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by José Fique

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Abstract

In complex and interconnected banking systems, counterparty risk does not depend only on the risk of the immediate counterparty but also on the risk of others in the network of exposures. However, frequently, market participants do not observe the actual network of exposures. I propose an approach that incorporates this network of exposures, among other factors, in a valuation model of credit default swaps. The model-implied spreads are then used to retrieve the set of networks that are consistent with market spreads. The approach is illustrated with an application to the UK banking system.

Bank topics: Financial institutions; Financial stability
JEL codes: C63, D85, G21

Résumé

Dans les systèmes bancaires complexes et interconnectés, le risque de contrepartie ne dépend pas uniquement du risque de la contrepartie immédiate, mais aussi du risque d'autres agents dans le réseau d'expositions. Cependant, fréquemment, les acteurs du marché n'observent pas le réseau réel d'expositions. Je propose une approche qui intègre ce réseau d'expositions, entre autres facteurs, dans un modèle d'évaluation des swaps sur défaillance de crédit. Les écarts implicites du modèle sont ensuite utilisés pour récupérer l'ensemble des réseaux qui sont compatibles avec les écarts du marché. L'approche est illustrée au moyen d'une application au système bancaire britannique.

Sujets : Institutions financières ; Stabilité financière
Codes JEL : C63, D85, G21

Non-Technical Summary

Banks are complex organizations that establish important economic relationships with their peers. Therefore, assessing from an isolated perspective the counterparty credit risk of a bank may fail to provide a complete picture.

Assessing counterparty risk in a truly system-wide manner poses difficult challenges since the network of economic links that ties banks together can seldom be observed by researchers and market participants. However, since this network is relevant for counterparty risk assessment, market participants may need to form expectations about it, which are reflected in the market prices of claims issued by banks.

In this paper, I propose an approach based on a valuation model that takes into account, among other factors, the interbank network of exposures. By comparing the model-implied prices with observed market prices for claims issued by banks, this approach helps allocate aggregate exposures, which can be observed from banks' balance sheets, at a bilateral level.

This approach is then illustrated with an empirical application to the UK banking system using data from the 2007–09 financial crisis period. This empirical application seems to suggest that in times of stress, market participants may find it difficult to discern the network structure of exposures from the maximum uncertainty benchmark.

1 Introduction

Simulations used to assess the risk of contagion in banking systems frequently include the interbank network of exposures (e.g., Gauthier et al. 2012). However, with some exceptions, researchers and market participants alike cannot observe the true matrix of exposures. Nevertheless, the total amount of exposures of a given financial institution is available from its balance sheet, albeit only at a relatively low frequency. Even though some information is available, this poses a missing data problem that affects the results of these simulations.

This paper proposes an approach to complement balance-sheet information with market data in order to enhance our understanding of the expected relevance of these exposures to market participants.

Here, I propose a valuation model that takes into account not only how common exposures affect the balance sheets of firms, but also the impact of the network of exposures on the market prices of contingent claims. The theoretical prices of these claims are then used to retrieve the set of networks that are consistent with observed prices.

Using an extension of the structural credit risk model of Merton (1974)[1] that incorporates the impact of contagious defaults on the market value of debt, I define the set of implied networks. These implied networks are defined as the ones that, while respecting the total amount loaned and borrowed obtained from balance-sheet information, are consistent with market data, according to the theoretical valuation model.

This translates into a network optimization problem. I use a genetic algorithm to find the network that improves upon the weighted mean squared error (WMSE) between the network-based theoretical and observed prices using the commonly used maximum entropy (ME) network as a benchmark.[2]

As Abbassi et al. (2017) show for German banks, market-based measures of interconnectedness correlate with the true interbank exposures. Therefore, the networks retrieved using the method that I propose have the potential to augment other approaches, such as ME, that rely only on aggregate exposures, leading to a more accurate assessment of systemic risk.

To the best of my knowledge, this is the first paper to draw on both aggregate exposures and market information to simulate bilateral exposures. The contribution of this paper is an applied one that illustrates how market information can be combined with balance-sheet information to infer the network of bilateral exposures. To do so, I subject the approach to an empirical application using data for four UK global systemically important banks (G-SIB) referring to the 2007–09 financial

[1] More precisely, I use the model proposed in Elsinger et al. (2006) to value defaultable debt via Monte Carlo simulation.

[2] The ME approach implicitly assumes that firms wish to diversify their exposures among counterparties as much as possible; see Upper (2011) for example.

crisis period based on balance sheet data retrieved from the 2008 interim (henceforth AUG2008), annual reports for 2008 (henceforth DEC2008) and 2009 (henceforth DEC2009), and corresponding credit default swaps (CDS) spreads.

The improvement of the WMSE over the ME benchmark varied substantially across the time slices considered in this empirical application. While the improvement was very similar for AUG2008 and DEC2009 – around 40%, it was lower for the DEC2008 slice – roughly 25%. These results seem to indicate that in DEC2008, market participants seemed to perceive a network of exposures closer to the maximum uncertainty benchmark (i.e., ME).

One potential explanation is that as the level of risk increases at the aggregate level, as it can be argued that was the case when looking at the CDS spreads for the four banks, it becomes more difficult to discern bilateral exposures. This then leads to market participants perceiving a network of exposures closer to the ME benchmark. Alternatively, it can be the case that since the ratio of internal (interbank) assets to total assets tends to be lower in DEC2008, the distribution of interbank assets across counterparties is less relevant for pricing of risk. This argument is less strong, however, when we notice that the ratio of interbank assets to equity tends to decline over time and not just for the DEC2008 slice.

While theoretical support for the valuation model can be found in the literature (Fischer 2014 proposes a valuation model that shares similarities with the one proposed in this paper) and the overall approach has been used to assess systemic risk (see, for example, Elsinger et al. 2006), which is the ultimate reason to be interested in these networks, the approach does come at the cost of imposing an *a priori* structure to the data.

The use of a modified Merton model brings with it some strong assumptions that constrain the ability of the approach to accurately model market prices.

The rest of the paper is structured as follows. Section 2 frames the paper in the literature. Section 3 presents a motivating example. Section 4 describes the asset pricing model used to derive model-implied prices for a given network. Section 5 explains the approach used to retrieve the set of implied networks. Section 6 shows the results of simulations used to analyze the sensitivity of the methodology to exogenous factors. Section 7 elaborates on the motivating example provided in Section 3 to provide some intuition behind the mechanisms at play in the simulations. Section 8 presents the empirical application. Finally, Section 9 concludes.

2 Literature review

This paper is related to several strands of literature. Firstly, the paper is closely related to papers that address the issue of reconstructing financial networks based on partial information, such as Anand et al. (2015), Anand et al. (2017), Gandy and Veraart (2016) and Montagna and Lux (2017). Even though in this paper the totals of internal assets and liabilities are also key pieces of information to determine the set of model-implied networks, this information is complemented with market-based signals to shed light on the market perception of the network of exposures. Secondly, since the set of model-implied networks is determined by an asset pricing model, this paper is related to the literature on asset pricing in financial networks (see, for example, Eisenberg and Noe 2001, Egloff et al. 2007, Gouriéroux et al. 2013, Barucca et al. 2016 and Abbassi et al. 2017). Finally, the paper is also related to the literature on the potential role that uncertainty about the network of exposures plays in financial contagion, which has been studied by Caballero and Simsek (2013) and Li et al. (2016).

While these papers focus on pricing contingent claims dependent on a true network, the focus of this paper is to search for a set of networks that reconciles model-implied and observed prices for those claims. The paper by Abbassi et al. (2017) closely relates to mine as the authors show that for German banks, market-based measures of interconnectedness correlate with the interbank exposures and these vary both at a cross-sectional level and over time. Thus, the information contained in market prices constitutes a valuable input in reconstructing networks from partial data.

3 A motivating example

Suppose that in a four-bank system, total internal assets and liabilities are given as follows:[3]

Table 1: Matrix of individually unknown exposures

	A	B	C	D	Liabilities
A	0	?	?	?	15
B	?	0	?	?	11
C	?	?	0	?	13
D	?	?	?	0	10
Assets	11	18	12	8	

Naturally, there is more than one matrix of exposures that is consistent with the total individual exposures displayed in Table 1. Consider, as an example, the following matrices:

[3]Note that by convention, the diagonal of such matrices is assumed to be a vector of zeros.

