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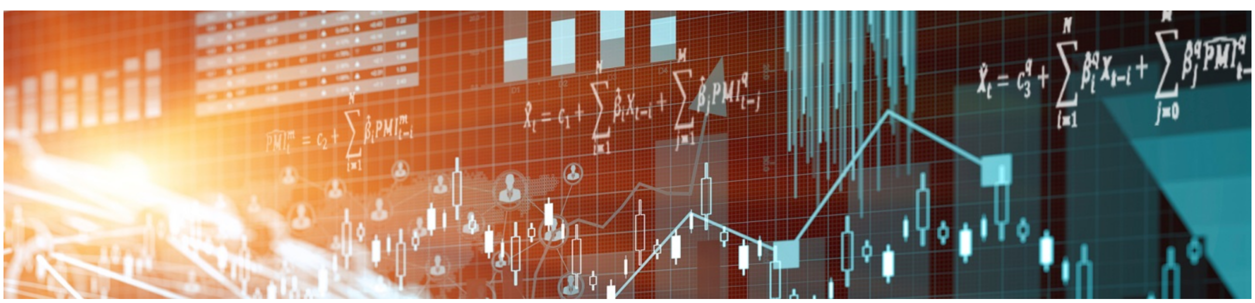

by Elena Goldman and Xiangjin Shen

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Analysis of Asymmetric GARCH Volatility Models with Applications to Margin Measurement

by

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Abstract

We explore properties of asymmetric generalized autoregressive conditional heteroscedasticity (GARCH) models in the threshold GARCH (GTARCH) family and propose a more general Spline-GTARCH model, which captures high-frequency return volatility, low-frequency macroeconomic volatility as well as an asymmetric response to past negative news in both autoregressive conditional heteroscedasticity (ARCH) and GARCH terms. Based on maximum likelihood estimation of S&P 500 returns, S&P/TSX returns and Monte Carlo numerical example, we find that the proposed more general asymmetric volatility model has better fit, higher persistence of negative news, higher degree of risk aversion and significant effects of macroeconomic variables on the low-frequency volatility component. We then apply a variety of volatility models in setting initial margin requirements for a central clearing counterparty (CCP). Finally, we show how to mitigate procyclicality of initial margins using a three-regime threshold autoregressive model.

Bank topics: Econometric and statistical models; Payment clearing and settlement systems
JEL codes: C58, G19, G23, G28

Résumé

Nous étudions les propriétés de modèles asymétriques d'hétéroscédasticité conditionnelle autorégressive généralisée (GARCH) de la famille des modèles GARCH à seuil (TGARCH) et proposons un modèle TGARCH à fonction spline plus général où les termes de l'hétéroscédasticité conditionnelle autorégressive (ARCH) et généralisée rendent compte de la volatilité des rendements de variables à forte fréquence, de la volatilité des variables macroéconomiques à faible fréquence et de la réaction asymétrique aux nouvelles défavorables passées. Une estimation des rendements des indices S&P 500 et S&P/TSX par la méthode du maximum de vraisemblance ainsi qu'une simulation numérique de Monte-Carlo permettent d'arriver à plusieurs constats : le modèle de volatilité asymétrique plus général que nous proposons est mieux adapté, les effets des nouvelles défavorables persistent plus longtemps, le degré d'aversion au risque est plus élevé et les variables macroéconomiques ont une incidence significative sur la composante à faible fréquence de la volatilité. Nous recourons ensuite à divers modèles de volatilité pour établir les marges initiales exigées par une contrepartie centrale. Enfin, nous montrons comment atténuer la procyclicité des marges initiales en faisant appel à un modèle autorégressif à seuil à trois régimes.

Sujets : Méthodes économétriques et statistiques; Systèmes de compensation et de règlement des paiements
Codes JEL : C58, G19, G23, G28

Non-technical summary

The mandatory use of clearing in certain markets is one of the cornerstone regulations introduced to prevent another global financial crisis. Central counterparties (CCPs) base their risk management systems on a tiered default waterfall relying on two types of resources provided by their members: margins and default fund contributions. The initial margins are typically set based on value at risk (VaR) calculations. Since VaR models are typically volatility-based, the properties of the underlying volatility models such as risk aversion are essential for setting initial margin requirements.

There is high degree of procyclicality of margin requirements based on VaR models. On the one hand, there is a need for margins to adjust to changes in the market and be responsive to risk. Thus, margins increase substantially in times of stress and go down when volatility is low. However, this practice produces big changes in margins when markets are stressed, which may lead to liquidity shocks. In addition, in stable times margins may be too low. CCPs try to reduce the procyclicality of their models by setting a floor on the margin.

The current study introduced a flexible volatility model that can capture a high degree of risk aversion as well as effects of macroeconomic variables that can be used for stress testing. The model is extended to reduce procyclicality using a three-regime model rather than ad hoc tools such as setting a 25% floor for the initial margin, as was suggested in the literature. Moreover, unlike other literature, we introduce not only the lower bound (floor) but also the upper bound (ceiling) for the initial margins, as the upper bound is essential at times of liquidity stress in the market. Finally, we define and use a loss function with different degrees of trade-off between two competing objectives of the CCP: risk sensitivity and mitigation of procyclicality.

1 Introduction

The generalized autoregressive conditional heteroscedasticity (GARCH) model and exponentially weighted moving average (EWMA) RiskMetrics model are popular for measuring and forecasting volatility by financial practitioners. Since the ARCH and GARCH models were introduced by Engle (1982) and Bollerslev (1986), there have been many extensions that resulted in better statistical fit and forecasts. For example, GJR-GARCH (Glosten, et al. (1993)) is one of the well-known extensions of GARCH models with an asymmetric term that captures the effect of negative shocks in equity prices on volatility, commonly referred to as the leverage effect. EGARCH introduced by Nelson (1991) is an alternative asymmetric model of the logarithmic transformation of conditional variance that does not require positivity constraints on parameters. Different volatility regimes can be captured by Markov Regime Switching ARCH and GARCH models, allowing for stochastic time variation in parameters. These models were introduced by Cai (1994) and Hamilton and Susmel (1994) respectively.

Since tail risk measures typically incorporate forecasts of volatility, model specification is important. Engle and Mezrich (1995) introduced a way to estimate value at risk (VaR) using a GARCH model, while Hull and White (1998) proved that a GARCH model has a better performance than a stochastic volatility model in the calculation of VaR. The GJR-GARCH model was also used by Brownlees and Engle (2017) among others for forecasting volatility and measurement of tail and systemic risks.

A typical feature of the GARCH family models is that the long-run volatility forecast converges to a constant level. An exception is the Spline-GARCH model of Engle and Rangel (2008) that allows the unconditional variance to change with time as an exponential spline and the high-frequency component to be represented by a unit GARCH process. This model may incorporate macroeconomic and financial variables into the slow-moving component and, as shown in Engle and Rangel (2008), improves long-run forecasts of international equity indices. In this model the unconditional volatility coincides with the low-frequency volatility. The Factor-Spline-GARCH model developed in Rangel and Engle (2012) is used to estimate high- and low-frequency components of equity correlations. Their model is a combination of the asymmetric Spline GJR-GARCH and the dynamic conditional correlations (DCC) models. Another application of an asymmetric Spline GJR-GARCH model for commodity volatilities is in Carpantier and Dufays (2012).

In this paper we generalize the asymmetric Spline-GARCH models using a more general threshold GARCH model as in Goldman (2017). The widely used asymmetric GJR-GARCH model has the problem that the unconstrained estimated coefficient of α often has a negative value for equity indices. A typical solution to this problem is setting the coefficient of α to zero in the constrained maximum likelihood or Bayesian estimation. Following Goldman (2017) we use a generalized threshold GARCH (GTARCH) model where both coefficients, α and β, in the GARCH model are allowed to change to reflect the asymmetry of volatility due to negative shocks. We use data for the US and Canadian equity indices, S&P 500 (SPX) and S&P/TSX (TSX), as well as a numerical example to estimate various asymmetric volatility models. We find that the most general GTARCH model fits better and does not have a negative alpha bias. We also find higher persistence and more risk aversion in the GTARCH models.

We add macroeconomic variables of GDP growth, inflation, overnight interest rate and exchange rate into the spline model for the slow-moving component. The Spline-Macro model results in a smaller number of optimal knots for SPX and has better fit for both SPX and TSX.

Next we apply GTARCH, Spline-GTARCH and Spline-Macro-GTARCH models for VaR and conditional value at risk (CVaR) or Expected Shortfall (ES) estimation. For comparison we also estimate RiskMetrics exponentially weighted moving average (EWMA), GARCH, GJR-GARCH and GTARCH0[1] models. In the latter model that we introduce, the asymmetric effect of negative news is in the GARCH term but not in the ARCH term. We perform backtests and compare the performance of VaR and ES models using the Kupiec (1995) test. We find that all asymmetric volatility models pass the Kupiec test for SPX and TSX data, while EWMA and GARCH fail the test.

The mandatory use of clearing in certain markets is one of the cornerstone regulations introduced to prevent another global financial crisis. However, the rules implemented have not been tested in crisis conditions. CCPs base their risk management systems on a tiered default waterfall relying on two types of resources provided by their members: margins and default fund contributions. The CCPs, by acting as intermediary, have exposure to both the buyer and the seller. The initial margins are typically set by CCPs based on VaR models (Murphy et al. (2016), Knott and Polenghi (2006)).

As documented in Murphy et al. (2014, 2016) and Glasserman and Wu (2017) margin models are typically procyclical and may negatively impact members' funding liquidity at the times of crisis. We explore the procyclicality of initial margin requirements based on VaR volatility models above. On the one hand, there is a need for margins to adjust to changes in the market and be responsive to risk. Thus margins are higher in times of stress and lower when volatility is low. However, this practice may produce big changes in margins when markets are stressed, which in turn may lead to liquidity shocks. In addition, in stable times margins may be too low. CCPs try to reduce the procyclicality of their models by using various methods, including setting floors on margin. Some such methods are discussed in white papers produced by the Bank of England (Murphy et al. (2016)). Their study suggests five tools, including a floor margin buffer of 25% or

[1] GTARCH0 is a subset of the GTARCH model and will be defined in the context below.

greater to be used in times of stressed conditions. We suggest placing both a floor and a ceiling on margins, by using a threshold autoregressive model with three regimes (3TAR), as well as expert judgement based on historical margin settings. We estimate 3TAR for each volatility-based VaR model and discuss the resulting regimes and settings for floor and ceiling. If the margins were allowed to be set within two bounds and the high-volatility regime was not persistent, margins would be stable. Such policy could be also useful to manage expectations at times of stressed liquidity.

The paper is organized as follows. Section 2 presents GTARCH and Spline GTARCH models, maximum likelihood estimation and tail risks. In Section 3 we perform data analysis for S&P500, S&P/TSX and a numerical example with Monte Carlo simulations. In Section 4 we compare tail risks and perform backtests of all models. Next we analyze procyclicality properties and estimate a three-regime TAR model for setting a floor and a ceiling on margins. Section 5 presents the conclusion and further work.

2 Asymmetric Threshold GARCH Models

In this section, we present the generalized threshold GARCH model (GTARCH) and a family of its subset models including GJR-GARCH, GTARCH0 and GARCH. Next we add spline to the GTARCH model extending the analysis of Engle and Rangel (2008).

2.1 The Generalized Threshold GARCH (GTARCH) Model

One of the stylized facts in empirical asset pricing is negative correlation between asset returns and volatility commonly explained by risk aversion and leverage effect. In a popular threshold ARCH or GJR-GARCH model (Glosten, Jagannathan, and Runkle (1993)), a negative return results in an asymmetrically higher effect on the next-day conditional variance compared to a positive return.

Consider time series of logarithmic returns r_t with constant mean μ and the GJR-GARCH conditional variance σ_t^2 given by

$$\begin{aligned} r_t &= \mu + u_t = \mu + \sigma_t \varepsilon_t \\ \sigma_t^2 &= \omega + \alpha u_t^2 + \gamma u_t^2 I(r_{t-1} - \mu < 0) + \beta \sigma_{t-1}^2, \end{aligned} \tag{1}$$

where ε_t are Gaussian (or other distribution) independent random variables with mean zero and unit variance, $I(r_{t-1} - \mu < 0)$ is a dummy variable equal to one when previous-day innovation u_{t-1} is negative, α and β are GARCH parameters, and γ is an asymmetric term capturing risk aversion. The stationarity condition for the GJR-GARCH model is approximately given by: $1 - \alpha - \beta - \frac{1}{2}\gamma > 0$. [2]

However, there is a problem with the threshold ARCH model above since coefficient α may take negative values in practice. In such a case, constrained optimization imposing positivity on all variance parameters results in α equal to zero. Goldman (2017) suggested using a more general Threshold GARCH (GTARCH) model:

$$\sigma_t^2 = \omega + \alpha u_t^2 + \gamma u_t^2 I(r_{t-1} - \mu < 0) + \beta \sigma_{t-1}^2 + \delta \sigma_{t-1}^2 I(r_{t-1} - \mu < 0), \tag{2}$$

where the added term δ reflects the degree of asymmetric response in the GARCH term. In this model both parameters γ and δ create the asymmetric response of volatility to negative shocks. Results below show that allowing both ARCH and GARCH parameters to change with negative news results in better statistical fit and smaller information criteria. Moreover, the GTARCH model not only better captures the leverage effect but also shows higher persistence for negative returns compared to its subset GJR-GARCH model. In addition, the coefficients of μ and ω could be

[2] To be more precise the stationarity condition is given by: $1 - \alpha - \beta - \theta\gamma > 0$, where θ is percentage of observations in the regime with negative innovations $u_t < 0$. In practice, θ is set to 0.5.

allowed to change with the regime of negative news to make the model even more flexible. The GTARCH is a generalized model with the following subset of models: GJR-GARCH ($\delta = 0$), GTARCH0 ($\gamma = 0$) and GARCH ($\gamma = 0$ and $\delta = 0$).

The stationarity condition for the GTARCH model is given by: $1 - \alpha - \beta - \frac{1}{2}\gamma - \frac{1}{2}\delta > 0$.[3] The more general GTARCH model, due to its flexibility of parameters, shows different dynamics for GARCH parameters when the news is negative and allows for higher persistence in the regime of negative news. This in turn takes away the negative bias from α, which measures the reaction to the positive news. At the same time, estimation of extra parameters using the maximum likelihood is a straightforward extension, as shown in Section 2.3.

