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The MacroFinancial Risk Assessment Framework (MFRAF), Version 2.0

by José Fique

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José Fique

Financial Stability Department
Bank of Canada
Ottawa, Ontario, Canada K1A 0G9
jpereirafique@bankofcanada.ca

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September 2017

Abstract

This report provides a detailed technical description of the updated MacroFinancial Risk Assessment Framework (MFRAF), which replaces the version described in Gauthier, Souissi and Liu (2014) as the Bank of Canada's stress-testing model for banks with a focus on domestic systemically important banks (D-SIBs). This new version incorporates the characteristics of the previous model and also includes fire-sale effects resulting from the regulatory leverage constraints faced by banks, as well as an enhanced treatment of feedback-loop effects between solvency and liquidity risks through both the pricing and costly asset-liquidation channels. These new features improve the model's ability to capture the non-linear effects of risk scenarios on D-SIBs' capital positions and shed light on the importance of additional channels of stress propagation. The model is also subject to a comprehensive sensitivity analysis.

Bank topics: Financial stability; Financial system regulation and policies

JEL codes: G01, G21, G28, C72, E58

Résumé

Ce rapport donne une description technique détaillée du Cadre d'évaluation des risques macrofinanciers (CERM) dans sa version actualisée. Cette nouvelle mouture, qui remplace la version présentée dans Gauthier, Souissi et Liu (2014), est le modèle de simulation que la Banque du Canada utilise pour soumettre les établissements bancaires à des tests de résistance, plus particulièrement les banques d'importance systémique nationale. Elle reprend les caractéristiques du précédent modèle, mais y incorpore les effets des ventes forcées d'actifs qui résultent des exigences réglementaires en matière de levier financier imposées aux banques. Par ailleurs, elle offre une meilleure prise en charge des effets de rétroaction qui se manifestent entre les risques de solvabilité et de liquidité par le canal du coût du financement et celui des liquidations onéreuses. Grâce à ces nouvelles caractéristiques, le modèle reproduit mieux les effets non linéaires des scénarios de risque sur le niveau des fonds propres des banques d'importance systémique nationale et révèle l'importance d'autres canaux de propagation des tensions. Le modèle est également soumis à une analyse de sensibilité approfondie.

Sujets : Stabilité financière; Réglementation et politiques relatives au système financier

Codes JEL : G01, G21, G28, C72, E58

1. Introduction

Over the past few years, financial sector authorities and financial institutions around the world have increasingly used stress testing to examine risks to the financial system. For example, stress testing is used by banks for internal risk management and by authorities to quantify the impact of large but plausible negative shocks on banks (see BCBS 2009 and Dees, Henry and Martin 2017). Stress testing has also become a central component of the bilateral and multilateral surveillance work undertaken by the International Monetary Fund (IMF).

For the Bank of Canada (the Bank), stress testing can contribute to the assessment of financial stability risks as part of the Bank's enhanced risk-assessment framework (Anand, Bédard-Pagé and Traclet 2014) to provide a quantitative assessment of the expected impact of financial stability risks on financial system participants should those risks materialize. For example, the Bank has developed a stress-testing model for the household sector that quantifies the impact of macro-risk scenarios on Canadian households (Peterson and Roberts 2016).

In this report, we describe the Bank's stress-test model for the banking sector, the MacroFinancial Risk Assessment Framework (MFRAF), which quantifies the impact of risk scenarios on domestic systemically important banks (D-SIBs), taking into account the second-round effects associated with the interaction of the various risks that banks face and the actions taken by banks.

Unlike some central banks (e.g., the Bank of England, Federal Reserve, European Central Bank), the Bank of Canada does not have supervisory responsibilities for banks, but instead focuses on systemic risk.[2] Consequently, the focus of stress testing at the Bank is not on the impact on

[2] Although a unique, commonly accepted definition of systemic risk does not exist, it is defined in this technical report as a confluence of events that leads to a substantial disruption in the functioning of the financial system with severe negative consequences to the real economy. This definition is in line with the one proposed by the Bank for International Settlements (BIS), the Financial Stability Board and the International Monetary Fund (2009): "the disruption to the flow of financial services that is caused by an impairment of all or parts of the financial system; and has the potential to have serious negative consequences for the real economy."

individual banks, but on the banking sector as a whole, in a manner that accounts for the contagion mechanisms that may exist among the various banks.