Table 2: Matrices consistent with aggregate exposures

	A	B	C	D	Liabilities
A	0	3	8	4	15
B	4	0	4	3	11
C	4	8	0	1	13
D	3	7	0	0	10
Assets	11	18	12	8	

	A	B	C	D	Liabilities
A	0	4	8	3	15
B	3	0	4	4	11
C	4	8	0	1	13
D	4	6	0	0	10
Assets	11	18	12	8	

Even though both of these matrices are consistent with the aggregate exposures, they may imply different outcomes when used for risk assessment purposes, as already pointed out by Mistrulli (2011). Thus, to inform this process, I propose a model that I describe in the next section.

4 The model

In this section, I present a valuation model, which is an extension of Merton (1974), that takes into account the impact of the network of exposures in the market price of defaultable debt. Fundamentally, I use the model proposed in Elsinger et al. (2006) to value defaultable debt via Monte Carlo simulation. The model-implied (or theoretical) prices are then used to determine the set of networks of exposures that are consistent with the observed prices.

The section is divided into three subsections. In Subsection 4.1, I show how the balance sheet of an interconnected firm changes according to the paths assumed for its own assets and liabilities and also via its connections with the value of other firms' assets and liabilities at maturity. Then, in Subsection 4.2, I present a valuation algorithm since a closed-form expression cannot be obtained. Finally, in Subsection 4.3, I formally define the concept of the networks that are consistent with market data conditional on the theoretical prices derived from the valuation model[4] proposed in Subsections 4.1 and 4.2.

4.1 Basic setup

Consider an industry where firms, in addition to establishing economic ties with firms in other industries, establish relevant connections within the same industry.[5] The balance sheet of a typical firm i in this industry, with $i \in I = \{1, ..., n\}$, can be decomposed into: (i) internal assets (IA_i) and liabilities (IL_i); (ii) external assets (EA_i) and liabilities (EL_i); and (iii) equity (EQ_i). In illustrative terms,[6]

[4]Similar asset pricing models have already been proposed by Fischer (2014).

[5]While the focus of this paper is on debt claims, an interesting extension would be to also treat equity claims, which is left for future research.

[6]Note that the decomposition of the balance sheet does not have a quantitative meaning.

Assets	Liabilities
EA_i	EL_i
	IL_i
IA_i	EQ_i
TA_i	$TL_i + EQ_i$

where total assets of firm i (TA_i) are equal to the sum of EA_i and IA_i, and total liabilities (TL_i) are equal to the sum of EL_i and IL_i.

In particular, in the case of the banking industry, internal assets and liabilities can be thought of as interbank loans and external assets as loans to the non-financial sector.

Following Elsinger et al. (2006), I assume the following dynamics for the external assets of firm i:

$$\frac{dEA_i(t)}{EA_i(t)} = rdt + \sigma_i dZ_i(t),$$

where Z_i is a one-dimensional Brownian motion defined in the filtered risk-neutral probability space $(\Omega, \mathbb{P}, \mathscr{F}, \mathscr{F}_t)$, r is the risk-free rate and σ_i is the volatility of the assets of i. Also, r and σ_i are assumed to be constant, and the instantaneous correlation between $Z_i(t)$ and $Z_j(t)$ is given by ρ_{ij} for all i and $j \in I$. In its turn, the value of the internal assets at $t = T$ is given by

$$IA_j(T) = \sum_{i=1}^{n} p_i^\star m_{ij},$$

where m_{ij} is the nominal liability of firm i to firm j, the typical element of the exposure matrix $\mathbf{M} \in \mathscr{M}$ with $\mathscr{M}$ being the space of all possible liabilities matrices. $\mathbf{M}$ is assumed to remain unaltered before the common maturity date of all internal claims, and $p_i^\star$ is the fraction of the liabilities of firm i recovered by all other creditors with the same priority of internal claims at maturity.[7] For the purpose of the methodology developed in this paper, it is critical that there exists a class of defaultable debt with observable market prices that has the same priority in liquidation as the bilateral exposures that I wish to retrieve. Thus, I make the following assumptions:

Assumption 1. *(Seniority structure of debt) Internal liabilities (IL_i) and external liabilities (EL_i) have the same priority in liquidation.*

Under this assumption, I define a new liabilities matrix, $\tilde{\mathbf{M}} \in \tilde{\mathscr{M}}$, with $\tilde{\mathscr{M}}$ defined analogously to $\mathscr{M}$, that summarizes the cross holdings of debt with the same priority in liquidation as the bilateral exposures among firms. This matrix extends M to include a (sink) node that has no liabilities to other nodes to accommodate the external liabilities of all other nodes and is given by

[7]Note that if all firms remain solvent, $\mathbf{p}^\star$ is simply a vector of ones. However, if the $i-$th firm is unable to repay its creditors fully, the $i-$th entry of this vector is lower than one reflecting the number of cents on the dollar each creditor ends up receiving.

	1	2	...	n	$n+1$	Liabilities
1	0	m_{12}	...	m_{1n}	EL_1	$EL_1 + \sum_{j=1}^{n} m_{1j}$
2	m_{21}	0	...	m_{2n}	EL_2	$EL_2 + \sum_{j=1}^{n} m_{2j}$
...	...	...	...	...	...	...
n	m_{n1}	m_{n2}	...	0	EL_n	$EL_n + \sum_{j=1}^{n} m_{nj}$
$n+1$	0	0	...	0	0	0
Assets	$\sum_{i=1}^{n} m_{i1}$	$\sum_{i=1}^{n} m_{i2}$	...	$\sum_{i=1}^{n} m_{in}$	$\sum_{k=1}^{n} EL_{in+1}$	

The formal presentation of their definition follows.

Let $e = EA$ be the firms' external assets, $d_i = \sum_j \tilde{m}_{ij}$ and $\bigodot$ denote the Hadamard product. Then, under assumptions:

Assumption 2. *(Limited liability)* $\forall\, i \in I$, $\mathbf{p}^\star = max\left\{\left(\tilde{M}'\mathbf{p}^\star + \mathbf{e}\right) \bigodot \left(\frac{1}{d_1}, \ldots, \frac{1}{d_{n+1}}\right), 0\right\}$.

Assumption 3. *(Proportional repayment)*

$$\forall\, i \in I,\ \mathbf{p}^\star = min\left\{\mathbf{1}, \left(\tilde{M}'\mathbf{p}^\star + \mathbf{e}\right) \bigodot \left(\frac{1}{d_1}, \ldots, \frac{1}{d_{n+1}}\right)\right\},$$

that is, if obligations are not paid in full, then creditors obtain a proportional share of the total value of the firm's assets with respect to the outstanding debt.

Definition 1. The clearing payment vector is given by

$$\mathbf{p}^\star = min\left\{\mathbf{1}, max\left\{\left(\tilde{M}'\mathbf{p}^\star + \mathbf{e}\right) \bigodot \left(\frac{1}{d_1}, \ldots, \frac{1}{d_{n+1}}\right), 0\right\}\right\}.$$

As Eisenberg and Noe (2001) (henceforth EN) show, this vector has some desirable properties, such as existence and uniqueness, under some regularity conditions. Even though this vector has the desirable property of reflecting how the successive rounds of contagious defaults affect the loss given default, it does not take into account the reduction in the value of the assets when a bank defaults. For this reason, I use the extended version of the Eisenberg and Noe (2001) algorithm proposed by Rogers and Veraart (2013) that allows for a fractional loss, $1-\alpha$, realized when e is liquidated.

In the next subsection, I propose the valuation algorithm that determines the theoretical price that will be used in the subsequent subsection to define the set of networks implied by the observed prices.

4.2 Valuation via Monte Carlo simulation

Given that loss-given default is endogenously determined by the clearing payment vector, the defaultable bonds are valued by Monte Carlo simulation. As described in Hull (Hull), assuming constant interest rates, the price of the asset can be found by taking the following steps:

1. Sample a random path for EA_i in a risk-neutral world.

2. Find $\mathbf{p}^\star$ via the fictitious default algorithm based on the sample path.

3. Repeat steps 1. and 2. K times.

4. Calculate the average of the payoff of the defaultable bond in a risk-neutral setting by computing the average of $\mathbf{p}^\star$ over all states of nature. Note that in this setting, $\mathbf{p}^\star$ is the recovery rate, which is also the normalized payoff of the defaultable debt.