In addition to GTARCH models we also estimate the EWMA model defined as:

$$\sigma_t^2 = (1 - \lambda) r_{t-1}^2 + \lambda \sigma_{t-1}^2, \tag{3}$$

where λ is a smoothing parameter estimated using maximum likelihood. This model is not stable but is a benchmark for 1-day volatility forecasts with a typical estimate of $\lambda = 0.94$ frequently used in the industry. The EWMA model is popular for measuring tail risks, as will be discussed below.

Finally, there has been growing literature in using intra-daily measures of variance such as realized variance (RV) computed as the sum of squared returns using 5-minute intervals. Andersen et al. (2003) showed that the ARFIMA model can be used for forecasting realized variance. The recent contributions on using these measures for predicting future variance are the HAR model of Corsi (2009) and the model using VIX by Bekaert and Hoerova (2014), among others. Following this literature we evaluate each model in this paper using RV as a benchmark observable variance.

[3] To be more precise, the stationarity condition is $1 - \alpha - \beta - \theta\gamma - \theta\delta > 0$.

2.2 The Spline Generalized Threshold GARCH (Spline-GTARCH) Model

The literature incorporating economic variables for modeling and forecasting financial volatility has been growing. For example, Officer (1973), Schwert (1989), Roll (1988), Balduzzi et al. (2001), and Anderson et al. (2007), among others, found that even though the linkages between aggregate volatility and economy are weak volatility is higher during recessions and post-recessionary stages, and lower during normal periods. Engle and Rangel (2008) introduced the Spline-GARCH model combining high-frequency financial returns and low-frequency macroeconomic variables. The latter paper analyzes the effects of macroeconomic variables on the slow-moving component of volatility using spline. This model releases the assumption of volatility mean reversion to a constant level, which is a property of a stable GARCH model. Instead, the long-run unconditional variance is dynamic.

The Engle and Rangel (2008) Spline-GARCH model is given by the following returns r_t, GARCH variance σ_t^2 and quadratic spline τ_t equations:

$$\begin{aligned} r_t - E_{t-1}r_t &= \sqrt{\tau_t \sigma_t^2} z_t, \\ \sigma_t^2 &= (1-\alpha-\beta) + \alpha\left(\frac{(r_{t-1}-E_{t-1}r_t)^2}{\tau_{t-1}}\right) + \beta\sigma_{t-1}^2, \\ \tau_t &= c\, exp(w_0 t + \sum_{i=1}^{k} w_i((t-t_{i-1})_+)^2 + m_t\gamma), \end{aligned} \tag{4}$$

$$(t-t_i)_+ = \begin{cases} (t-t_i), & \text{if } t \geq t_i, \\ 0, & \text{otherwise,} \end{cases}$$

where z_t is a standard Gaussian white noise process, σ_t^2 is a GARCH process with an unconditional mean of one, m_t is the set of weakly exogenous variables (i.e., macroeconomic variables), and ($t_0 = 0,\ t_1, t_2, ...,\ t_k = T$) is a partition of total number of observations T into k equal subintervals. The constant term in the GARCH equation is equal to $(1-\alpha-\beta)$ due to the normalization of the GARCH process. Since the constant term in the GARCH variance equation is normalized, the long-run (unconditional) variance is determined by the spline. A higher number of knots (k) implies more cycles in the low-frequency volatility, while parameters $w_1,...,w_k$ represent the sharpness of the cycles.

We propose the Spline-GTARCH model that accounts for both asymmetric effect in high-frequency volatility and the slow-moving spline component. Combining the Spline model (4) with the general GTARCH asymmetric volatility model in equation (2) we get:

$$\begin{aligned} r_t &= \mu + \sqrt{\tau_t \sigma_t^2} z_t, \\ \sigma_t^2 &= \omega + \alpha \left(\frac{(r_{t-1}-\mu)^2}{\tau_{t-1}} \right) + \gamma \left(\frac{(r_{t-1}-\mu)^2}{\tau_{t-1}} \right) I(r_{t-1}-\mu<0) \\ &\quad + \beta \sigma_{t-1}^2 + \delta \sigma_{t-1}^2 I(r_{t-1}-\mu<0), \\ \tau_t &= c\, exp(\sum_{i=1}^{k} w_i((t-t_{i-1})_+)^2 + m_t \gamma), \end{aligned} \tag{5}$$

where $\omega = (1-\alpha-\beta-\frac{1}{2}\gamma-\frac{1}{2}\delta)$ and $omega > 0$ if the GTARCH process is stable.

In equation (5) we simplified the return process with a constant μ instead of the time variant conditional mean (which could be easily extended for a different process). In practice we also dropped the constant w_0 in the quadratic spline as it was never significant.[4]

The vector of all jointly estimated parameters in the most general model is $\theta = \{\mu, \alpha, \beta, \gamma, \delta, c, w_1, ..., w_k\}$. We note that Spline-GJR-GARCH, Spline-GARCH and Spline-GTARCH0 (the latter has asymmetry only in the GARCH term) are subsets of the model in equation (4) and will be estimated as part of the analysis.

[4] A similar Spline-GARCH specification with constant μ and $w0 = 0$ is used by NYU Stern VLAB Institute at vlab.stern.nyu.edu.

2.3 Maximum Likelihood Estimation

We use the maximum likelihood estimation (MLE) to jointly estimate parameters in the Spline-GTARCH model: $\theta = \{\mu, \alpha, \beta, \gamma, \delta, c, w_1, ..., w_k\}$. The positivity and stability restrictions on the parameters are given by $\alpha, \beta, \gamma, \delta \geq 0$ and $\alpha + \beta + 0.5\gamma + 0.5\delta < 1$.

Even though we use a Gaussian process for returns in the likelihood function below, the normality assumption is not crucial since asymptotically, a quasi-maximum likelihood approach can be used if returns are not Gaussian.

The likelihood function is the product of probability density functions:

$$f(r_t; \mu, \sigma_t, \tau_t) = \frac{1}{\sqrt{2\pi\tau_t\sigma_t^2}} e^{-\frac{1}{2\sigma_t^2}\frac{(r_t-\mu)^2}{\tau_t}}.$$

We maximize the log likelihood function below to find estimates of $\hat{\theta}$:

$$L(\hat{\theta}) = log(L(r_t|\theta)) = -\frac{T}{2} log 2\pi - \frac{1}{2}\sum_{t=1}^{T}\left(log\sigma_t^2 + log\tau_t + \frac{(r_t-\mu)^2}{\sigma_t^2\tau_t}\right). \tag{6}$$

The number of knots, k, is chosen by minimizing information criteria: Bayesian-Schwartz information criterion $BIC = -2L(\hat{\theta}) + d \times ln(T)/T$ and Akaike information criterion $AIC = -2L(\hat{\theta}) + d \times 2/T$, where d is the dimension of $\hat{\theta}$. In addition to selecting the number of knots in the spline, we use the above criteria for comparing the overall fit of various volatility models discussed in this paper. Due to a higher penalty on a number of parameters, a model choice based on BIC criterion can result in a more parsimonious model.

2.4 Tail Risks

One of the most popular tail risk measures is $q\%$ value at risk (VaR), which is defined as a loss of the portfolio that can be exceeded only with probability $1-q$. For a unit value of the portfolio, essentially VaR is the negative of $(1-q)$ quantile of the distribution of returns, where q is the upper tail probability:[5]

$$P\left(r_t < -VaR_q\right) = 1 - q.$$

Both in-sample and out-of-sample daily VaR can be computed based on the volatility model used for estimating and forecasting of portfolio returns. The VaR is typically computed using either a parametric assumption for the distribution of returns or bootstrapped standardized residuals (also called "filtered historical simulation").

If a parametric assumption is used with a cumulative density function F, the 1-day $q\%$ VaR is given by:

$$VaR_{t+1} = \sigma_{t+1} \times F^{-1}_{(1-q)},$$

where $F^{-1}_{(1-q)}$ is the $(1-q)$ quantile of the distribution of F. If a standard normal distribution is used for F, the daily VaR can be estimated based on $F^{-1}_{(1-q)} = 1.282, 1.645, 2.326$ for $q = 90\%, 95\%$ and 99% respectively.

If the standardized residuals $e_t = \frac{r_t - \mu}{\sigma_t}$ after adjustment for time-varying volatility still have fat tails, the alternative approach is to use bootstrap or filtered historical simulation (FHS) based on Hull and White (1998). They suggest estimating the daily VaR through a filtered process by estimating the F's quantile instead of using the parametric distributional assumption. The estimate of F^{-1}_{1-q} is the $1-q$ quantile of the empirical distribution of the standardized residuals e_t.

In an extreme outcome of $1-q$ probability, the actual loss (L) is larger than VaR, especially when the loss distribution has a very long tail. An alternative commonly used tail risk is conditional VaR (CVaR) or expected shortfall (ES), which measures the expected value of the portfolio loss given that the loss actually exceeded the VaR.

[5] Since VaR is reported as a positive number it is typically measured as negative of a 1%, 5% or 10% quantile.

The ES is given by

$$ES_{1-q} = E(L|L > VaR_{1-q}).$$

Similar to VaR we can apply a parametric or Hull and White (1998, HW) method to estimate the expected shortfall. In the case of normal distribution it is given by

$$ES_{1-q} = \frac{\phi(VaR_{1-q})}{q} \times \sigma_t,$$

where ϕ is standard normal probability density function.

For the HW method, we sort the standardized residuals and find the average of them in the $1-q$ percent tail. Then we multiply this value by the 1-day forecast of volatility.

3 Data Analysis

In this section we perform data analysis for S&P500 (SPX), S&P/TSX (TSX) and a numerical example with Monte Carlo simulations. The results of thirteen estimated volatility models are discussed below. The daily SPX data for the period between 10/08/2002 and 12/30/2016 was obtained from CRSP in the Wharton Database, while the TSX data for the period between 03/17/2003 and 03/31/2017 was obtained from Bloomberg. For both series we found logarithmic returns that resulted in 3500 observations. Realized variances (RVs) computed using 5-minute returns were obtained from the Oxford-Man Institute of Quantitative Finance.[6] The number of observations for the RV was slightly smaller than for daily returns: 3487 observations for SPX and 3480 observations for TSX.

For the spline model with macroeconomic variables we used similar data to Engle and Rangel (2008), including quarterly nominal GDP growth rates for both countries, daily US federal funds effective rate and Canadian overnight money market financing rate, monthly CPI inflation for both

[6] http://realized.oxford-man.ox.ac.uk/

countries, daily Trade Weighted U.S. Dollar Index and USD/CAD exchange rates. We also added monthly unemployment rates for each country. Table 1 provides the description and sources of data for all variables.

Table 1 here

We transformed macroeconomic variables for the Spline-Macro model in the following way. For the CPI, GDP and exchange rates we used log differences, while interest rates and unemployment rates were used at observed levels. We ran the AR(1) model for each variable and found the squared residuals. Using the squared residuals we computed the moving average volatility for each variable. For quarterly GDP data, monthly CPI and monthly unemployment data, we used a 250-day moving average window, while for daily data we used a 25-day window.[7]

Table 2 in Panel A presents the results of estimated simple GTARCH family models without spline for SPX data. We estimate GTARCH with all parameters $(\alpha, \beta, \gamma, \delta)$; GJR-GARCH $(\delta = 0)$, GTARCH0 $(\gamma = 0)$ and GARCH ($\gamma = 0$ and $\delta = 0$). First we performed unconstrained optimization without imposing a positivity constraint on parameters and then we constrained all parameters to be positive. For the unconstrained results we see that $\alpha = -0.0139$ and is statistically significant in the GJR-GARCH model. Clearly the GJR-GARCH model does not effectively capture the risk aversion in a single asymmetric parameter γ shifting α to a negative value in order to distinguish better negative and positive news. However, the interpretation of negative α that positive news reduces volatility in the next period is unintuitive. At the same time, α is positive and not significant in the more general GTARCH model. Since GARCH parameters need to be positive we impose constraints in optimization, which results in estimated α being positive but very close to zero in these models. Most model parameters are not affected by imposing the positivity constraint, except

[7] We experimented with several windows for moving average, including 100 days for daily data, but we presented a 250-day window based on Engle and Rangel (2008) settings with annual estimates.

for the GJR-GARCH model.

In the GTARCH model both coefficients $\gamma = .13$ and $\delta = .16$ are highly significant, showing the asymmetric effect present in both ARCH and GARCH terms and showing higher persistence in the regime of negative news. Based on both AIC and SIC criteria, a GTARCH model is chosen among alternatives. All models satisfy the stationarity condition with overall $persistence = \alpha + \beta + \frac{1}{2}\gamma + \frac{1}{2}\delta < 1$.

In order to check the robustness of the MLE algorithm, we performed Monte Carlo experiments and present an example in Panel B of Table 1. We used previously estimated GTARCH parameters for the SPX as true parameters of the data-generating process in this example. We generated data using equation (2) with 5000 observations and 500 replications. For each replication of the data, we estimated each model in the GTARCH family and presented means and standard errors of the overall results. For the unconstrained optimization we still find a negative α for the GJR-GARCH model, while the GTARCH model has a small positive α. The parameters of the constrained GTARCH model are very close to true values (within one standard deviation). The parameters of subset models (GJR-GARCH, GTARCH0 and GARCH) produce biases due to some dropped parameters in their specification.

Table 2 here

Tables 3 and 4 show results of estimated GTARCH family models including specifications without spline, with spline and with spline and macroeconomic variables in equation (5). Table 3 is for SPX, while Table 4 is for TSX. Only results with imposed positivity constraints are reported.

In addition to the volatility models presented in the table, we estimated the RiskMetrics EWMA volatility model that is commonly used as a benchmark in volatility forecasting and VaR estimation. The MLE for the EWMA model resulted in the following smoothing parameters with standard error given in brackets and information criteria:

SPX: $\lambda = 0.9409$ (0.0049), $AIC = 2.7262, BIC = 2.7279$

TSX: $\lambda = 0.9369$ (0.0055), $AIC = 2.8222, BIC = 2.8240$

The smoothing parameter is very close to .94 in both cases, which is frequently used in practice. Thus, the US and Canadian indices have similar EWMA volatility dynamics.