As the 2007–09 global financial crisis clearly illustrated, banks can be affected by various sources of risk—credit, market and liquidity risk, as well as contagion effects—that interact with each other. Given the Bank's focus on systemic risk, MFRAF has been designed to take these different sources of risk and their interactions into account in a consistent modular framework that allows us to decompose the ultimate impact on D-SIBs' capital positions into its drivers.

Although the Bank's focus is on the whole banking sector, not individual banks, MFRAF considers individual banks separately to take into account their interactions and contagion effects (e.g., fire sales and network effects). MFRAF contributes to the Bank's ability to assess systemic risk and its transmission channels by enhancing our understanding of how these different risks interact and how the actions of individual banks under stress affect the overall banking system.

While MFRAF has been developed as a top-down stress-test model to be used by authorities for purposes of systemic risk assessment, it has also been used as a "hybrid" in the context of the Office of the Superintendent of Financial Institutions (OSFI)–Bank of Canada macro stress test (MST).[3] In this context, the results of bottom-up stress tests generated from banks' internal models are entered as input into MFRAF; the results from MFRAF complement those of individual banks by accounting for systemic risk considerations that individual banks may not necessarily capture well.[4]

MFRAF has been under development at the Bank for several years and is subject to regular enhancements to improve the methods for taking risk into account in the model, capitalizing on the progress in the literature, and to account for the evolution of banking regulation and its

[3] See page 23 in http://www.bankofcanada.ca/wp-content/uploads/2017/06/fsr-june2017.pdf and page 4 in http://www.osfi-bsif.gc.ca/Eng/Docs/jh20160505.pdf.

[4] See Anand, Bédard-Pagé and Traclet (2014) for an illustration of an MFRAF hybrid application.

implications for banks. This technical report complements previous publications by providing a detailed description of recent developments in the model.[5]

The new version of MFRAF (MFRAF v2.0) includes, in addition to the characteristics of the model presented in Gauthier, Souissi and Liu (2014), fire-sale effects resulting from the leverage constraints that banks are facing, as well as an enhanced treatment of feedback-loop effects between solvency and liquidity risks through both the cost-of-funding and costly liquidation-of-assets channels.

The updated liquidity-risk module provides a more-comprehensive quantification of liquidity losses than did the previous version, which addresses a recommendation for improvement identified in the 2013 IMF Financial Sector Assessment Program (FSAP) for Canada (see IMF 2014).

In the model, when banks become leverage-constrained as a result of credit and market shocks, they sell securities to repay maturing liabilities in order to return to regulatory compliance with the leverage ratio requirements. These sales introduce a downward pressure in securities prices, which leads to mark-to-market (MTM) losses at other banks through balance-sheet commonalities.

As in previous versions, the model captures the effect of solvency on liquidity risk, whereby bank creditors consider solvency in their funding rollover decisions, but it now also reflects how the composition of banks' assets and liabilities influences solvency risk endogenously through both the cost-of-funding and costly asset-liquidation channels. All else being equal, an increase in solvency risk leads to an increase in banks' cost of funding, which in turn feeds back negatively into their solvency positions through the profit-and-loss (P&L) account. Moreover, this effect is not independent of banks' asset composition and may lead to fire sales, as described above, which this time are triggered by funding considerations: banks that rely substantially on short-term funding may be forced to prematurely liquidate securities at fire-sale prices to meet withdrawals. Since the model determines banks' funding costs and fire-sale