5. Discount the expected payoffs using the risk-free rate, r, to obtain the theoretical price of defaultable debt.

4.3 The set of implied networks

The entire analysis as presented thus far implicitly assumes that the network of claims is observable. However, this is only true for a reduced number of cases. In most instances, only the total amount borrowed/loaned is known. All studies based on the fictitious default algorithm are either based on data available strictly to central banks or on reconstruction methods, such as ME, that rely mostly on aggregate exposures (see Anand et al. 2017 for a comparative study on the different reconstruction methods). Entropy maximization roughly translates into assuming that firms wish to diversify their exposures as much as possible, which may be a strong assumption with potentially important consequences for risk assessment, as pointed out by Mistrulli (2011).

A natural extension of the valuation algorithm presented in Subsection 4.2 is to use observed market prices to infer the network that is likely to generate them, much like the method to recover the option-implied volatility (see, for example, Jackwerth and Rubinstein (1996)). A possible method of doing so is to minimize the WMSE between the observed and theoretical prices of a zero coupon bond with a face value of \$1 at time t with maturity T.

Definition 2. Let $\mathbb{F}$ denote the set of feasible liabilities matrices, i.e.,

$$\mathbb{F} := \left\{ \tilde{\mathbf{M}} \in \tilde{\mathscr{M}} : \sum_{k=1}^{n} \tilde{m}_{ik} = \tilde{m}_{i.} \cap \sum_{k=1}^{n} \tilde{m}_{kj} = \tilde{m}_{.j} \cap \tilde{m}_{ii} = 0 \cap \tilde{m}_{in+1} = EL_i \, \forall \, i \in I \right\},$$

where $\tilde{m}_{i.}$ and $\tilde{m}_{.j}$ are the total amount of internal liabilities of i and the total amount of internal assets of j, respectively. Note that $\tilde{m}_{i.}$, $\tilde{m}_{.j}$ and EL_i can be observed from balance sheets, albeit at a relatively low frequency.

Definition 3. Let $\mathbb{IN}$, the set of implied networks, denote the set of liabilities matrices that are a solution to **Problem 1**:

$$\mathbb{IN} := \underset{\tilde{\mathbf{M}} \in \mathbb{F}}{argmin} \left\{ \Xi \left(\tilde{\mathbf{M}} \mid \theta \right) \right\},$$

where

$$\Xi \left(\tilde{\mathbf{M}} \mid \theta \right) := \frac{1}{n} \left[p_t^{obs} - e^{-r_f \tau} E_t^Q \left(\mathbf{p}^\star \left(\tilde{\mathbf{M}} \mid \theta \right) \right) \right]' W \left[p_t^{obs} - e^{-r_f \tau} E_t^Q \left(\mathbf{p}^\star \left(\tilde{\mathbf{M}} \mid \theta \right) \right) \right],$$

with

$$W = \begin{bmatrix} \frac{TA_1}{\sum_{j=1}^n TA_j} & 0 & \dots & 0 \\ 0 & \frac{TA_2}{\sum_{j=1}^n TA_j} & \dots & 0 \\ \dots & \dots & \dots & 0 \\ 0 & 0 & 0 & \frac{TA_n}{\sum_{j=1}^n TA_j} \end{bmatrix},$$

$$\theta = \left(EA(t), r, \sigma, \rho \right)$$

and $\tau = T - t$. The choice of the weighting matrix based on the relative size of a bank is justified by higher liquidity, and thus informativeness, of the market signal.

Alternatively, a similar expression can be derived using CDS spreads instead of bond prices. The set of implied networks is now given by

$$\mathbb{IN}_{cds} := \underset{\tilde{\mathbf{M}} \in \mathbb{F}}{argmin} \left\{ \Xi_{cds} \left(\tilde{\mathbf{M}} \mid \theta \right) \right\}, \tag{1}$$

where

$$\Xi_{cds} \left(\tilde{\mathbf{M}} \mid \theta \right) := \frac{1}{n} \left[cds_t^{obs} + \frac{ln \left[e^{-r_f \tau} E_t^Q \left(\mathbf{p}^\star \left(\tilde{\mathbf{M}} \mid \theta \right) \right) \right]}{\tau} - r_f \right]' W \left[cds_t^{obs} + \frac{ln \left[e^{-r_f \tau} E_t^Q \left(\mathbf{p}^\star \left(\tilde{\mathbf{M}} \mid \theta \right) \right) \right]}{\tau} - r_f \right].$$

5 The genetic algorithm

I propose an approach based on a genetic algorithm adapted to network optimization to solve Problem 1.[8] The algorithm works as follows:

1. Use the ME network as a starting point.
2. Generate *ngen* "mutations" of the ME matrix and compute the model predicted prices. This set of matrices is referred to as the children set.
3. Evaluate the fitness, i.e., compute $\Xi\left(\tilde{\mathbf{M}} \mid \theta\right)$, for all matrices generated in 2.
4. Preserve the *npar* matrices with the best fitness. This set of matrices is referred to as the set of parents.
5. Create *ngen* "mutations" of the matrices generated in 4. and proceed as in step 2.
6. Evaluate the fitness of all matrices generated in step 4.
7. Repeat steps 4-6 until the fitness measure improves a specified percentage over the fitness measure (i.e., WMSE) obtained for the ME network.

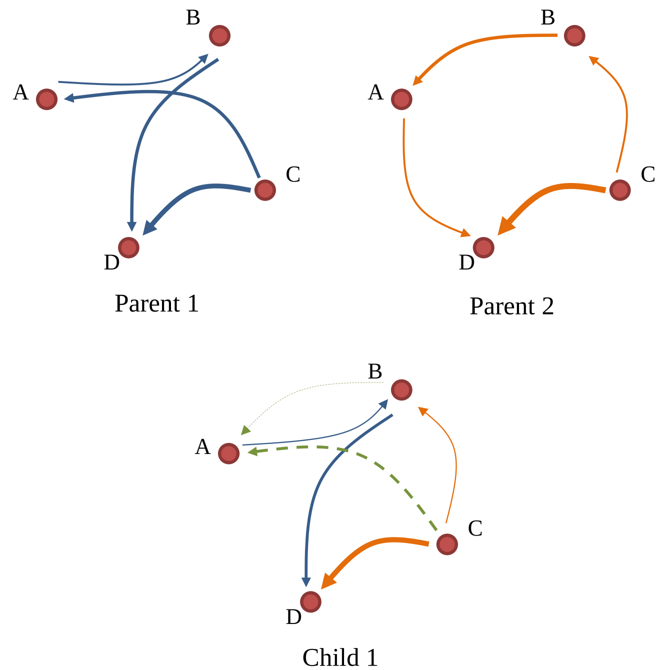
The success of the algorithm depends on the ability of each new generation to preserve the best fitting connections of the parent matrices while "mutating" in such a way that the fitness improves. In order for this to be possible, the mutation process takes place as follows:

1. Uniformly select two parents and replace one of the nodes of parent 1 in the copy of parent 2 (see Figure 1 for an illustration).
2. Add an innovation to all off-diagonal elements of each child matrix and re-balance the resulting matrix to ensure that total loaned/borrowed amounts are preserved.[9]

[8]Genetic algorithms have been used previously in network science to detect communities in graphs. See, for example, Pizzuti (2008).

[9]This is achieved by employing the RAS algorithm; see Schneider and Zenios (1990).

Figure 1: Mutation process



This figure depicts the creation of Child 1 based on Parents 1 and 2. Note that Child 1 is the result of copying Parent 1 and replacing node C's connections with the corresponding ones obtained from Parent 2 and then re-balancing the resulting matrix to ensure that total loaned/borrowed amounts are preserved. Additionally, the exposure of node C to nodes A and B are introduced as innovations.