Based on the BIC information criterion with heavier penalty for extra parameters, the GTARCH model without spline is preferred, while using the AIC criterion, the Spline-Macro-GTARCH is the superior model. This result holds for both SPX and TSX. Note that both selected models include the most general GTARCH specification with the presence of asymmetry in both ARCH and GARCH terms. Moreover, the asymmetric term δ goes up to 0.24 in the SPX Spline GTARCH model, making the response to negative news even more asymmetric compared to GTARCH without spline with $\delta = 0.16$. The optimal number of knots in the SPX Spline model is 17, while the number of knots goes down to 8 when we add macroeconomic variables. Macroeconomic variables are useful in modeling the low-frequency component, as their presence reduces the number of knots for cycles and they have a statistically significant effect on long-run volatility dynamics. Engle and Rangel (2008) use macroeconomic variables for panel regressions of 48 countries using annual volatility data. In our paper, we model macroeconomic variables for daily volatility forecasting in the slow-moving component. Thus, statistically significant macroeconomic variables in the low-frequency component could be used for stress testing of VaRs, which is a typical regulatory requirement. The following variables are statistically significant at 10% for predicting the low-frequency volatility component for SPX:

- Interest rate ($InterestR$) and interest rate volatility ($InterestR_V$), both which have a positive effect on SPX volatility

- Volatility of the unemployment rate ($unempv_V$), which has a negative effect on SPX volatility

- Volatility of USD trade weighted index (USD_V), which has a positive effect on SPX volatility

- GDP growth, which has a negative effect on SPX volatility

All the signs are as expected except for the volatility of the unemployment rate. It might be the case that the reduction in unemployment rate rather than an increase in it is driving these results.

As for the Canadian data, fewer macroeconomic variables were found significant at 10% and the optimal number of knots stayed the same (15 knots) after the macroeconomic variables were added:

- Inflation Volatility ($Inflation_V$), which has a negative effect on TSX volatility

- Interest rate volatility ($InterestR_V$), which has a negative effect on TSX volatility

- Volatility of USD/CAD exchange rate ($USDCAD_V$), which has a positive effect on TSX volatility

While the effect of Canadian dollar exchange rate volatility has an expected positive sign, the negative signs for volatilities of inflation and interest rate are not intuitive. The Canadian overnight interest rate was volatile before the crisis in our sample between 2003 and 2007. During the 2008-2009 financial crisis and afterwards, the interest rate was stable. Thus, during the time of high volatility for equities, interest rates were not volatile compared to the previous period. Similarly, while the US experienced the largest volatility of inflation (from inflation to deflation) during the 2008-2009 financial crisis, the largest drops in consumer prices in Canada happened in Quarter 1 2008 (before the crisis) and in Quarter 3 2012, which were relatively calm periods in the equity market.

The Spline-Macro model has lower persistence than the Spline model and No-Spline model. Spline-Macro models thus have faster convergence of variance to the long-run spline macroeconomic component. This is because the long-run component is not as smooth as in the simple spline model.

Table 5 shows the degree of risk aversion in each model measured by the correlation between returns r_{t-1} and log difference of fitted conditional variance $log(\sigma_t^2/\sigma_{t-1}^2)$ for each model. The more negative correlation implies a higher degree of risk aversion because of asymmetrically higher volatility for negative returns. For comparison, the log difference in the VIX index has a correlation with the S&P 500 return of about -0.7. Table 5 shows that the highest degree of risk aversion is captured by the GTARCH models and the smallest correlation is for the EWMA, GARCH and RV that are symmetrical.

Tables 3, 4, and 5 here

Figures 1, 2, 3, and 4 here

Figures 1 and 2 show annualized GTARCH volatilities for SPX data while Figures 3 and 4 present similar graphs for TSX. First we present the Spline-GTARCH volatility compared to a simple GTARCH in Figures 1 and 3. Next we compare Spline-Macro-GTARCH volatility to a simple GTARCH in Figures 2 and 4. We can see that the low-frequency component is smooth for both SPX and TSX data in the Spline-GTARCH model and the high-frequency component is close but generally higher than GTARCH. Once the macroeconomic variables are added, the dynamics of low-frequency volatility become much less smooth. This is due to reaction to macroeconomic volatility in turbulent times affecting the long-run volatility component. The reaction to the negative news is also amplified by the asymmetric effect in the GTARCH model.

Overall, the US and Canadian market volatilities have similar dynamics and peaks; however, the Canadian market has a lower level of volatility. For the low-frequency spline component, the highest level during the financial crisis was 17% for TSX compared to over 40% for SPX. Similarly, the high-frequency TGARCH volatility peaks in the US market are more than twice those of the Canadian market.

Figures 5 and 6 show RV versus GTARCH for SPX and TSX. We observe that RV is more procyclical compared to GTARCH as RV graphs exhibit higher peaks and lower levels in calm periods.

Figures 5 and 6 here

4 Initial Margin Measures

In this section we compute tail risks and perform backtests of all models. Next we analyze initial margin models' procyclicality and estimate a three-regime threshold autoregressive model (3TAR) for setting a floor and a ceiling on margins.

4.1 Properties of Tail Risks for Setting Margin Requirements

Figures 7 and 8 show the logarithmic returns in red and negative values of 1-day 99% value at risk (VaR) for SPX and TSX respectively. We generated 1-day 99% VaRs using the Hull and White (1998) bootstrap method (the blue line) and the normal distribution (the red line). We used the Spline-GTARCH model on these graphs while all other models are reported in Tables 6 and 7. The margin requirements with the Hull and White method are higher because this method uses the actual returns distribution with fat tails compared to normal distribution.

Tables 6 and 7 here

Figures 7 and 8 here

Table 6 presents 1- to 3-day forecasts of all volatility models, VaR and expected shortfall (ES) produced by each model for SPX and TSX at the time of low volatility at arbitrarily selected dates in 2016 and 2017. Table 7 reports the same results at the time of high volatility in the fall of 2008. Margins are usually computed over some period of time greater than one day. For example, exchange traded assets are cleared within 2-3 days in the US. Thus, in Tables 6 and 7 we presented 1- to 3-day tail risks that can be easily extended to longer periods. One-day VaR and ES at $q = (90\%, 95\%, 99\%)$ are reported using the Hull and White (1998) method.[8] In order to compute t-day VaR and ES we used $\sqrt{t}$ adjustment based on Basel requirement and common practice.[9] Monte Carlo simulations would be an interesting extension of the method, especially for longer time horizons.

The results for the SPX and TSX data in Table 6 show increasing volatility forecasts from 1 to 3 days since the starting point is at the time of low volatility and volatility is mean-reverting.[10] Similarly, volatility forecasts go down in Table 7 when we start in a high-volatility period. While there is no one specific model that always has the highest volatility forecast and tail risks among reported models, those with asymmetric terms (GTARCH, GJR-GARCH and GTARCH0) produce higher forecasts and tail risks than symmetrical GARCH and EWMA models. Thus, models accounting for risk aversion such as GTARCH are useful to make sure that volatility is well measured and sufficient margin requirements are set.

Tables 6 and 7 also illustrate that models with spline have smaller volatility forecasts than models with Spline-Macro at the time of low volatility. The opposite is true at the time of high

[8] To save space, we did not report the results with normal distribution which have similar patterns but lower estimates, as shown in Figures 7 and 8.

[9] The $\sqrt{t}$ multiplier is correct only under the assumption of independence in returns.

[10] This is true for all models except EWMA, which is not stationary and thus can produce only 1-day volatility forecast.

volatility. Thus, Spline-Macro models turn out to be less procyclical. This could be explained by countercyclical monetary policy as well as faster convergence to a less smooth long-run spline macroeconomic component as was found from lower persistence of Spline-Macro models in Tables 3 and 4.

4.2 Model Validation

Backtesting is often used in practice for model validation. The testing window is set to evaluate the number of VaR violations (or breaches) and compare it to the expected number of violations for a specific VaR quantile. For example, if VaR is measured with $q = 99\%$ the expected number of violations is 1%. Considering the whole sample size of N=3500 observations, we would expect 35 violations.[11] If the actual breach rate turns out to be too high the VaR margin model underestimates risk, which creates a loss for the CCP. Alternatively, if the breach rate is too low the VaR model overestimates risk and results in unnecessary high margin charges for the members of the CCP. Thus, margins can be set based on VaR that has a reasonable number of backtest violations falling within some confidence interval.

The most popular backtesting statistical test used in practice is the Kupiec (1995) proportion of failures (POF) test, with the null hypothesis that the breach rate is equal to expected $(1-q)\%$ quantile. The two-sided test has asymptotic likelihood ratio statistics with chi-square distribution and one degree of freedom $X^2(1)$.

Table 8 presents the results of backtesting with the number of breaches for the 90%, 95% and 99% VaRs of SPX and TSX produced by each volatility model for the whole sample period. The table also shows 95% confidence intervals with lower and upper bounds for the number of allowed breaches using the Kupiec test. We report the results for VaRs using the Hull and White (1998)

[11] A common criticism of backtesting is the small expected number of violations if the testing window is not large enough.

filtered historical simulations method to make sure that risk is not underestimated compared to models that use normal distribution.

Table 8 here

The results in Table 8 show that all VaRs that use asymmetric volatility models (GTARCH, GJR-GARCH and GTARCH0) with various quantiles ($q = 90\%, 95\%, 99\%$) pass the Kupiec test at a 5% significance level for both SPX and TSX. At the same time, we find that the EWMA model failed the test underestimating risk for both SPX and TSX for each quantile q. The GARCH model fails the Kupiec test only for SPX data with $q = 90\%$ quantile overestimating risk.[12]

Conditional coverage backtests introduced by Christoffersen and Pelletier (2004) can be added.[13] However, past research showed low power of all backtests above (see, e.g., Lopez (1998)). Moreover, backtesting is only concerned with the number of exceptions and their independence. Regulators are also concerned with the magnitude of exceptions (margin shortfall) as well as excessive procyclicality of VaR models that increase the speed of margin calls at the time of crisis, as will be discussed in Section 4.3. Recognizing the limitations of backtests, we use the above results in conjunction with other model validation criteria.

In addition to backtesting we ran realized volatility regressions for assessing model performance similar to Corsi (2009) and Bekaert and Hoerova (2014). Table 9 shows one-day-ahead in-sample performance using the regressions of log realized variance (RV) on log variances estimated by each model. This table presents the mean square error (MSE), the mean absolute error (MAE) and Mincer-Zarnowitz (MZ) adjusted R^2. Since the spline model has low- and high-frequency components, both were used for spline model regressions and adjusted R^2 are reported. We find

[12] We note that if a 1% significance level is used, the GARCH model passes the Kupiec test but the EWMA model still fails.

[13] We performed the Christoffersen test as well and found that all models pass the conditional coverage test but GARCH models fail the independence test at a 5% significance level.

that the TGARCH model performs better than all other models for SPX using all three statistics: MSE, MAE and MZ R^2. For TSX the results are mixed, with GTARCH and GJR-GARCH models performing better than other models.

The backtesting results and forecast evaluations using MSE, MAE and MZ R^2 statistics reinforce the need to use asymmetric volatility models capturing risk aversion to make sure that margins are set adequately.

Table 9 here

4.3 Procyclicality of the CCP's Initial Margin Requirements

Central counterparties (CCPs) base their risk management systems on a tiered default waterfall relying on two types of resources provided by their members: margins and default fund contributions. The initial margins are typically set based on VaR calculations. The CCPs, by acting as intermediary, have exposure to both the buyer and the seller. Since VaR and ES calculations are typically volatility-based, the properties of the underlying volatility models such as risk aversion are essential for setting initial margin requirements.

This section explores the procyclicality of margin requirements based on VaR models and suggests remedies to reduce procyclicality. On the one hand, there is a need for margins to adjust to changes in the market and be responsive to risk. Thus, margins increase substantially in times of stress and go down when volatility is low. However, this practice may produce big changes in margins when markets are stressed which, in turn, may lead to liquidity shocks. Brunnermeier and Pedersen (2009) showed that margins can be destabilizing, with stresses in market and funding liquidity leading to liquidity spirals. In addition, in stable times margins may be too low. CCPs try to reduce the procyclicality of their models by using various methods, including setting floors on margin. Some such methods are discussed in white papers produced by the Bank of England

(Murphy et al. (2016)) and the European Securities and Markets Authority (EMIR) Regulatory Technical Standards (2015). They suggest several ad hoc tools to create margin buffers and reduce procyclicality. The first tool is setting a floor margin buffer of 25% or greater to be used in times of stressed conditions.[14]

We suggest placing both a floor and a ceiling on margins, by using a threshold autoregressive model with three regimes, as well as expert judgement based on historical margin settings. For example, we can evaluate the appropriateness of the suggested 25% margin buffer for maintaining funding liquidity under a stressed market. We illustrate the use of this method below.

The threshold autoregressive model (TAR) (also called self-exciting threshold model (SETAR)) was first introduced by Tong (1983) and Tong and Lim (1980). A smooth transition model (STAR) was later developed by Terasvirta (1994).

Consider a time series of logarithm of VaR, $y_t = log(VaR)$, with three regimes. A simple threshold autoregressive model (TAR) with p lags for y_t is given by:

$$\begin{aligned} y_t &= \phi_0^j + \phi_1^j y_{t-1} + \ldots + \phi_p^j y_{t-p} + \varepsilon_t \\ \varepsilon_t &\sim \mathrm{N}\left(0, \sigma^2\right), \end{aligned} \tag{7}$$

where $j = 1, \ldots K$ with number of regimes $K = 3$. The regimes are determined by an observable threshold variable z_{t-d} with delay parameter d and sorted threshold values $\theta_1, \ldots, \theta_{K-1}$, such that

$$\begin{cases} j = 1 & z_{t-d} < \theta_1, \\ j = 2 & \theta_1 \leq z_{t-d} \leq \theta_2, \\ j = 3 & z_{t-d} > \theta_2. \end{cases}$$

In practice, we use $z_{t-d} = y_{t-d}$ and we set the delay parameter for the threshold variable equal to one ($d = 1$). We also use $p = 2$ for the order of the autoregressive model. Alternatively, these

[14] Other suggested tools are to assign at least 25% weight to stressed observations, setting a floor based on the maximum volatility over a 10-year historical look-back period and setting speed limits on how fast the margins can be raised and lowered.

parameters as well as the number of regimes K could be found by minimizing information criteria.

While there are various methods[15] to estimate this model, the commonly used classical method is a grid search for optimal thresholds $\theta_1, ..., \theta_{K-1}$ by minimizing the sum of squared residuals.

We estimated a TAR with three regimes (3TAR), and two corresponding thresholds for the logarithm of VaR for each model. VaR was previously estimated using the Hull and White (1998) method. We used log(VaR) for estimation of the 3TAR model since log transformation smoothes the peaks. Then we exponentially transformed the threshold values and reported them in Table 10.

The results for thresholds for all volatility models are given in Table 10 and are presented graphically as horizontal lines in Figures 9 and 10 for log(VaR) in the Spline-GTARCH model for SPX and TSX respectively.