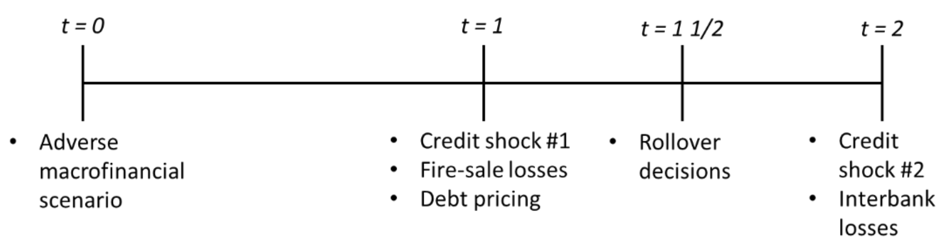
[5] See http://www.bankofcanada.ca/wp-content/uploads/2010/11/wp10-29.pdf; http://www.bankofcanada.ca/wp-content/uploads/2015/08/wp2015-32.pdf.

losses jointly, it provides a decomposition of the feedback effects between the solvency and liquidity risks. In addition, since the funding rate is determined in equilibrium, the model also sheds light on the conditions under which banks can no longer access the unsecured funding market.

The rest of the report is organized as follows. Section 2 provides an overview of the model. Section 3 describes MFRAF in detail. Section 4 lists the data needed to run the model. Section 5 explains the calibration requirements of the model. Section 6 illustrates the main mechanisms at play through the use of comparative statics exercises. Finally, Section 7 concludes.

2. Overview of MFRAF

Figure 1: Timeline of Events



MFRAF is a three-period model, as **Figure 1** illustrates. As in any other stress-testing model, we assume that an adverse macrofinancial risk scenario materializes at $t = 0$. This scenario, which is defined as a set of paths for key macrofinancial variables (e.g., unemployment rate, GDP growth, etc.), translates into credit and market shocks that affect banks' capital positions at $t = 1$ and $t = 2$.[6]

[6] The choice of the macrofinancial scenario considered, and how its direct effects are reflected on banks' balance sheets, constitutes the most crucial element for gauging the associated amplification effects. Breuer and Csiszar (2013) and Bidder, Giacomini and McKenna (2016) suggest some approaches to increase the robustness of the stress-testing process to this step.

If these first-round losses are large enough, banks become leverage-constrained, which would lead them to de-lever to meet the regulatory leverage ratio.[7] Even though these actions may seem optimal at an individual level, they can result in downward pressures on securities prices and lead to system-wide MTM losses, given balance-sheet common exposures across banks.

In turn, these losses lead to further deterioration in banks' solvency positions, resulting in an increase in the probability of default. A higher probability of default implies that banks' access to unsecured wholesale funding becomes more difficult. In other words, solvency and access to funding (or its conditions) are intrinsically linked.

In MFRAF, banks access the unsecured funding market at $t = 1$ to replace liabilities that mature over the stress horizon. They do so by offering short-term debt contracts to investors. Given their nature, these contracts expose banks to rollover risk. That is, a bank is exposed to the risk that a significant fraction of its liabilities come due without new sources of liquidity becoming available to repay them.

Since early liquidation of assets is costly, the ability of a bank to honour its obligations may depend on the actions taken by its creditors. MFRAF determines the fraction of creditors who decide to withdraw, and the associated premature liquidation costs, as the outcome of a coordination game where banks' creditors face uncertainty with respect to the actions that will be taken by other creditors.

As will be shown later, the outcome of this game and the conditions of the contract offered by banks will depend on the following factors:

- The cost of liquidating assets prior to maturity. These costs depend on the assets available for sale and the corresponding market liquidity conditions. The higher these costs, the less likely it is that a bank will be able to access unsecured funding (or the bank will do so at a higher cost).

[7] We assume that de-leveraging actions can be in the form of securities sales only, not loan sales, for two reasons. First, the sale of securities can be accomplished in a relatively expedient manner, which would make it a desirable option for banks. Second, from a practical viewpoint, determining an asset-selection process and asset-price calibration is easier to implement for securities than it would be for loans.

- The bank's solvency prospects. The better these prospects are, the more likely it is that the bank will be able to access unsecured funding (or the bank will do so at a lower cost).
- The degree of conservatism of the bank's creditors. The more conservative creditors are, the less likely it is that the bank will be able to access unsecured funding (or the bank will do so at a higher cost).