6 Simulations

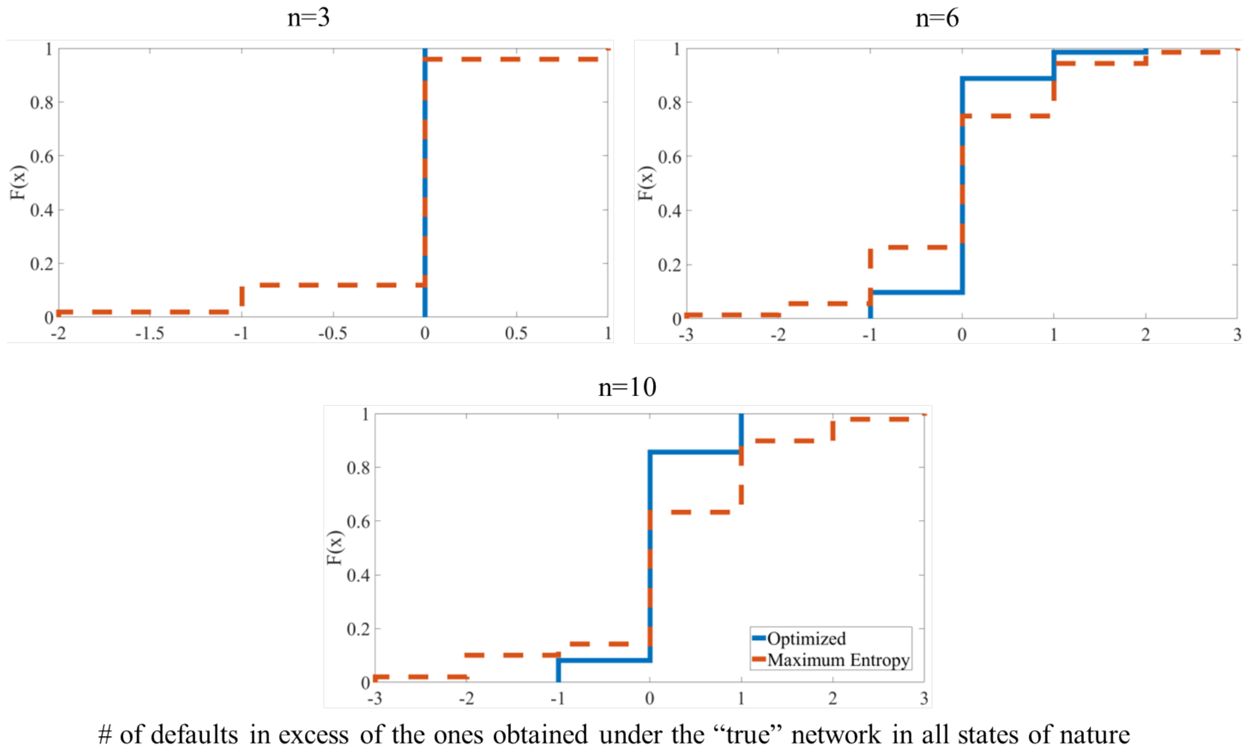
The model can be more easily understood with the help of some illustrative simulations that reveal the sensitivity of the results to changes in key parameters. To do so, I randomly generated 50 networks that are treated as the "true" network and compared against these simulated networks the performance of both the ME and implied (or optimized) networks approaches.[10]

Interbank assets were calibrated to 5% of total assets, volatility of banks' assets ranged between 2.7% and 4%, risk-free rate was set at around 2%, the minimum initial leverage ratio was set at 3%, and for each network 500 states of nature were simulated.

The following figures show the simulated cumulative distributions of the deviation of the number of defaults in excess of the ones associated with the assumed network when one of the reconstructed networks (ME or optimized) is used instead.

[10]These 50 networks were generated separately for each comparison.

Figure 2: Simulated CDF: 5% bankruptcy costs ($\alpha = 0.95$)

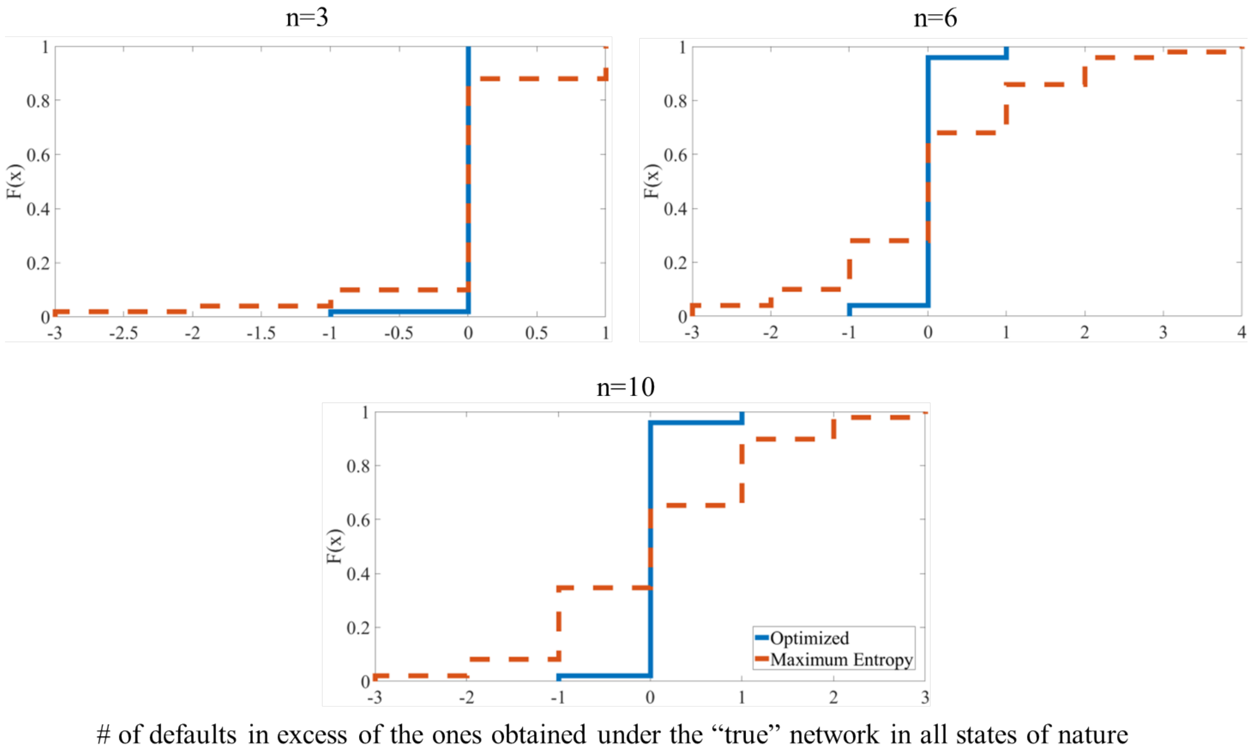


Source: author’s calculations.

Figure 2 shows that when bankruptcy costs are set at 5% (or $\alpha = 0.95$), in about 51 instances the ME approach implies a deviation of at least one default event in all simulated states of nature, compared with what the EN’s algorithm implies using 150 generated networks with 3, 6 and 10 banks. Moreover, this method implies at least two deviations in about 15 instances. In comparison, the optimized network shows a better performance. In about 24 (1) instances it produces one deviation (two deviations) compared with the number of bankruptcies implied by the EN algorithm when applied to the generated networks.

These results are as expected since the optimized networks approach uses additional information to reconstruct the “true” network. Overall, the increase in the size of the network leads to a deterioration in the ability of the ME network to reproduce the number of “true” bankruptcies, while the optimized network approach performance only decreases slightly for the simulated networks.