Table 10 here

Figures 9 and 10 here

The 3TAR model provides a straightforward method of setting both the floor and the ceiling for the initial margin that is stable and not too procyclical: the one-day margins are on average bounded between 1.84% and 2.58% for SPX and between 0.77% and 1.01% for TSX. This way, when volatility is low the margins are fixed at a conservative floor level that corresponds historically to about 29% quantile of the lowest margins for SPX, and at the time of market stress they can't go above the upper threshold. It is an interesting coincidence that the estimated lower threshold for SPX using the EWMA model corresponds to the 25% of observations in the lower regime, as was also suggested by Murphy et al. (2016). For TSX, the margin buffer is a higher 32% of observations on average. On the other hand, at the time of stress the higher regime thresholds on

[15]For example, Goldman et al. (2013) introduced a Bayesian method for measuring thresholds and long memory parameters in a more sophisticated threshold model.

average correspond to 38% of the observation points for both SPX and TSX, which may not appear too conservative. One could add here actual historical margins set by CCPs at the time of stress, to see if the upper bound was historically higher and would have resulted in a lower percentage of observations for the high regime.[16] For comparison we also estimated a three-regime model for the VaR constructed using realized volatility, which resulted in thresholds of 1.57% and 2.27% for SPX and 0.52% and 1.02% for TSX. These thresholds result in approximately 25% of observations in the upper regime, which might be more conservative. Having said this, compared to other models the VaRs based on realized volatility measures are much more procyclical with higher peaks and potentially higher margin calls exactly when the market is in distress.

In order to guarantee that the margin floors and ceilings would be sufficient at the time of crisis, we need to make sure that the time series of VaRs in the regime of high volatility are stationary and revert back inside the bounds. The unit root test results indicate that all models pass the stability test for the SPX, while EWMA and all spline models without macroeconomic variables could be unreliable to set a sustainable ceiling.

We set the floor and the ceiling on the VaR using estimated thresholds in Table 10. If the margins were allowed to be set within two bounds and the high-volatility regime was not persistent, margins would be stable. Such policy could also be useful to manage expectations at times of stressed liquidity. This model is the limiting case for mitigating procyclicality while sacrificing risk sensitivity.

In order to evaluate initial margin models in addition to the backtesting and volatility forecast evaluation performed in section 4.2, we minimize a loss function with two competing objectives: risk sensitivity (model accuracy) and mitigation of procyclicality. We note that in a recent paper

[16] Actual margins data are confidential and we are not able to make this comparison. However, we use two competing objectives: risk sensitivity and mitigation of procyclity, when we compute loss functions in Table 11.

Wong and Ge (2017) used time series similarity relative to a theoretical "regulatory target model" for measuring the initial margin model performance. We do not use any assumed "correct" model and minimize loss function while performing a sensitivity analysis of the loss to a different trade-off parameter w discussed below.

We introduce a parameter $0 \leq w \leq 1$ that measures the degree of trade-off between these two objectives. The higher the w, the more weight is given to procyclicality correction with the limiting case of $w = 1$. If $w = 0$ there is no correction of procyclicality and the whole weight is given to the objective of model accuracy with the most risk-sensitive model. The regulator/CCP may have different preferences for the trade-off parameter w that may result in more or less model accuracy versus mitigation of procyclicality.

Let the loss function $L(w)$ be defined as a quadratic measure of margin shortfall. We define the overall loss function as the weighted sum of two components with trade-off parameter w:

$$L(w) = (1-w)L_1 + wL_2. \tag{8}$$

Here, L_1 is the loss for the unconstrained VaR, while L_2 is the loss for the Threshold VaR (TVaR) bounded between the floor and the ceiling estimated in Table 10.

$$L_1 = \sum_{t=t_0}^{T} (r_t + VaR_t)^2 \, I(r_t < -VaR_t),$$

$$L_2 = \sum_{t=t_0}^{T} (r_t + TVaR_t)^2 \, I(r_t < -TVaR_t),$$

where r_t is one-day logarithmic return, VaR_t is a one-day 99% VaR forecasted for time t, T is the number of observations in the sample, t_0 is the first observation for which a VaR forecast is available, and $I()$ is an indicator function equal to one when there is a violation of VaR and zero otherwise.

Table 11 presents the results of loss functions and the number of violations produced by each volatility model. Overall ranks for loss functions L_1 and L_2 are presented in columns 7 and 8. These ranks correspond to loss L with $w = 0$ and $w = 1$ respectively. We also compute loss function in equation (8) for various other settings of $w = .25, .5, .75$. We rank the overall loss function within each group of models: without spline, with spline and with spline and macroeconomic variables. Ranks for each group using various trade-off parameters w ranging from 0 to 1 are presented in columns 9-13.

The realized volatility is the most risk-sensitive measure for SPX and it is ranked as #1 for L_1. However, as we observe RV ex-post and do not produce a forecast for it, it is not surprising that it performs the best.[17] [18] For the loss L_2 with the highest preference for correcting procyclicality we find that GTARCH0 has the best rank. For each group of models (no spline, spline, spline-macro), as we increase the trade-off parameter (columns 9-13) we can see that in the beginning, model ranks change between $w = 0$ and $w = .25$, but then ranks are stable for all other weights: $w = .25, .75, 1$. Thus, the exact value of w is not crucial for model selection and even a small weight for mitigating procyclicality (.25) produces sufficient information. The best performing models for SPX in each group using some degree of procyclicality mitigation ($w = .25$ or above) are: GTARCH0, Spline-GTARCH and Spline-Macro-GARCH. The best performing models for TSX in each group are: GTARCH0, Spline-GTARCH0 and Spline-Macro GJR-GARCH.

Table 11 here

With the exception of Spline-Macro-GARCH model for SPX, we find that asymmetric volatility models in the GTARCH family are generally preferred for trade-off parameters varying from

[17] In this paper we use RV simply as observable volatility and are not concerned with modeling and forecasting it, which is extensively studied in the literature.

[18] Surprisingly, RV for TSX was actually performing worse than other models for L_1.

$w = .25$ to 1. Adding macroeconomic variables in the GARCH model helps reduce model procyclycality.[19]

The mandatory use of CCPs in certain markets is one of the cornerstone regulations introduced to prevent another global financial crisis. However, the rules implemented have not been tested in crisis conditions. Above, we presented a simple approach to test the sustainability of margin models using a three-regime threshold autoregressive model.

5 Conclusion and Further Development

In this paper we considered asymmetric GARCH models in the threshold GARCH family and proposed a more general Spline GTARCH model that captures high-frequency return volatility, low-frequency macroeconomic volatility as well as an asymmetric response to past negative news in both ARCH and GARCH terms.

Based on maximum likelihood estimation of S&P 500 returns, S&P/TSX returns and the Monte Carlo numerical example, we found that the proposed more general asymmetric volatility model has better fit, higher persistence of negative news, higher degree of risk aversion and significant effects of macroeconomic variables on the low-frequency volatility component.

We then applied a variety of volatility models including asymmetric GARCH, GARCH and EWMA in setting initial margin requirements for central clearing counterparties (CCPs). Since VaR and ES calculations are typically volatility-based, the properties of the underlying volatility models such as risk aversion are essential for setting initial margin requirements.

Finally, we showed how to mitigate procyclicality of initial margins using a three-regime threshold autoregressive model. We set the floor and the ceiling on the VaR using estimated

[19] At the same time, simpler models with procyclicality correction may be occasionally preferred to models with more parameters.

thresholds. This model is the limiting case for mitigating procyclicality while sacrificing risk sensitivity. In order to evaluate initial margin models in addition to backtesting and volatility forecast evaluation, we introduced a loss function with two competing objectives: risk sensitivity and mitigation of procyclicality. The trade-off parameter between these objectives can be selected by regulator/CCP depending on the specific preferences. We found that asymmetric volatility models generally perform better under various trade-off parameters.

In future research, more international equity markets can be tested and additional macroeconomic variables can be added to the spline. The VaR bootstrap algorithm can be modified to one with rolling windows. The multi-day VaR and ES multiplier could be computed using Monte Carlo simulations. In terms of margin procyclicality mitigation, a three-regime threshold autoregressive model with changing volatility and a Markov-Switching model with three regimes could be applied as well.

Table 1: **Definitions of Variables and Data Sources**

Definition	Frequency	Source
US Data		
S&P 500 Index	Daily (business)	CRSP Wharton Database
US federal funds effective rate	Daily (business)	FEDL01 Index (Bloomberg)
US nominal GDP	Quarterly	U.S. Department of Commerce, BEA
US CPI, chained	Monthly	U.S. Bureau of Labor Statistics
Unemployment rate	Monthly	U.S. Bureau of Labor Statistics
Trade Weighted U.S. Dollar Index: Major Currencies	Daily	DTWEXM St Louis FED
Canadian Data		
S&P/TSX Composite Index	Daily (business)	SPTSX Index (Bloomberg)
Canadian overnight money market financing rate	Daily (business)	CAOMRATE Index (Bloomberg)
Canadian nominal GDP	Quarterly	CANSIM table 380-0064
Canadian CPI	Monthly	CANSIM table 326-0022
Unemployment rate	Monthly	CANSIM table 282-0087
Units of USD per CAD	Daily(business)	CAD-USAD X-RATE-Price (Bloomberg)

Table 2: **Estimation Results for GTARCH Models: SPX and Monte Carlo Example**

Parm		GTARCH				GTARCH0				GJR-GARCH				GARCH			
		unconstrained		constrained		unconstrained		constrained		unconstrained		constrained		unconstrained		constrained	
		parm	Std	parm	Std	parm	Std	parm	Std	parm	Std	parm	Std	parm	Std	parm	Std
Panel A: SPX Results																	
μ		0.0076	(0.0126)	0.0003	(0.0000)	0.0256	(0.0126)	0.0006	(0.0000)	0.0164	(0.0137)	0.0184	(0.0134)	0.0546	(0.0130)	0.0546	(0.0140)
ω		0.0218	(0.0029)	0.0226	(0.0037)	0.0220	(0.0030)	0.0226	(0.0031)	0.0227	(0.0032)	0.0238	(0.0039)	0.0238	(0.0039)	0.0238	(0.0040)
α		0.0007	(0.0081)	0.0000	(0.0130)	0.0833	(0.0085)	0.0780	(0.0082)	-0.0139	(0.0069)	0.0000	(0.0194)	0.1015	(0.0110)	0.1015	(0.0110)
β		0.8357	(0.0153)	0.8374	(0.0187)	0.7823	(0.0150)	0.7887	(0.0153)	0.8978	(0.0114)	0.8879	(0.0187)	0.8755	(0.0123)	0.8755	(0.0125)
γ		0.1370	(0.0173)	0.1398	(0.0197)					0.1849	(0.0189)	0.1745	(0.0213)				
δ		0.1634	(0.0240)	0.1596	(0.0248)	0.2460	(0.0237)	0.2485	(0.0246)								
Persistence		0.9866		0.9871		0.9886		0.9909		0.9763		0.9752		0.9770		0.9770	
BIC		2.3232		2.3239		2.3444		2.3442		2.3344		2.3353		2.3714		2.3714	
AIC		2.3126		2.3133		2.3356		2.3354		2.3256		2.3265		2.3644		2.3644	
Panel B: Monte Carlo Simulations																	
μ	0.0076	0.0030	(0.0395)	0.0100	(0.0111)	0.0196	(0.0156)	0.0205	(0.0135)	0.0145	(0.0150)	0.0164	(0.0129)	0.0594	(0.0129)	0.0592	(0.0131)
ω	0.0218	0.0275	(0.0516)	0.0222	(0.0027)	0.0244	(0.0229)	0.0221	(0.0036)	0.0231	(0.0037)	0.0231	(0.0036)	0.0262	(0.0057)	0.0259	(0.0062)
α	0.0007	0.0026	(0.0227)	0.0039	(0.0058)	0.0725	(0.0090)	0.0728	(0.0098)	-0.0095	(0.0046)	0.0006	(0.0022)	0.1282	(0.0139)	0.1267	(0.0188)
β	0.8357	0.8290	(0.0622)	0.8313	(0.0121)	0.7691	(0.0627)	0.7749	(0.0155)	0.8911	(0.0183)	0.8841	(0.0177)	0.8539	(0.0141)	0.8555	(0.0202)
γ	0.1370	0.1375	(0.0177)	0.1342	(0.0138)					0.0145	(0.0104)	0.0164	(0.0086)				
δ	0.1634	0.1612	(0.0392)	0.1676	(0.0224)	0.2669	(0.0454)	0.2710	(0.0240)								
Persistence	0.9866	0.9809		0.9860		0.9751		0.9832		0.8888		0.8929		0.9821		0.9822	
BIC		2.3869		2.3812		2.4094		2.3997		2.3932		2.3939		2.4394		2.4418	
AIC		2.3763		2.3707		2.4006		2.3909		2.3844		2.3851		2.4323		2.4347	

Notes: Panel A presents the results of estimation of GTARCH models for SPX data between 10/08/2002 and 12/30/2016 with 3500 observations. Panel B presents results of Monte Carlo simuations using parameters of estimated SPX model for data generating processes. We used a sample size of N=5000 and 500 replications.

