And since those might be in between the grid points, I use trilinear interpolation on the values of the neighboring grid points in this three-dimensional state space.

Step 2: If the difference between the values of the two subsequent iterations for $\bar{\Gamma}(\mathcal{S})$ is smaller than the stopping criterion, i.e., if

$$\| \bar{\Gamma}_{l+1}(\mathcal{S}) - \bar{\Gamma}_l(\mathcal{S}) \| < \varepsilon,$$

then I move to Step 3; otherwise, I go back to Step 1 with the iteration counter l increased by one.

Step 3: I declare $\bar{\Gamma}_{l+1}(\mathcal{S})$ as the final approximate solution of the model and compute the remaining endogenous variables in the model.

[39]Note that the subscript for $\bar{\Gamma}$ denotes the iteration number.