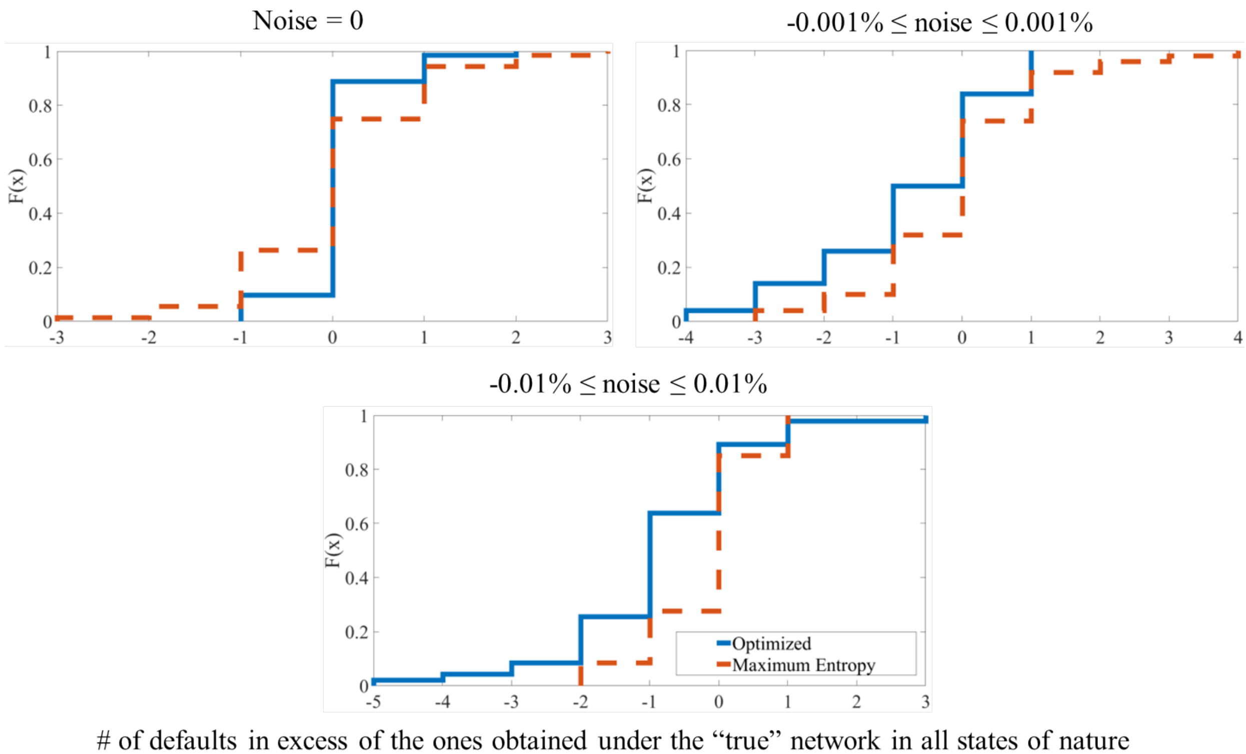
Figure 3: Simulated CDF: 15% bankruptcy costs ($\alpha = 0.85$)



Source: author’s calculations.

A natural question to ask is how the performance of the optimized network approach changes when bankruptcy costs increase. Since the network structure is likely to be more relevant when bankruptcy costs increase, one would expect to see an increase in the gap in performance of the optimized networks approach over ME. This is what Figure 3 confirms. When bankruptcy costs increase from 5% to 15%, the ME (optimized network) approach produces biased results in 50% (6%) of the instances when compared with the results of the “true” network.

Figure 4: Simulated CDF: noisy prices for $n = 6$ and $\alpha = 0.95$



Source: author’s calculations.

However, the effectiveness of the optimized network approach relies substantially on the ability of market signals to convey information about the network. Since market signals are likely to contain a noise component, the performance of the optimized network approach deteriorates as the noise in the market signal increases.

To understand how sensitive the approach is, I added a uniformly distributed random variable to each “fundamental” price (i.e., the one that results from the application of the pricing model to the “true” network) such that the prices used to retrieve the optimized network deviate from the average “fundamental” price in a specified percentage.

Figure 4 shows that when a noise term is added to the theoretical bond prices, ME can outperform the optimized network approach. For 6 banks and 5% bankruptcy costs, the optimized network approach greatly outperforms ME when noise is non-existent. However, when the noise term is set between -0.001% and 0.001% of the average “fundamental” price, the performance of both approaches is somewhat comparable. Finally, ME largely outperforms the optimized network approach when the noise term is between -0.01% and 0.01%.

7 A numerical example (revisited)

In the motivating example of Section 3, I showed that there is more than one matrix that satisfies the total exposures constraints. I now revisit the example to present the differences among network topologies and predicted bond prices of the assumed “true”, ME and the implied liabilities matrices in order to provide some intuition on the simulations conducted in Section 6. The purpose of this numerical example is to provide some intuition on how changes in how exposures are reallocated affect prices.

I assume $r_f = 0$ to make the explanation more intuitive. This allows me to match bond prices with expected recovery rates at maturity (i.e., the Eisenberg and Noe’s clearing payment vector) for each Monte Carlo draw. Given simulated paths for the $EA(T)$ vector, and assuming that the “true” liabilities matrix that results from market participants’ expectations is

	A	B	C	D	Liabilities
A	0	3	8	4	15
B	4	0	4	3	11
C	4	8	0	1	13
D	3	7	0	0	10
Assets	11	18	12	8	

the predicted vector of bond prices is $[.8919, .8940, .9502, .9082]$. However, based on the same simulated paths for $EA(T)$, the predicted price vector is $[.8919, .8940, .9505, .9082]$ when the ME network is considered (left table) and $[.8920, .8940, .9501, .9082]$ when the implied network approach is used (right table):

	A	B	C	D	Liabilities
A	0	7.28	4.83	2.89	15
B	4.14	0	4.29	2.57	11
C	4.08	6.38	0	2.54	13
D	2.78	4.34	2.88	0	10
Assets	11	18	12	8	

	A	B	C	D	Liabilities
A	0	4.89	7.20	2.91	15
B	3.64	0	4.09	3.23	11
C	4.07	7.07	0	1.86	13
D	3.25	6.04	0.71	0	10
Assets	11	18	12	8	

The key takeaways are that reallocating the exposures has an effect on the prices of an instrument (specifically, bond prices), which can be understood as the extent to which shocks to all nodes affect a particular node.

Under the ME network, node C is less affected by shocks to all banks than under the “true” network. Conversely, under the network retrieved using the algorithm, node C (A) is more (less) affected by shocks to all banks than under the “true” network, albeit the deviations are smaller than the ones under the ME network.

More precisely, when considering the ME matrix, the extent to which contagion affects C is underestimated. This is the case because the ME network, in line with the implied assumption that firms wish to diversify their exposures as much as possible, puts relatively more weight on the exposures of C to B and D, which have a higher expected recovery value than A. Note also that there is no significant difference in the payment vector entries for all other firms. This can be explained as follows. First, the ME solution is almost identical to the original for the internal assets of A. Second, even though the ME solution substantially affects the distribution of the internal claims of B, the fact that B's internal assets largely exceed its internal liabilities makes B's payment vector entry largely independent of the liabilities matrix. Finally, D's payment vector entry also remains largely unaltered, even though the ME approach redistributes D's interbank assets by reducing the weight in A and B (which have low recovery value) to C (which has a high recovery value). One potential explanation is that the external assets of D are more than sufficient to fulfill its obligations when there is a default by one or more of its counterparties.