Table 3: **Estimation Results for GTARCH, Spline-GTARCH and Spline-Macro-GTARCH Models: SPX**

Parm	GTARCH						GTARCH0						GJR-GARCH						GARCH					
	SMacro		Spline		No Spline		SMacro		Spline		No Spline		SMacro		Spline		No Spline		SMacro		Spline		No Spline	
	parm	Std	parm	Std	parm	Std	parm	Std	parm	Std	parm	Std	parm	Std	parm	Std	parm	Std	parm	Std	parm	Std	parm	Std
μ	0.013	(0.015)	0.008	(0.015)	0.000	(0.000)	-0.726	(0.154)	0.026	(0.014)	0.001	(0.000)	0.028	(0.013)	0.025	(0.012)	0.018	(0.013)	0.058	(0.013)	0.058	(0.013)	0.055	(0.014)
ω					0.023	(0.004)					0.022	(0.003)					0.023	(0.003)					0.023	(0.003)
α	0.000	(0.023)	0.000	(0.022)	0.000	(0.013)	0.061	(0.018)	0.065	(0.006)	0.078	(0.008)	0.000	(0.000)	0.000	(0.000)	0.000	(0.019)	0.089	(0.010)	0.097	(0.009)	0.102	(0.011)
β	0.781	(0.030)	0.771	(0.028)	0.837	(0.019)	0.729	(0.033)	0.734	(0.015)	0.789	(0.015)	0.841	(0.016)	0.840	(0.013)	0.888	(0.019)	0.826	(0.018)	0.833	(0.015)	0.875	(0.012)
γ	0.122	(0.023)	0.131	(0.025)	0.140	(0.020)							0.177	(0.020)	0.189	(0.017)	0.175	(0.021)						
δ	0.230	(0.035)	0.243	(0.033)	0.160	(0.025)	0.316	(0.031)	0.310	(0.025)	0.249	(0.025)												
c	0.936	(0.289)	2.718	(0.222)			0.846	(0.513)	1.800	(0.300)			0.754	(0.193)	2.246	(0.418)			0.640	(0.158)	1.873	(0.373)		
w_1	-0.711	(0.118)	-1.973	(0.236)			-0.726	(0.154)	-1.454	(0.370)			-0.760	(0.106)	-2.215	(0.426)			-0.753	(0.107)	-1.994	(0.462)		
w_2	0.854	(0.251)	4.110	(0.626)			0.821	(0.448)	2.860	(1.030)			1.032	(0.241)	4.587	(1.211)			0.891	(0.222)	4.196	(1.281)		
w_3	2.106	(0.567)	-2.430	(0.712)			2.365	(0.838)	-1.300	(1.157)			1.854	(0.618)	-2.706	(1.591)			2.301	(0.566)	-2.589	(1.516)		
w_4	-4.443	(0.743)	-0.008	(0.700)			-4.796	(0.936)	-0.894	(0.900)			-4.267	(0.804)	0.185	(1.765)			-4.759	(0.759)	0.099	(1.424)		
w_5	2.831	(0.455)	1.669	(0.679)			2.918	(0.468)	2.948	(0.872)			2.752	(0.486)	1.645	(1.704)			2.917	(0.464)	2.109	(1.436)		
w_6	-1.002	(0.398)	-1.607	(0.687)			-0.872	(0.569)	-3.167	(1.101)			-0.972	(0.380)	-2.343	(1.519)			-0.964	(0.353)	-2.689	(1.521)		
w_7	0.736	(0.527)	1.628	(1.152)			0.651	(0.791)	3.230	(1.157)			0.837	(0.405)	3.429	(1.782)			0.911	(0.365)	3.071	(1.652)		
w_8	-0.859	(0.728)	-5.859	(1.744)			-1.153	(1.161)	-7.984	(1.153)			-1.130	(0.559)	-9.085	(2.101)			-1.532	(0.506)	-8.526	(2.104)		
w_9			6.568	(1.585)					8.707	(0.996)					10.785	(1.950)					10.411	(2.108)		
w_{10}			-1.421	(0.907)					-2.060	(0.763)					-5.566	(1.875)					-4.571	(1.650)		
w_{11}			-1.728	(0.815)					-2.658	(0.803)					2.000	(2.214)					0.363	(1.501)		
w_{12}			1.752	(0.815)					2.842	(0.783)					-1.081	(2.132)					-0.282	(1.476)		
w_{13}			-1.128	(0.782)					-1.278	(1.194)					0.239	(1.876)					0.904	(1.412)		
w_{14}			0.175	(0.796)					-0.011	(1.153)					-0.013	(1.834)					-0.960	(1.667)		
w_{15}			2.089	(1.145)					1.689	(1.088)					1.971	(2.014)					2.481	(1.911)		
w_{16}			-3.193	(1.194)					-3.076	(1.212)					-3.218	(2.222)					-4.310	(1.951)		
w_{17}			0.005	(1.105)					0.523	(1.220)					-0.201	(2.488)					1.848	(2.378)		
$Inflation$	0.084	(0.109)					0.081	(0.150)					0.112	(0.103)					0.062	(0.092)				
$Inflation_V$	-1.309	(0.829)					-1.983	(0.748)					-1.190	(1.150)					-1.983	(1.122)				
$InterestR$	0.675	(0.152)					0.749	(0.165)					0.666	(0.152)					0.768	(0.149)				
$InterestR_V$	2.077	(1.166)					2.050	(2.079)					4.326	(2.033)					4.972	(1.541)				
$unemp_V$	-1.456	(0.779)					-5.099	(1.911)					-5.132	(4.081)					-6.893	(3.957)				
USD_V	0.992	(0.332)					1.176	(0.329)					1.028	(0.315)					1.167	(0.305)				
GDP	-0.218	(0.088)					-0.250	(0.154)					-0.245	(0.087)					-0.269	(0.076)				
GDP_V	0.224	(0.216)					0.500	(0.247)					0.403	(0.235)					0.601	(0.231)				
$Persistence$	0.957		0.958		0.987		0.948		0.953		0.991		0.929		0.935		0.975		0.915		0.930		0.977	
BIC	2.330		2.333		2.324		2.350		2.357		2.344		2.343		2.346		2.335		2.382		2.390		2.371	
AIC	2.291		2.292		2.313		2.313		2.319		2.336		2.308		2.309		2.335		2.347		2.353		2.326	

Notes: This table presents the results of all volatility models for SPX. SPX data are for the period between 10/08/2002 and 12/30/2016. The sample size is 3500 observations. Only results with positivity constraints are reported. In addition to volatility models presented in the table, we estimated the EWMA model that resulted in a smoothing parameter estimate and standard error given in parentheses: $\lambda = 0.9409$ (0.0049) and information criteria: $AIC = 2.7262, BIC = 2.7279$.

Table 4: **Estimation Results for GTARCH, Spline-GTARCH and Spline-Macro-GTARCH Models: TSX**

Parm	GTARCH						GTARCH0						GJR-GARCH						GARCH					
	SMacro		Spline		No Spline		SMacro		Spline		No Spline		SMacro		Spline		No Spline		SMacro		Spline		No Spline	
	parm	Std	parm	Std	parm	Std	parm	Std	parm	Std	parm	Std	parm	Std	parm	Std	parm	Std	parm	Std	parm	Std	parm	Std
μ	0.013	(0.006)	0.010	(0.005)	0.000	(0.000)	-1.256	(0.772)	0.016	(0.005)	0.000	(0.000)	0.016	(0.005)	0.015	(0.005)	0.014	(0.005)	0.026	(0.005)	0.025	(0.005)	0.024	(0.005)
ω					0.002	(0.000)					0.002	(0.000)					0.003	(0.001)					0.002	(0.001)
α	0.000	(0.062)	0.000	(0.011)	0.019	(0.009)	0.045	(0.010)	0.046	(0.005)	0.058	(0.007)	0.000	(0.000)	0.000	(0.000)	0.010	(0.009)	0.077	(0.008)	0.099	(0.010)	0.085	(0.010)
β	0.826	(0.058)	0.841	(0.014)	0.869	(0.013)	0.796	(0.023)	0.808	(0.020)	0.844	(0.013)	0.861	(0.015)	0.872	(0.013)	0.914	(0.011)	0.838	(0.017)	0.890	(0.011)	0.902	(0.012)
γ	0.079	(0.047)	0.076	(0.016)	0.069	(0.015)							0.122	(0.013)	0.130	(0.016)	0.107	(0.015)						
δ	0.168	(0.031)	0.192	(0.022)	0.137	(0.024)	0.227	(0.045)	0.239	(0.026)	0.194	0.000												
c	1.334	(0.362)	0.871	(0.122)			1.314	(1.305)	0.713	(0.095)			0.530	(0.113)	0.466	(0.063)			0.736	(0.038)	0.962	(0.094)		
w_1	-1.332	(0.218)	-0.674	(0.140)			-1.256	(0.772)	-0.632	(0.218)			-0.805	(0.330)	-0.286	(0.252)			-0.783	(0.241)	-0.418	(0.506)		
w_2	3.000	(0.437)	2.026	(0.314)			2.911	(1.450)	1.938	(0.636)			1.466	(0.854)	0.722	(0.798)			1.308	(0.669)	1.117	(1.509)		
w_3	-0.468	(0.492)	-1.882	(0.498)			-0.408	(1.480)	-1.692	(0.681)			1.060	(1.104)	-0.029	(1.165)			1.662	(0.863)	-0.373	(1.732)		
w_4	-4.362	(0.980)	0.370	(0.985)			-4.445	(6.642)	0.235	(0.544)			-5.151	(1.582)	-1.166	(1.343)			-5.982	(1.057)	-0.668	(1.844)		
w_5	5.727	(1.627)	0.738	(1.138)			5.771	(10.502)	0.605	(0.772)			6.368	(1.716)	1.635	(1.373)			6.999	(1.298)	0.440	(2.174)		
w_6	-1.189	(1.090)	0.564	(0.773)			-1.575	(4.702)	0.474	(1.171)			-0.871	(1.125)	0.173	(1.283)			-2.058	(1.531)	1.001	(1.863)		
w_7	-6.600	(1.753)	-5.860	(1.009)			-5.827	(3.282)	-5.225	(1.311)			-8.453	(2.149)	-5.863	(1.389)			-6.957	(2.570)	-5.400	(1.665)		
w_8	8.539	(1.679)	8.342	(1.637)			7.997	(1.943)	7.426	(1.262)			10.000	(1.689)	8.688	(1.551)			9.785	(2.378)	7.829	(1.922)		
w_9	-3.104	(0.968)	-4.116	(1.457)			-2.804	(3.835)	-3.014	(1.217)			-3.269	(1.066)	-4.278	(1.481)			-3.972	(1.353)	-3.860	(1.928)		
w_{10}	-1.983	(0.993)	-0.486	(1.081)			-2.405	(4.409)	-1.745	(0.998)			-2.301	(1.086)	-1.089	(1.386)			-1.900	(0.982)	-1.057	(1.910)		
w_{11}	3.062	(1.029)	2.021	(0.868)			3.600	(3.912)	3.212	(1.035)			3.018	(1.252)	2.575	(1.351)			3.009	(1.007)	2.359	(2.149)		
w_{12}	-2.623	(0.880)	-1.660	(0.744)			-2.871	(2.801)	-2.686	(1.083)			-1.785	(1.251)	-1.462	(1.340)			-1.866	(1.072)	-1.136	(2.138)		
w_{13}	4.839	(1.106)	1.716	(0.739)			4.603	(3.546)	3.417	(0.950)			3.933	(1.325)	2.481	(1.643)			4.095	(1.268)	1.975	(2.456)		
w_{14}	-7.824	(1.689)	-1.846	(0.908)			-7.622	(4.738)	-5.281	(0.866)			-7.644	(1.583)	-5.155	(1.943)			-8.343	(1.480)	-4.909	(2.715)		
w_{15}	8.409	(2.383)	0.286	(0.999)			8.602	(6.186)	4.938	(1.715)			8.482	(2.208)	4.835	(2.432)			10.000	(1.696)	5.256	(3.321)		
w_{16}																								
w_{17}																								
$Inflation$	-0.191	(0.145)					-0.281	(0.138)					-0.263	(0.125)					-0.395	(0.105)				
$Inflation_V$	-10.000	(2.537)					-9.358	(19.123)					-9.413	(3.119)					-10.000	(1.798)				
$InterestR$	0.163	(0.140)					0.119	(1.013)					0.332	(0.140)					0.191	(0.091)				
$InterestR_V$	-2.888	(1.116)					-2.279	(6.446)					-0.806	(1.012)					-6.055	(1.211)				
$unemp_V$	1.604	(1.196)					0.853	(2.361)					1.197	(1.172)					5.960	(4.520)				
$USDCAD_V$	0.412	(0.143)					0.417	(0.148)					0.501	(0.137)					0.578	(0.107)				
GDP	-0.036	(0.054)					-0.036	(0.063)					-0.023	(0.040)					-0.032	(0.034)				
GDP_V	0.057	(0.059)					0.048	(0.240)					0.028	(0.044)					-0.022	(0.060)				
$Persistence$	0.949		0.974		0.991		0.955		0.974		0.999		0.922		0.938		0.978		0.915		0.989		0.987	
BIC	2.326		2.313		2.306		2.337		2.324		2.312		2.332		2.321		2.312		2.354		2.373		2.327	
AIC	2.275		2.276		2.296		2.287		2.289		2.303		2.284		2.288		2.303		2.307		2.340		2.320	

Notes: This table presents the results of all volatility models for TSX. TSX data are for the period between 03/17/2003 and 03/31/2017. The sample size is 3500 observations. Only results with positivity constraints are reported. In addition to volatility models presented in the table, we estimated the EWMA model that resulted in a smoothing parameter estimate and standard error given in parentheses: $\lambda = 0.9369$ (0.0055) and information criteria: $AIC = 2.8222, BIC = 2.8240$.

Table 5: **Degree of Risk Aversion: SPX and TSX**

		GTARCH	GTARCH0	GJR-GARCH	GARCH	EWMA	RV
SPX	Spline+Macro	-0.726	-0.57	-0.658	-0.158		
	Spline	-0.764	-0.6	-0.659	-0.183		
	No Spline	-0.755	-0.544	-0.659	-0.192	-0.146	-0.111
TSX	Spline+Macro	-0.744	-0.614	-0.628	-0.213		
	Spline	-0.762	-0.638	-0.661	-0.245		
	No Spline	-0.715	-0.584	-0.632	-0.255	-0.190	-0.057

Notes: This table presents the correlation between returns r_t and log difference of fitted conditional variance $log(\sigma_t^2/\sigma_{t-1}^2)$ for each model. The last column presents correlation results for realized volatility (RV) estimate of σ_t. The more negative correlation implies higher degree of risk aversion in the model.