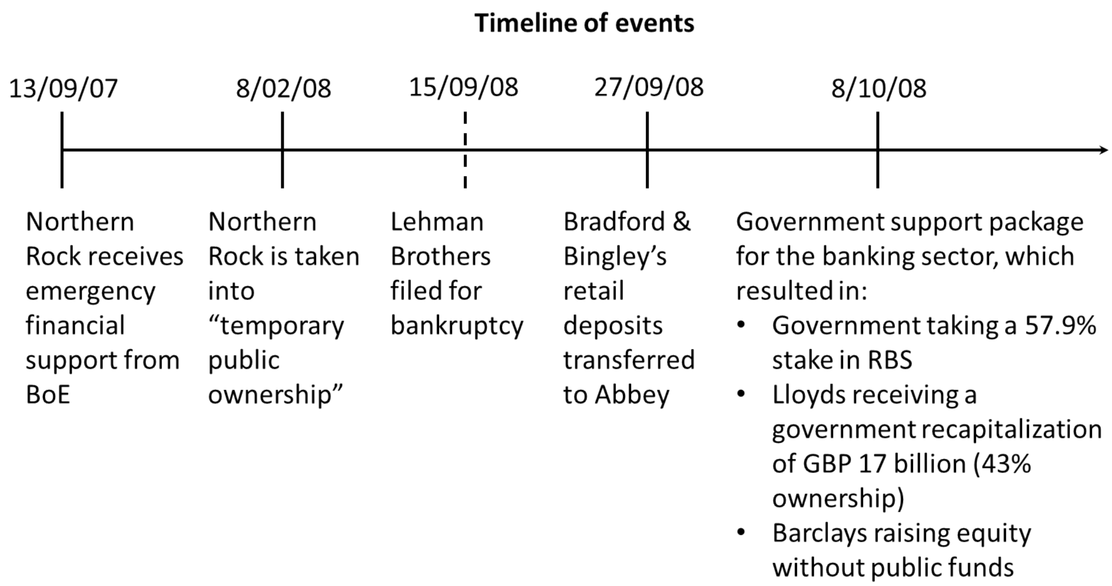
Analogously, the price vector obtained from the implied network indicates that the extent to which contagion affects firms A (C) is underestimated (overestimated). Note that under this methodology, A's internal assets are relatively more (less) concentrated in D (B) than under ME, thus the expected effects of contagion are less severe under the proposed methodology since the expected recovery of D is higher than of B. However, since in the implied network C's internal assets are more concentrated in A rather than in D, in contrast to the ME solution, C's clearing vector entry is lower than both under the ME and "true" networks.

8 Empirical application

The model is now brought to the data to investigate to what extent the structure of the interbank network plays a role in the pricing of risk. To that end, the model is calibrated to the observed data for the UK banking system corresponding to the 2007–09 financial crisis period. The UK banking system has some properties that are aligned with the methodology proposed in this paper: it is highly concentrated,[11] which means that the relevant network has a limited number of nodes, reducing the computational complexity; and it experienced a substantial level of stress during the 2007–09 financial crisis, a fundamental requirement given that the network structure is expected to become more relevant during these periods. Figure 5 presents a timeline of selected events that unfolded in the UK banking system during the financial crisis.

[11] In 2011, the seven largest banks held 71% of total banking assets (IMF, 2011 Financial System Stability Assessment).

Figure 5: Timeline of selected events that unfolded in the UK banking system during the financial crisis



Source: author's summary based on House of Commons (2009).

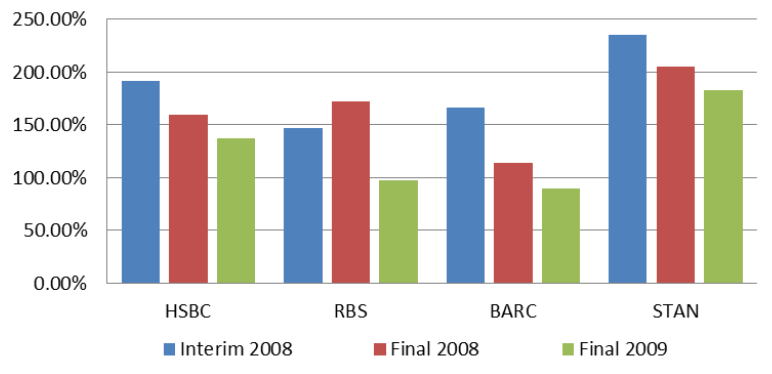
Based on these two criteria and the systemic importance as defined by the Financial Stability Board, the banks included in the condensed network are HSBC, Royal Bank of Scotland (RBS), Barclays (BARC) and Standard Chartered (STAN). To understand to what extent uncertainty plays a role in market participants' views regarding the network of exposures, the model is calibrated using data corresponding to periods that surrounded the support provided by the UK's government to the banking sector in October 2008. Moreover, the UK banking system offers the additional advantage of having been studied extensively from a financial networks' perspective (see, for example, Langfield et al. (2014)).

8.1 Calibration

Since the objective of the proposed methodology is to shed light on market participants' views regarding the interbank network of exposures, the model is calibrated using strictly publicly available data. This requirement poses some natural challenges, as the data needed to estimate the stochastic processes for the market value of assets are not available at the bank level but rather at a group level, at least for some years. Thus, the data used to calibrate the model are at the group level of aggregation.

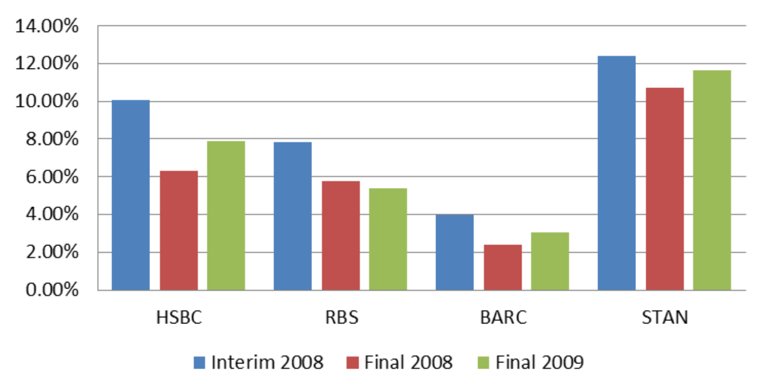
Aggregate interbank assets and liabilities are obtained from the 2008 interim, and 2008 and 2009 annual reports and defined as "Loans and advances to banks" and "Deposits by banks," respectively. The ratios of interbank assets over book value of equity and book value of total assets ranged from 90% to 230% and 3% to 12%, respectively (see Figures 6 and 7, respectively).

Figure 6: Ratio of internal (interbank) assets to equity (book values)



Source: banks' annual reports.

Figure 7: Ratio of internal (interbank) assets to total assets (book values)



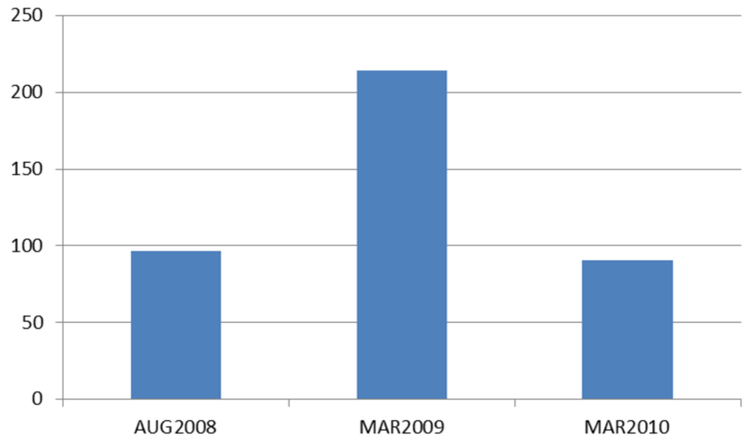
Source: banks' annual reports.

Even though the UK banking system is highly concentrated, a condensed network with only four banks is naturally a simplification made for tractability reasons. Consequently, a sink node, denoted by all other creditors (AllOthers), is added to the network. Importantly, this sink node has no liabilities to any of the four banks chosen, and aggregates all other liabilities, both to financial and non-financial counterparties.[12] The fractional loss of e realized on liquidation, $1-\alpha$, was set to 10% as in Chatterjee (2013).

[12]As Langfield et al. (2014) show, the UK banking system can be described as a "core-periphery" network. That is, core nodes are highly interconnected among themselves and with peripheral nodes, and nodes in the periphery are weakly connected amongst themselves. However, the fit of the "core-periphery" model varies substantially across the different types of liabilities, albeit the identified number of "core" banks is algined with the number of banks included in the condensed network.

The market signals used to complement balance-sheet information are the peak 5-year CDS spreads observed when the reports were released. Both CDS spreads and market values of equity are obtained from Bloomberg (see Figure 8).

Figure 8: Mean of 5-Year CDS spreads (in bps)



Source: Bloomberg.

Estimation of the parameters of the stochastic processes for banks' market value of assets was conducted independently of the interbank network via the Maximum Likelihood method as in Duan (1994) and Duan (2000).[13]

8.2 Results

Figures 9–11 show the results for the networks obtained using ME and the ones implied by the model based on data from the 2008 interim (henceforth AUG2008), and annual reports for 2008 (henceforth DEC2008) and 2009 (henceforth DEC2009), respectively.[14] An arrow from node A to node B (->) represents an exposure of A to B. Weights of the arrows represent the natural logarithm of the exposures. Tables 3–5 display the relative differences expressed in percentage terms of the ME exposures.

All the following results are evaluated against the ME benchmark. This benchmark was chosen not only because it is a frequently used method to reconstruct networks from partial information, but mainly because it represents the maximum uncertainty view of the distribution of bilateral exposures.

[13]Naturally, it is expected that banks' interconnectedness also affects the estimates of the stochastic processes' parameters. However, allowing for this extension would lead to a considerable increase in the complexity of the model. Consequently, it is left for future research.

[14]In this exercise, the size of each generation of *child* networks, *ngen*, was set at 5000 and the number of *parent* networks, *npar*, was set at 5. The stopping criterion was determined by experimentation and is reported in the results.

The improvement of the error measure defined in equation (1) varied substantially across the time slices considered in this empirical application. While the improvement of the error measure over the ME benchmark was very similar for AUG2008 and DEC2009 – around 40%, it was lower for the DEC2008 slice – roughly 25%. These results seem to indicate that in DEC2008, market participants seem to perceive a network of exposures closer to the maximum uncertainty benchmark - ME.

One potential explanation is that as the level of risk increases at the aggregate level, as it can be argued that was the case when looking at the CDS spreads for the four banks (see Figure 8), it becomes more difficult to discern bilateral exposures. This then leads to market participants perceiving a network of exposures closer to the ME benchmark. However, it can also be the case that since the ratio of internal (interbank) assets to total assets tends to be lower in DEC2008 (see Figure 7), the distribution of interbank assets across counterparties is less relevant for pricing of risk. This argument is less strong, however, when we notice that the ratio of interbank assets to equity tends to decline over time and not just for the DEC2008 slice (see Figure 6).

Table 3: AUG2008

	HSBC	RBS	BARC	STAN	Liabilities
HSBC		-18	28	221	
RBS	-8		-18	-83	
BARC	8.6	17		-87	
STAN	15	-3	-6		

Source: author's calculations.

Table 4: DEC2008

	HSBC	RBS	BARC	STAN	Liabilities
HSBC		19	2	48	
RBS	-19		8	-42	
BARC	27	-33		-9	
STAN	-33	71	-80		

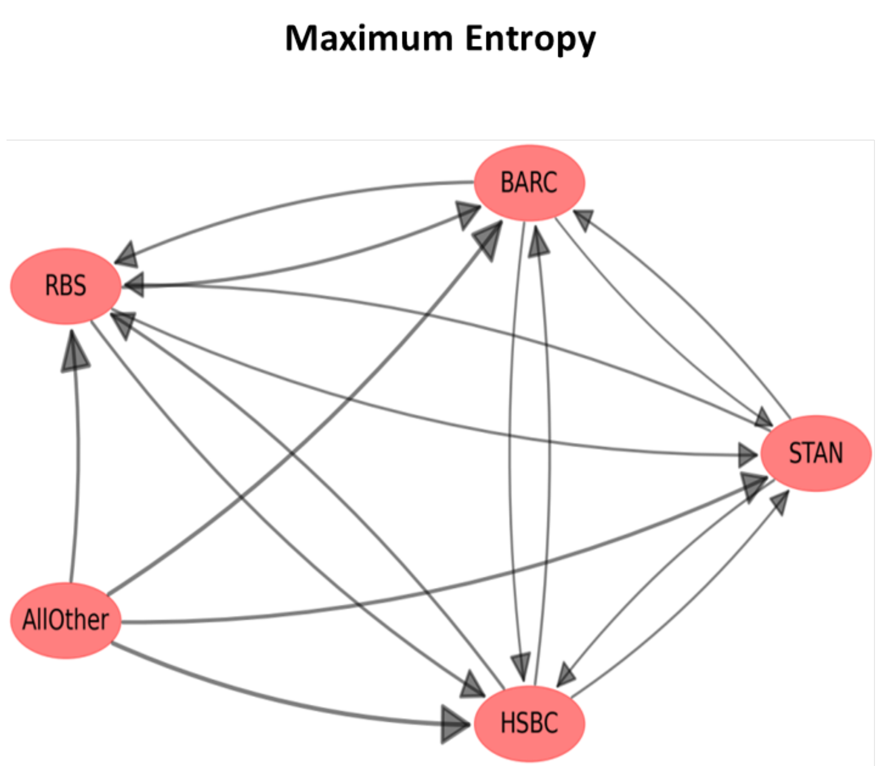
Source: author's calculations.

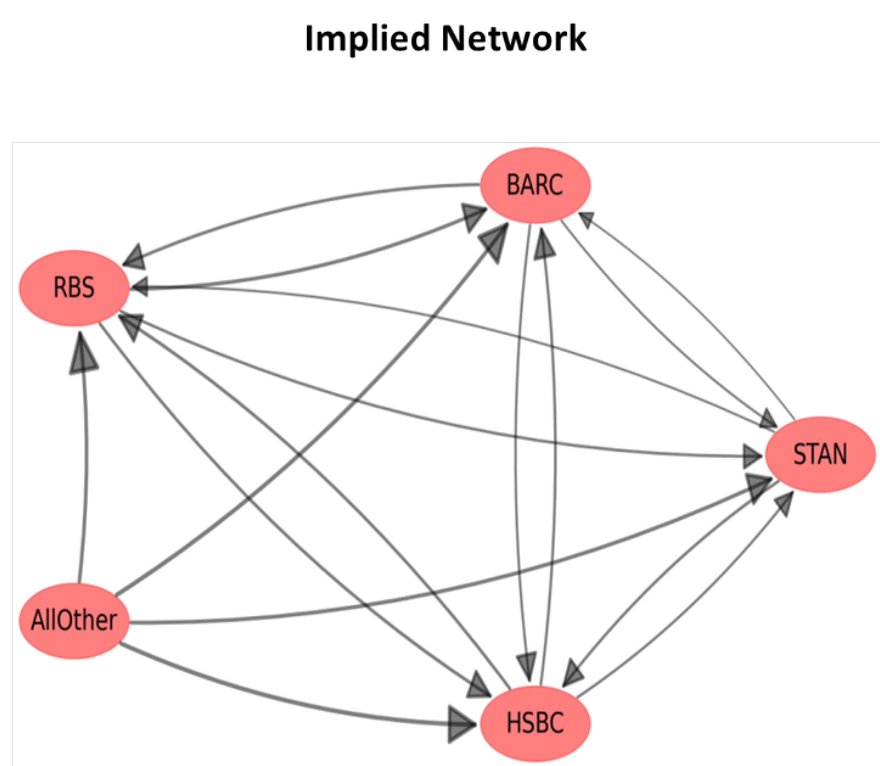
Table 5: DEC2009

	HSBC	RBS	BARC	STAN	Liabilities
HSBC		7	6	-68	
RBS	-22		-1	66	
BARC	29	4		-9	
STAN	-15	-58	-27		

Source: author's calculations.