Table 6: **Forecasts of Volatility and Tail Risk: Low Volatility**

		GTARCH			GTARCH0			GJR-GARCH			GARCH			EWMA
	Forecast	SMacro	Spline	NoSpline	SMacro	Spline	NoSpline	SMacro	Spline	NoSpline	SMacro	Spline	NoSpline	
Panel A:	SPX	t = December 30, 2016												
	t+1day	10.841	9.656	10.875	9.409	8.399	10.762	10.383	9.328	10.178	9.116	8.481	9.615	8.044
σ	t+2day	10.949	9.717	11.065	9.468	8.437	10.976	10.516	9.391	10.345	9.256	8.595	9.814	
	t+3day	11.051	9.775	11.249	9.524	8.473	11.183	10.638	9.450	10.506	9.382	8.699	10.004	
VaR_{1day}	$q = 90\%$	0.867	0.772	0.854	0.771	0.683	0.849	0.858	0.765	0.820	0.759	0.713	0.787	0.647
	$q = 95\%$	1.185	1.065	1.199	1.033	0.930	1.179	1.138	1.032	1.113	0.995	0.947	1.073	0.892
	$q = 99\%$	1.822	1.647	1.842	1.601	1.430	1.841	1.741	1.550	1.735	1.540	1.436	1.655	1.414
VaR_{2day}	$q = 90\%$	1.226	1.092	1.208	1.090	0.966	1.201	1.214	1.082	1.160	1.074	1.008	1.113	
	$q = 95\%$	1.675	1.506	1.696	1.461	1.315	1.668	1.610	1.459	1.574	1.407	1.339	1.518	
	$q = 99\%$	2.576	2.330	2.605	2.264	2.022	2.604	2.462	2.192	2.454	2.178	2.030	2.341	
VaR_{3day}	$q = 90\%$	1.501	1.338	1.480	1.335	1.183	1.471	1.486	1.325	1.421	1.315	1.235	1.363	
	$q = 95\%$	2.052	1.844	2.077	1.789	1.611	2.043	1.971	1.787	1.928	1.723	1.640	1.859	
	$q = 99\%$	3.155	2.853	3.190	2.773	2.476	3.189	3.015	2.685	3.006	2.667	2.487	2.867	
ES_{1day}	$q = 90\%$	0.963	0.858	0.949	0.856	0.759	0.944	0.953	0.850	0.911	0.844	0.792	0.875	0.719
	$q = 95\%$	1.247	1.121	1.262	1.087	0.979	1.241	1.198	1.086	1.171	1.047	0.997	1.130	0.939
	$q = 99\%$	1.840	1.664	1.861	1.617	1.444	1.860	1.758	1.566	1.753	1.556	1.450	1.672	1.429
ES_{2day}	$q = 90\%$	1.362	1.214	1.342	1.211	1.073	1.335	1.348	1.202	1.289	1.193	1.120	1.237	
	$q = 95\%$	1.764	1.585	1.785	1.538	1.384	1.756	1.694	1.536	1.657	1.481	1.410	1.597	
	$q = 99\%$	2.602	2.353	2.631	2.287	2.042	2.630	2.487	2.215	2.479	2.200	2.051	2.364	
ES_{3day}	$q = 90\%$	1.668	1.486	1.644	1.483	1.314	1.635	1.651	1.472	1.579	1.461	1.372	1.515	
	$q = 95\%$	2.160	1.941	2.186	1.883	1.696	2.150	2.075	1.881	2.029	1.814	1.727	1.956	
	$q = 99\%$	3.187	2.882	3.223	2.801	2.501	3.222	3.046	2.712	3.036	2.694	2.512	2.896	
Panel B:	TSX	t = March 31, 2017												
	t+1day	4.111	3.711	4.273	4.018	3.679	4.132	4.108	3.659	4.262	4.089	3.625	4.011	3.627
σ	t+2day	4.149	3.738	4.325	4.073	3.724	4.191	4.133	3.666	4.290	4.142	3.653	4.052	
	t+3day	4.184	3.764	4.375	4.126	3.767	4.250	4.157	3.672	4.317	4.190	3.680	4.093	
VaR_{1day}	$q = 90\%$	0.344	0.307	0.346	0.335	0.307	0.339	0.336	0.308	0.356	0.351	0.310	0.340	0.299
	$q = 95\%$	1.185	0.413	0.472	0.454	0.414	0.456	0.461	0.413	0.478	0.474	0.416	0.459	0.410
	$q = 99\%$	1.822	0.611	0.724	0.685	0.625	0.722	0.681	0.625	0.743	0.720	0.636	0.728	0.646
VaR_{2day}	$q = 90\%$	1.226	0.434	0.490	0.473	0.434	0.479	0.475	0.436	0.503	0.496	0.438	0.481	
	$q = 95\%$	1.675	0.583	0.668	0.642	0.585	0.644	0.653	0.584	0.677	0.670	0.588	0.648	
	$q = 99\%$	2.576	0.865	1.024	0.968	0.883	1.021	0.963	0.884	1.050	1.018	0.900	1.029	
VaR_{3day}	$q = 90\%$	1.501	0.531	0.600	0.580	0.532	0.587	0.582	0.533	0.616	0.608	0.536	0.589	
	$q = 95\%$	2.052	0.715	0.818	0.787	0.717	0.789	0.799	0.715	0.829	0.821	0.720	0.794	
	$q = 99\%$	3.155	1.059	1.254	1.186	1.082	1.251	1.180	1.083	1.286	1.247	1.102	1.261	
ES_{1day}	$q = 90\%$	0.382	0.341	0.385	0.372	0.341	0.377	0.379	0.342	0.395	0.390	0.344	0.378	0.332
	$q = 95\%$	0.481	0.434	0.497	0.478	0.436	0.480	0.493	0.435	0.504	0.499	0.438	0.483	0.432
	$q = 99\%$	0.688	0.618	0.731	0.692	0.631	0.729	0.699	0.632	0.750	0.727	0.643	0.735	0.653
ES_{2day}	$q = 90\%$	0.541	0.482	0.544	0.526	0.482	0.533	0.535	0.484	0.559	0.552	0.487	0.535	
	$q = 95\%$	0.680	0.614	0.703	0.676	0.616	0.678	0.697	0.615	0.712	0.705	0.619	0.683	
	$q = 99\%$	0.973	0.873	1.034	0.978	0.892	1.032	0.988	0.893	1.061	1.028	0.909	1.040	
ES_{3day}	$q = 90\%$	0.662	0.590	0.667	0.644	0.591	0.652	0.656	0.593	0.685	0.676	0.596	0.655	
	$q = 95\%$	0.832	0.752	0.861	0.828	0.755	0.831	0.854	0.753	0.872	0.864	0.758	0.836	
	$q = 99\%$	1.192	1.070	1.267	1.198	1.093	1.263	1.210	1.094	1.299	1.259	1.114	1.273	

Notes: This table presents 1- to 3-day forecasts of volatility, VaR and ES produced by each volatility model for SPX and TSX at the time of low volatility. VaR and ES were estimated using the Hull and White (1998) method.

Table 7: **Forecasts of Volatility and Tail Risk: High Volatility**

		GTARCH			GTARCH0			GJR-GARCH			GARCH			EWMA
	Forecast	SMacro	Spline	NoSpline	SMacro	Spline	NoSpline	SMacro	Spline	NoSpline	SMacro	Spline	NoSpline	
Panel A:	SPX	t = November 11, 2008												
	t+1day	84.678	88.806	91.214	79.610	79.724	83.340	77.198	80.449	81.014	62.656	64.261	66.241	71.063
σ	t+2day	83.405	87.345	90.654	78.561	78.529	82.996	75.390	78.526	80.039	61.794	63.063	65.520	
	t+3day	82.168	85.924	90.098	77.553	77.374	82.654	73.670	76.685	79.077	60.994	61.927	64.808	
VaR_{1day}	$q = 90\%$	6.771	7.130	7.385	6.562	6.628	6.787	6.412	6.669	6.611	5.285	5.451	5.560	5.717
	$q = 95\%$	9.044	9.726	10.119	8.475	8.683	9.138	8.113	8.845	8.893	6.581	7.205	7.490	7.881
	$q = 99\%$	13.050	14.461	14.558	12.886	12.585	13.962	11.703	12.163	12.726	9.714	9.938	10.608	12.494
VaR_{2day}	$q = 90\%$	9.575	10.083	10.444	9.280	9.373	9.599	9.068	9.431	9.350	7.475	7.709	7.863	
	$q = 95\%$	12.791	13.755	14.311	11.986	12.280	12.923	11.473	12.508	12.577	9.307	10.189	10.593	
	$q = 99\%$	18.455	20.451	20.589	18.223	17.799	19.746	16.550	17.202	17.997	13.738	14.054	15.001	
VaR_{3day}	$q = 90\%$	11.727	12.349	12.792	11.365	11.480	11.756	11.106	11.551	11.451	9.155	9.441	9.630	
	$q = 95\%$	15.665	16.846	17.527	14.679	15.040	15.828	14.052	15.320	15.404	11.399	12.479	12.973	
	$q = 99\%$	22.603	25.048	25.216	22.319	21.799	24.183	20.270	21.068	22.042	16.825	17.213	18.373	
ES_{1day}	$q = 90\%$	7.523	7.922	8.206	7.291	7.364	7.541	7.124	7.410	7.346	5.873	6.057	6.177	6.352
	$q = 95\%$	9.520	10.238	10.652	8.921	9.140	9.619	8.540	9.310	9.361	6.928	7.584	7.884	8.296
	$q = 99\%$	13.181	14.607	14.705	13.016	12.713	14.103	11.821	12.286	12.855	9.812	10.038	10.715	12.620
ES_{2day}	$q = 90\%$	10.639	11.203	11.605	10.311	10.414	10.665	10.075	10.479	10.388	8.305	8.565	8.736	
	$q = 95\%$	13.464	14.479	15.064	12.616	12.926	13.604	12.077	13.167	13.239	9.797	10.725	11.150	
	$q = 99\%$	18.641	20.658	20.797	18.407	17.978	19.945	16.717	17.375	18.179	13.877	14.196	15.153	
ES_{3day}	$q = 90\%$	13.030	13.721	14.213	12.628	12.755	13.062	12.340	12.834	12.723	10.172	10.490	10.700	
	$q = 95\%$	16.490	17.733	18.449	15.452	15.832	16.661	14.792	16.126	16.214	11.999	13.136	13.656	
	$q = 99\%$	22.831	25.301	25.471	22.544	22.019	24.428	20.474	21.280	22.265	16.995	17.387	18.558	
Panel B:	TSX	t = September 16, 2008												
	t+1day	16.305	17.452	14.694	15.233	16.265	13.449	17.229	17.760	15.724	15.911	16.287	14.554	13.363
σ	t+2day	16.149	17.404	14.649	15.183	16.295	13.459	16.913	17.541	15.570	15.760	16.218	14.477	
	t+3day	15.999	17.358	14.604	15.135	16.325	13.469	16.617	17.333	15.417	15.621	16.152	14.400	
VaR_{1day}	$q = 90\%$	1.322	1.366	1.183	1.236	1.319	1.091	1.388	1.415	1.279	1.313	1.330	1.210	1.101
	$q = 95\%$	1.185	1.862	1.648	1.657	1.749	1.533	1.880	1.936	1.771	1.785	1.827	1.660	1.511
	$q = 99\%$	1.822	2.794	2.573	2.486	2.689	2.377	2.856	2.834	2.810	2.706	2.625	2.648	2.382
VaR_{2day}	$q = 90\%$	1.226	1.933	1.674	1.748	1.865	1.543	1.963	2.002	1.808	1.857	1.880	1.711	
	$q = 95\%$	1.675	2.634	2.331	2.344	2.473	2.169	2.658	2.738	2.504	2.524	2.584	2.347	
	$q = 99\%$	2.576	3.951	3.639	3.515	3.803	3.361	4.039	4.008	3.974	3.827	3.713	3.745	
VaR_{3day}	$q = 90\%$	1.501	2.367	2.050	2.141	2.284	1.890	2.404	2.451	2.215	2.274	2.303	2.096	
	$q = 95\%$	2.052	3.225	2.855	2.870	3.029	2.656	3.256	3.354	3.067	3.091	3.165	2.875	
	$q = 99\%$	3.155	4.838	4.456	4.306	4.658	4.117	4.947	4.908	4.867	4.687	4.547	4.587	
ES_{1day}	$q = 90\%$	1.469	1.518	1.315	1.373	1.465	1.213	1.393	1.573	1.421	1.459	1.477	1.344	1.223
	$q = 95\%$	1.855	1.960	1.735	1.744	1.841	1.614	1.930	2.038	1.864	1.879	1.923	1.747	1.590
	$q = 99\%$	2.595	2.822	2.599	2.511	2.717	2.401	2.880	2.862	2.838	2.734	2.652	2.675	2.406
ES_{2day}	$q = 90\%$	2.077	2.147	1.859	1.942	2.072	1.715	1.970	2.224	2.009	2.063	2.089	1.901	
	$q = 95\%$	2.624	2.772	2.454	2.467	2.604	2.283	2.729	2.882	2.636	2.657	2.720	2.471	
	$q = 99\%$	3.670	3.991	3.675	3.551	3.842	3.395	4.073	4.048	4.014	3.866	3.750	3.783	
ES_{3day}	$q = 90\%$	2.544	2.630	2.277	2.379	2.538	2.100	2.412	2.724	2.461	2.527	2.559	2.329	
	$q = 95\%$	3.214	3.395	3.005	3.021	3.189	2.796	3.342	3.530	3.228	3.254	3.331	3.026	
	$q = 99\%$	4.495	4.887	4.501	4.349	4.705	4.158	4.988	4.958	4.916	4.735	4.593	4.633	

Notes: This table presents 1- to 3-day forecasts of volatility, VaR and ES produced by each volatility model for SPX and TSX at the time of high volatility. VaR and ES were estimated using the Hull and White (1998) method.

Table 8: **Backtesting for SPX and TSX VaR Models**

Upper and lower bound from the Kupiec Test		
Breaches allowed at 95% CI	LB	UB
VaR_{Q90}	310	350
VaR_{Q95}	146	175
VaR_{Q99}	22	35

Breaches for SPX Data

		GTARCH	GTARCH0	GJR-GARCH	GARCH	EWMA
Spline	90% VaR	343	336	336	307	
	95% VaR	169	166	167	160	
	99% VaR	32	33	33	29	
Spline + Macro Variable	90% VaR	336	327	326	308	
	95% VaR	171	167	166	160	
	99% VaR	34	29	32	30	
No Spline	90% VaR	336	327	326	308	354
	95% VaR	171	167	166	160	176
	99% VaR	34	29	32	30	36

Breaches for TSX Data

		GTARCH	GTARCH0	GJR-GARCH	GARCH	EWMA
Spline	90% VaR	341	328	327	321	
	95% VaR	168	163	162	155	
	99% VaR	34	32	31	30	
Spline + Macro Variable	90% VaR	330	329	332	312	
	95% VaR	164	152	162	151	
	99% VaR	33	33	33	27	
No Spline	90% VaR	330	329	332	312	356
	95% VaR	164	152	162	151	182
	99% VaR	33	33	33	27	36

Notes: This table presents the number of backtest breaches for VaR of SPX and TSX produced by each volatility model. VaR was estimated using the Hull and White (1998) method.

Table 9: **One-day-ahead in-sample performance using log realized variance: SPX and TSX**

	Model	MSE	MAE	R^2
Panel A: SPX Data				
No Spline	GTARCH	0.106	0.252	0.635
	GTARCH0	0.113	0.260	0.612
	GJR-GARCH	0.107	0.253	0.632
	GARCH	0.119	0.267	0.592
	EWMA	0.130	0.280	0.553
Spline +	GTARCH	0.096	0.241	0.671
	GTARCH0	0.106	0.254	0.635
	GJR-GARCH	0.097	0.241	0.666
	GARCH	0.112	0.261	0.613
Spline +	GTARCH	0.095	0.240	0.673
Macro	GTARCH0	0.107	0.256	0.631
Variables	GJR-GARCH	0.097	0.241	0.665
	GARCH	0.114	0.262	0.609
Panel B: TSX Data				
No Spline	GTARCH	0.108	0.256	0.661
	GTARCH0	0.113	0.263	0.642
	GJR-GARCH	0.109	0.257	0.662
	GARCH	0.115	0.266	0.631
	EWMA	0.122	0.274	0.601
Spline +	GTARCH	0.114	0.264	0.667
	GTARCH0	0.115	0.266	0.646
	GJR-GARCH	0.110	0.258	0.673
	GARCH	0.114	0.263	0.636
Spline +	GTARCH	0.109	0.258	0.656
Macro	GTARCH0	0.112	0.261	0.636
Variables	GJR-GARCH	0.106	0.253	0.653
	GARCH	0.110	0.258	0.621

Notes: This table presents the mean square error (MSE), the mean absolute error (MAE) and Mincer-Zarnowitz (MZ) adjusted R^2 for the regressions of log realized variance (RV) on log variances estimated by each model above. Since the spline model has low- and high-frequency components, both were used for spline models regressions and adjusted R^2 are reported. The number of observations for the RV was slightly smaller and the resulting sample is: 3487 observations for SPX and 3480 observations for TSX.

Table 10: **Threshold Regimes: SPX and TSX**

	Model	Thresholds (in %)		Proportion			Unit Root		
				Low regime	Middle regime	High regime	Low regime	Middle regime	High regime
Panel A: SPX Data									
No Spline	GTARCH	1.90	2.44	33%	25%	42%	NO	NO	NO
	GTARCH0	1.91	2.95	30%	41%	28%	NO	NO	NO
	GJR-GARCH	1.88	2.39	31%	25%	44%	NO	NO	NO
	GARCH	1.93	2.47	27%	29%	44%	NO	NO	NO
	EWMA	1.82	2.90	25%	44%	31%	NO	NO	NO
	Average	1.89	2.63	29%	33%	38%			
Spline	GTARCH	1.74	2.66	26%	39%	35%	NO	NO	NO
	GTARCH0	1.92	2.47	33%	26%	41%	NO	NO	NO
	GJR-GARCH	1.87	2.47	36%	26%	38%	NO	NO	NO
	GARCH	1.86	2.29	27%	25%	48%	NO	NO	NO
	Average	1.85	2.48	31%	29%	40%			
Spline + Macro Variables	GTARCH	1.74	2.74	28%	40%	32%	NO	NO	NO
	GTARCH0	1.78	2.74	25%	41%	34%	NO	NO	NO
	GJR-GARCH	1.74	2.18	26%	25%	49%	NO	NO	NO
	GARCH	1.85	2.85	27%	43%	30%	NO	NO	NO
	Average	1.78	2.63	26%	37%	36%			
Overall	**Average**	1.84	2.58	29%	33%	38%			
	Realized Vol	1.57	2.27	49%	25%	26%	NO	NO	NO
Panel B: TSX Data									
No Spline	GTARCH	0.74	0.97	29%	29%	42%	NO	NO	NO
	GTARCH0	0.77	1.10	27%	41%	32%	NO	NO	NO
	GJR-GARCH	0.74	1.01	26%	33%	40%	NO	NO	NO
	GARCH	0.83	1.05	32%	29%	40%	NO	NO	NO
	EWMA	0.79	1.02	34%	26%	40%	NO	YES	YES
	Average	0.77	1.03	30%	32%	39%			
Spline	GTARCH	0.73	0.94	33%	26%	41%	NO	YES	YES
	GTARCH0	0.87	1.15	48%	25%	27%	NO	YES	YES
	GJR-GARCH	0.71	0.93	26%	30%	44%	NO	YES	YES
	GARCH	0.74	0.95	25%	27%	48%	NO	YES	YES
	Average	0.76	0.99	33%	27%	40%			
Spline + Macro Variables	GTARCH	0.82	1.12	47%	26%	27%	NO	YES	NO
	GTARCH0	0.82	1.04	41%	26%	33%	NO	YES	NO
	GJR-GARCH	0.70	0.87	25%	27%	47%	NO	NO	NO
	GARCH	0.75	1.05	26%	39%	36%	NO	YES	NO
	Average	0.77	1.02	35%	29%	36%			
Overall	**Average**	0.77	1.01	32%	29%	38%			
	Realized Vol	0.52	1.02	25%	49%	25%	NO	NO	NO

Notes: This table presents results of the three-regime threshold autoregressive (3TAR) model applied to VaR produced by each volatility model for SPX and TSX. VaR was estimated using the Hull and White (1998) method. In the last row of each panel we present the results of applying the 3TAR model to VaR using realized volatility (RV) and the Hull and White (1998) method. The number of observations for the RV was slightly smaller than for daily returns: 3487 observations for SPX and 3480 observations for TSX.

Table 11: **Loss Functions: SPX and TSX**

	Model	L_1	# violations	L_2	# violations	Rank for L_1	Rank for L_2	Group Ranks				
								w=0	w=0.25	w=0.5	w=0.75	w=1
Panel A: SPX Data												
No Spline	GTARCH	22.39	34	374.55	100	8	10	3	4	4	4	4
	GTARCH0	25.40	34	275.26	71	13	1	5	1	1	1	1
	GJR-GARCH	20.23	31	386.80	104	5	11	2	6	5	5	5
	GARCH	24.95	32	367.14	97	12	8	4	5	3	3	3
	EWMA	33.54	35	285.47	77	14	2	6	2	2	2	2
	Realized Volatility	2.23	28	414.19	119	1	13	1	3	6	6	6
Spline	**GTARCH**	19.18	32	328.41	85	3	6	2	1	1	1	1
	GTARCH0	20.82	33	366.29	97	6	7	3	3	3	2	2
	GJR-GARCH	18.09	33	367.75	99	2	9	1	2	2	3	3
	GARCH	22.25	29	410.60	115	7	12	4	4	4	4	4
Spline +	GTARCH	22.72	33	313.90	81	9	5	2	3	3	3	3
Macro	GTARCH0	23.13	28	311.95	80	10	4	3	2	2	2	2
Variables	GJR-GARCH	19.37	31	442.11	124	4	14	1	4	4	4	4
	GARCH	23.48	30	291.86	78	11	3	4	1	1	1	1
Panel B: TSX Data												
No Spline	GTARCH	1.93	35	65.48	98	9	9	4	6	6	6	6
	GTARCH0	1.63	35	54.35	78	7	3	2	1	1	2	2
	GJR-GARCH	1.70	31	61.77	89	8	8	3	4	5	5	5
	GARCH	1.34	30	58.04	82	2	4	1	2	3	3	3
	EWMA	2.06	36	60.73	88	12	7	5	5	4	4	4
	Realized Volatility	3.14	32	52.96	71	14	2	6	3	2	1	1
Spline	GTARCH	2.38	34	68.49	98	13	12	4	3	3	3	3
	GTARCH0	1.95	32	50.77	74	10	1	2	1	1	1	1
	GJR-GARCH	2.04	31	69.51	107	11	13	3	4	4	4	4
	GARCH	1.37	30	67.21	99	3	10	1	2	2	2	2
Spline +	GTARCH	1.62	33	59.26	84	6	6	4	2	2	2	2
Macro	GTARCH0	1.52	33	75.92	119	4	14	2	4	4	4	4
Variables	**GJR-GARCH**	1.61	33	58.55	84	5	5	3	1	1	1	1
	GARCH	1.28	27	68.20	97	1	11	1	3	3	3	3

Notes: This table presents results of loss functions L_1=Loss for VaR, L_2=loss for TVaR and the number of violations produced by each volatility model for SPX and TSX. VaR was estimated using the Hull and White (1998) method and Threshold VaR (TVaR) was set between the bounds estimated in Table 10. Overall ranks for loss functions are presented in columns 7 and 8. Ranks for each group using various trade-off parameters w ranging from 0 to 1 are presented in columns 9-13.