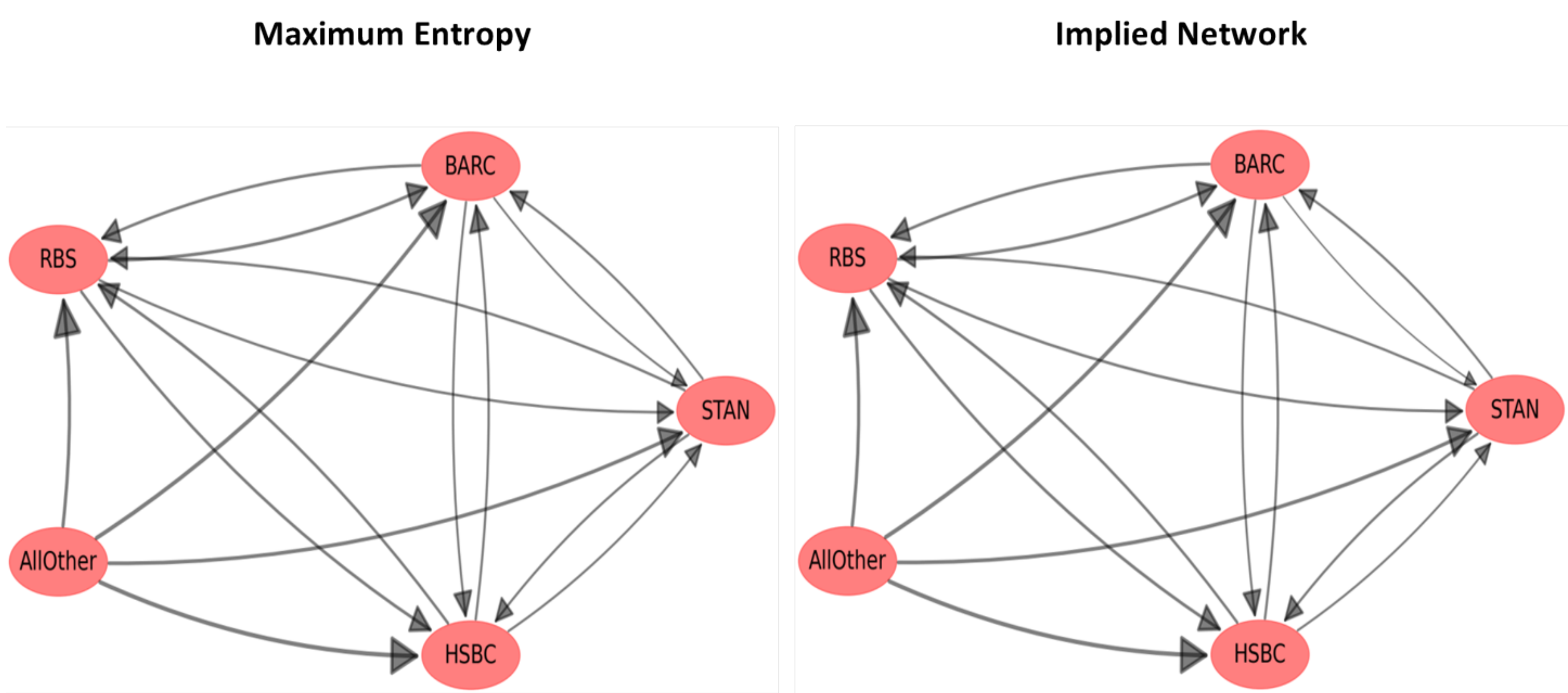
Figure 9: AUG2008





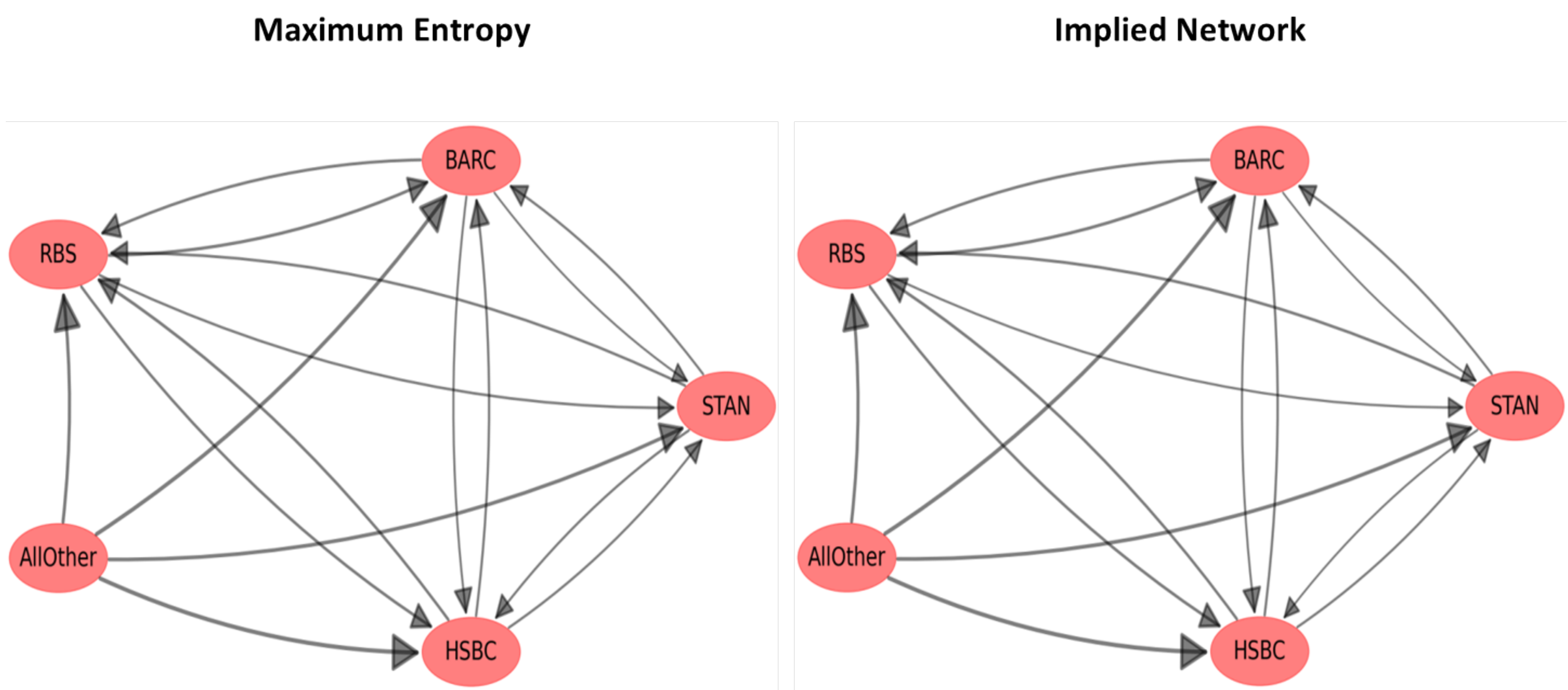
Source: author’s calculations.

Figure 10: DEC2008



Source: author's calculations.

Figure 11: DEC2009



Source: author's calculations.

9 Conclusion

In this paper, I propose an approach to complement balance-sheet information with market data in order to enhance our understanding regarding the expected relevance of the exposures established among banks. This approach translates into finding the set of networks that respect the total amount loaned/borrowed obtained from balance-sheet information and are consistent with market data according to a structural valuation model. As an illustration, I present an empirical application using data for four UK global systemically important banks (G-SIB) referring to the 2007–09 financial crisis period. The empirical application to the UK banking system suggests that in times of stress market participants may find it difficult to discern the network structure of exposures from the maximum uncertainty benchmark (i.e., ME).

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Appendix – Maximum Entropy approach

Consider the following matrix of exposures M, with typical element m_{ij} – the nominal liability of firm i to firm j,

	1	2	...	n	Liabilities $\left(l_i = \sum_{j=1}^{n} m_{ij}\right)$
1	0	m_{12}	...	m_{1n}	$\sum_{j=1}^{n} m_{1j}$
2	m_{21}	0	...	m_{2n}	$\sum_{j=1}^{n} m_{2j}$
...	...	...	...	...	...
n	m_{n1}	m_{n2}	...	0	$\sum_{j=1}^{n} m_{nj}$
Assets $\left(a_j = \sum_{i=1}^{n} m_{ij}\right)$	$\sum_{i=1}^{n} m_{i1}$	$\sum_{i=1}^{n} m_{i2}$	...	$\sum_{i=1}^{n} m_{in}$	

The ME approach approach roughly translates into assuming that banks wish to maximize the dispersion of their interbank assets. Formally, this is achieved by searching for a matrix $\hat{M}$ that solves the following cross-entropy problem:

$$\min_{\hat{m}_{ij}} \sum_{i=1}^{n} \sum_{j=1}^{n} ln\left(\frac{\hat{m}_{ij}}{m^{\star}_{ij}}\right)$$

$$s.t.\ \sum_{j=1}^{n} \hat{m}_{ij} = l_i,\ \sum_{i=1}^{n} \hat{m}_{ij} = a_j,\ \hat{m}_{ij} \geq 0 \forall i \neq j,$$

where $m_{ij}^{\star} = \begin{cases} a_j l_i & \forall\, i \neq j \\ 0 & otherwise \end{cases}$.

This problem can be solved using the RAS algorithm (see Schneider and Zenios 1990).