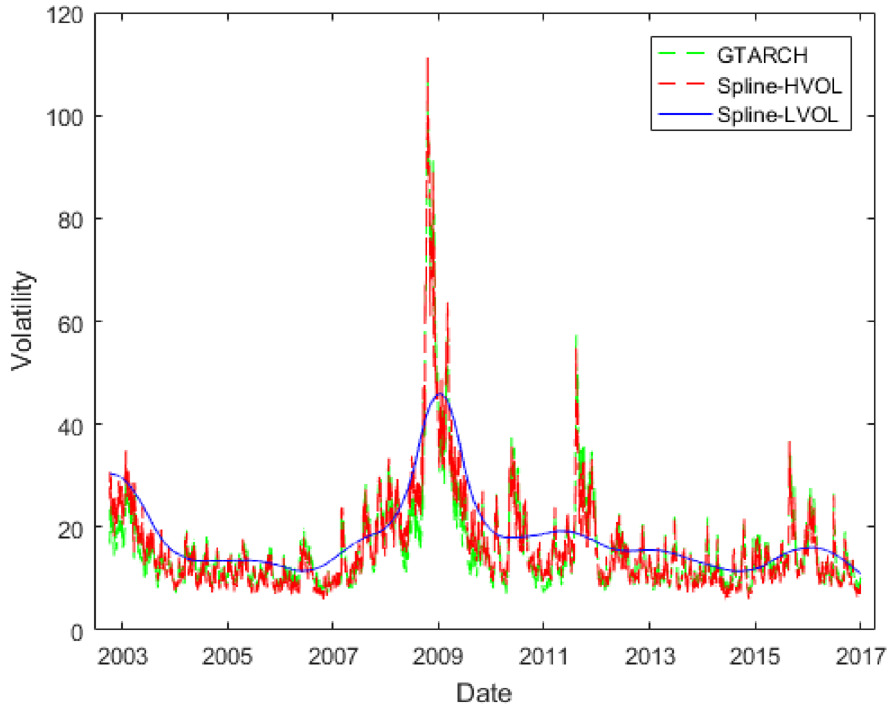

Figure 1: **High- and Low-Frequency Volatility: Spline-GTARCH for SPX**

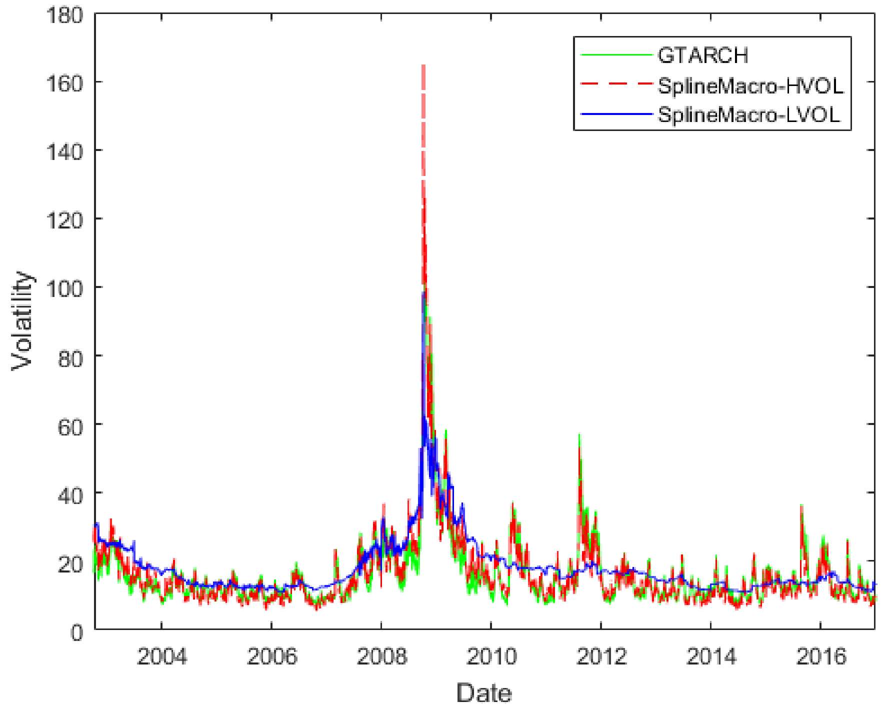

Figure 2: **High- and Low-Frequency Volatility: Spline-Macro-GTARCH for SPX**

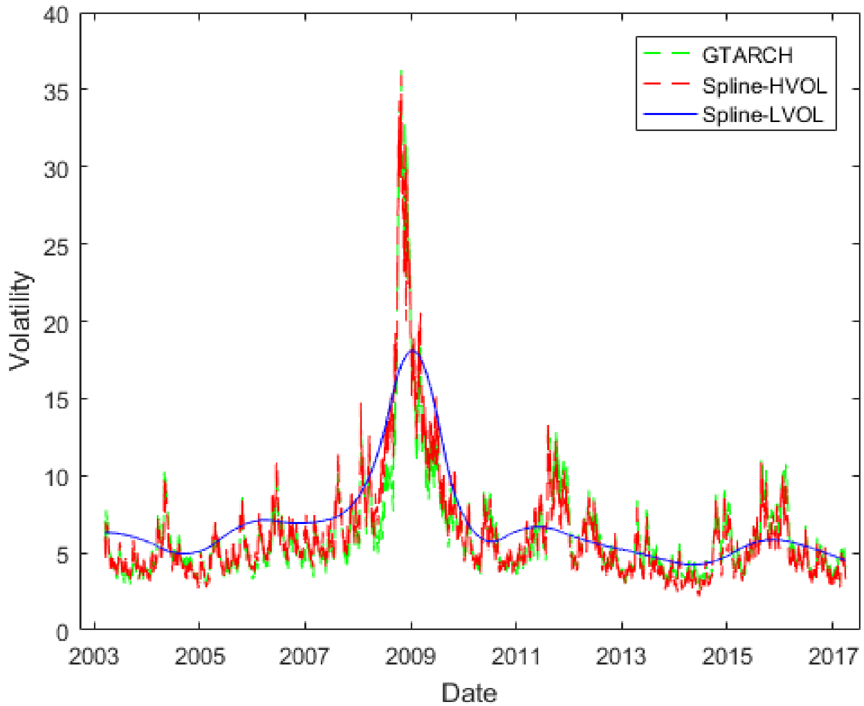

Figure 3: **High- and Low-Frequency Volatility: Spline-GTARCH for TSX**

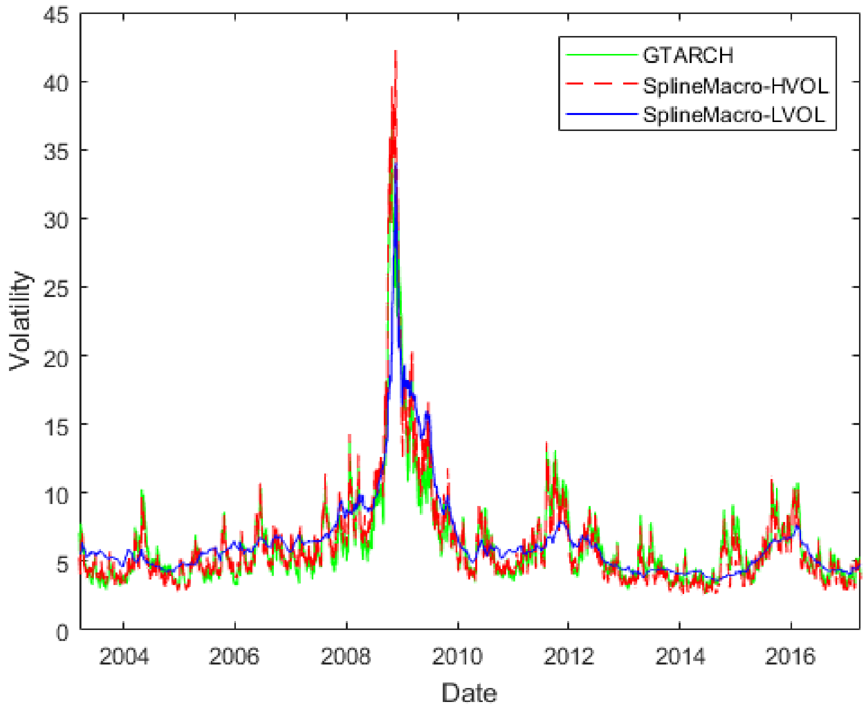

Figure 4: **High- and Low-Frequency Volatility: Spline-Macro-GTARCH for TSX**

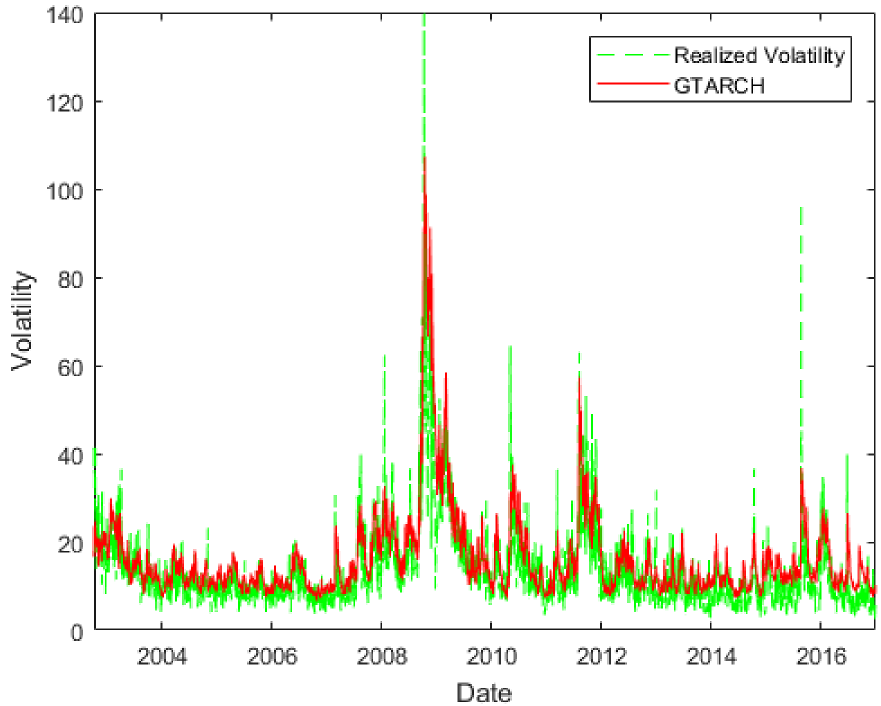

Figure 5: **Realized Volatility and GTARCH for SPX**

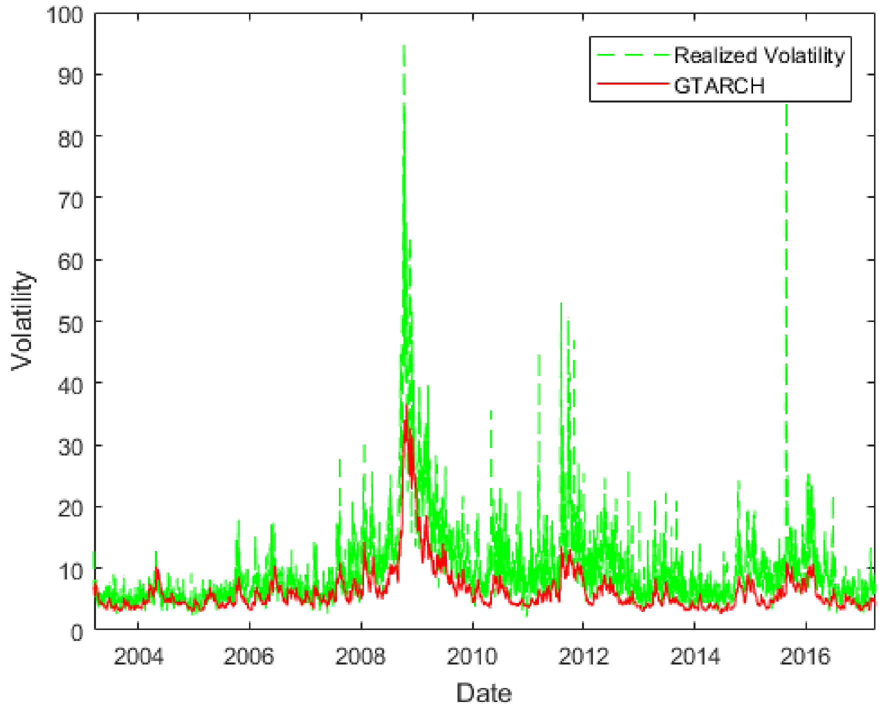

Figure 6: **Realized Volatility and GTARCH for TSX**

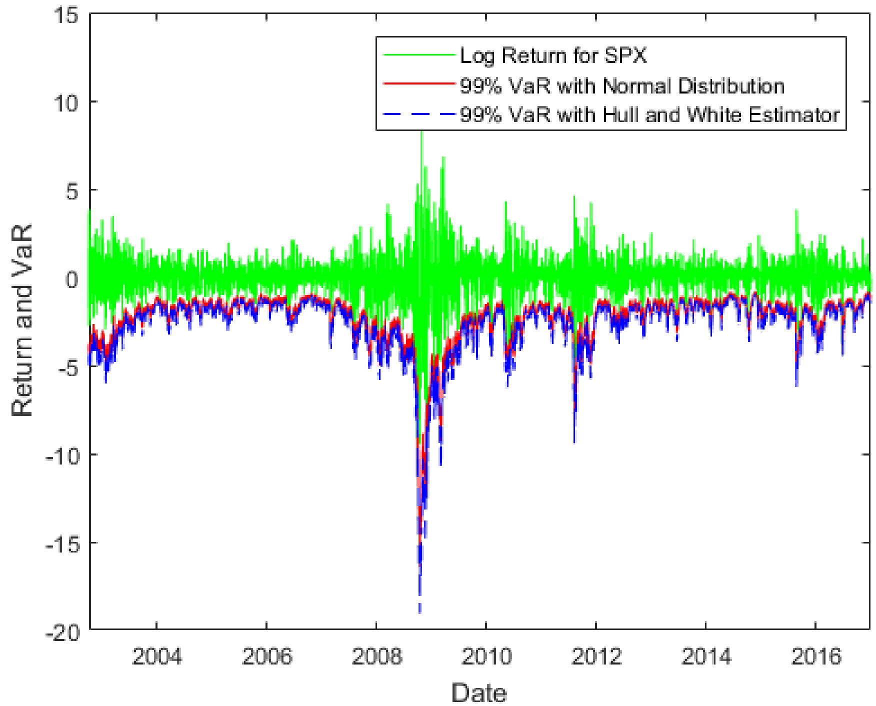

Figure 7: **SPX Log Returns and 1-day VaR: Spline-GTARCH**

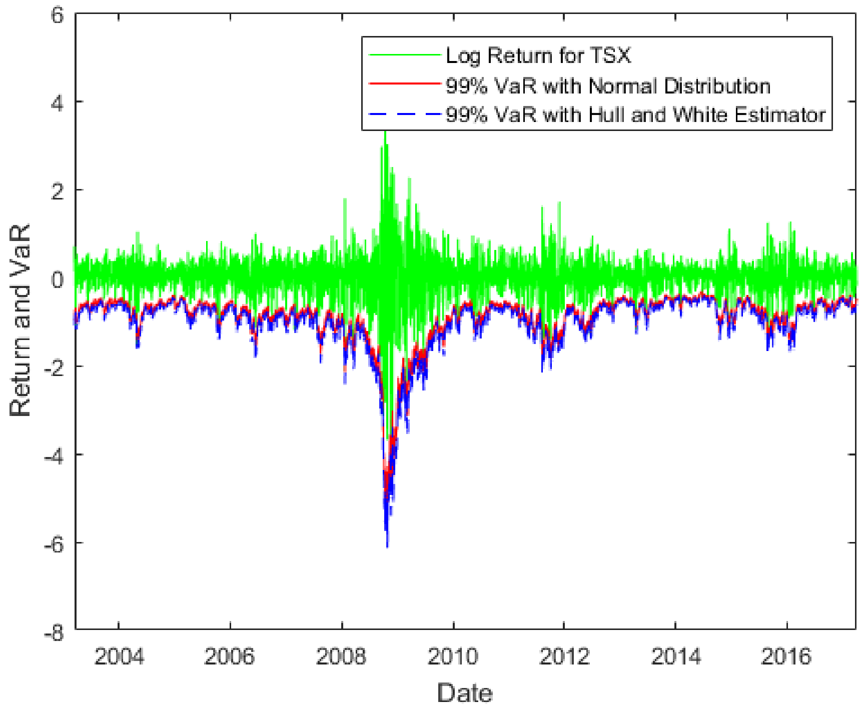

Figure 8: **TSX Log Returns and 1-day VaR: Spline-GTARCH**

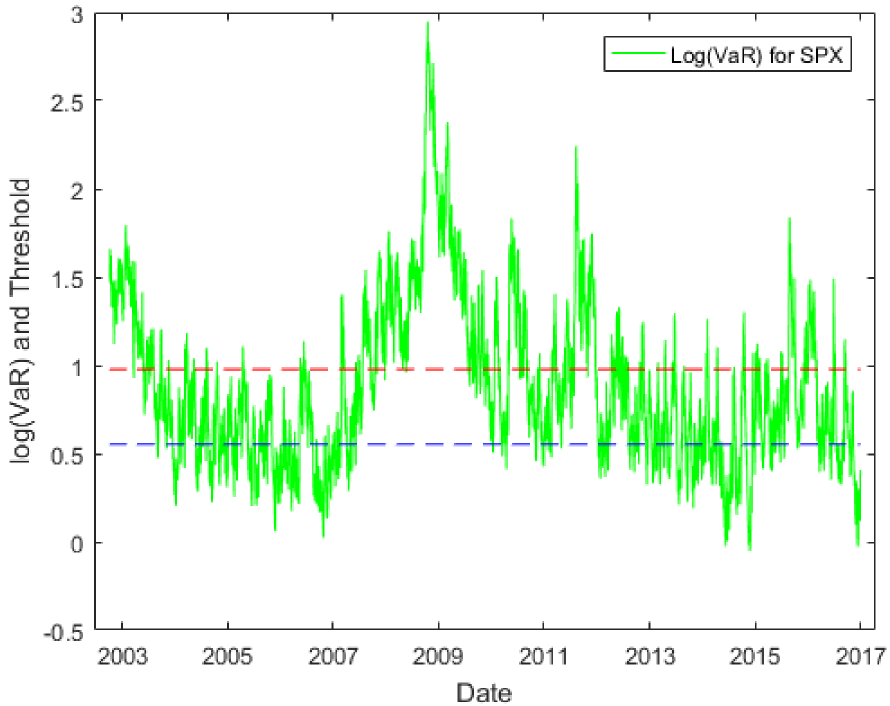

Figure 9: **Estimated Thresholds with 3 Regimes: Log(VaR) Spline-GTARCH for SPX**

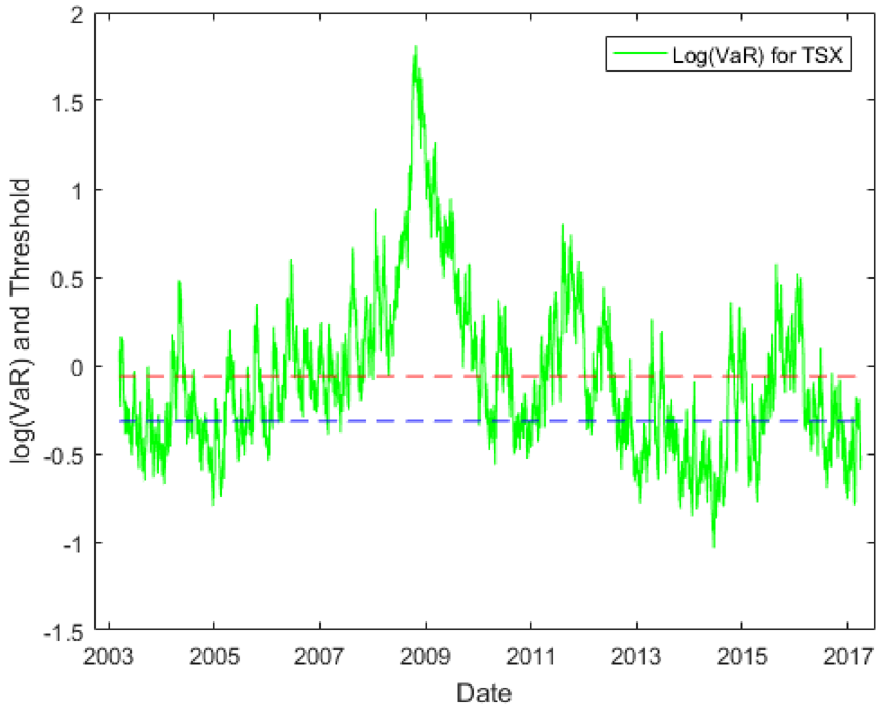

Figure 10: **Estimated Thresholds with 3 Regimes: Log(VaR) Spline-GTARCH for TSX**

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