Finally, losses from banks defaulting on their interbank obligations could lead to a further deterioration in the solvency positions of surviving banks.[8] After all these effects have been accounted for, MFRAF generates banks' distributions of common equity Tier 1 (CET1) capital ratios.[9]

3. MFRAF Modules

MFRAF is a modular framework[10] that operates sequentially to quantify the impact of the various types of risks that banks are exposed to on their CET1 ratios. This implies that the output of each module is used as input for the next one. Thus, even though the framework allows for substantial flexibility in mapping specific dates into individual modules, the sequence of events is predefined by design.

All of these effects are reflected on the banks' stylized balance sheets, as shown in **Figure 2**.

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[8] Losses on interbank loans are not the only type of contagion that banks can be exposed to. See Anand, Gauthier and Souissi (2015) for a version of MFRAF that accounts for information contagion.

[9] MFRAF focuses on simulating the capital component of banks' CET1 ratios. A satellite model that reproduces the logic of the regulatory framework in a manner consistent with the evolution of credit losses at the numerator is used to generate the values for risk-weighted assets (RWA).

[10] MFRAF's implementation is also modular in nature. The model is implemented in MATLAB, with each section of the main script corresponding to each module. The use of MATLAB's parallel computing capabilities, although helpful, is not an absolute requirement.

Figure 2: Balance Sheet for Bank *i* in MFRAF

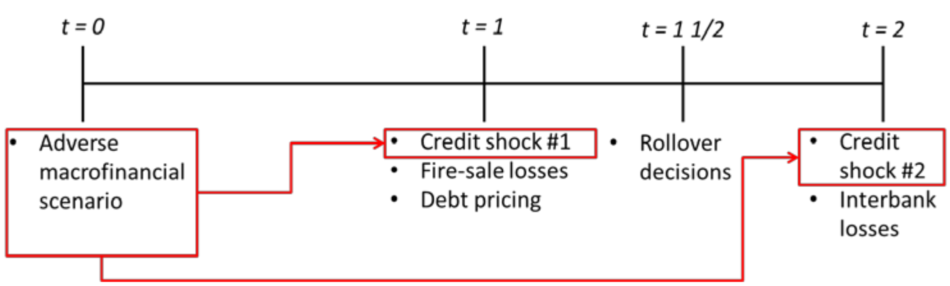
Assets		Liabilities and shareholders' equity
M_i − *Cash*		D_{L_i} − *Long-term Debt*
X_i − *Securities*	HR Govt Secs	
	MR & LR Govt Secs	
	CMBs	
	NHA MBS & Agency	
	CMBS, MBS and ABS	
	Corp Bonds IG	
	Corp Bonds HY	D_{S_i} − *Maturing Liabilities*
	Equities	
Y_i − *Illiquid Assets [Loans]*[a]	Consumer[#]	
	Corporate[##]	E_i − *Equity*
	Other[###]	

[a] Illiquid assets represent a residual category that contains loans, among other categories of assets.
[#] Broken down into: Residential Mortgages (Uninsured); HELOCs (Uninsured); Consumer Loans; Credit Cards.
[##] Broken down into: Agriculture; Construction and Real Estate; Financial Institutions; Fishing and Trapping; Lease Receivables; Logging and Forestry; Manufacturing; Mining, Quarrying and Oil; Multiproduct Conglomerates; Non-Residential Mortgages; Other; Retail Trade; Service; Transportation, Communication and Other Utilities; Wholesale Trade; Retail Trade.
[###] Broken down into: Canadian Governments; Foreign Governments.

A detailed description of each module follows.

3.1. Solvency-risk module

Figure 3: Timeline—Solvency-Risk Module



The solvency-risk module generates the distributions of credit losses that, together with the initial market losses, drive the other channels of risk propagation. These losses are assumed to affect banks' balance sheets at two dates, as shown in **Figure 3**, making MFRAF a multi-period model where expectations of future losses drive the behaviour of agents.

MFRAF does not rely on a single value for credit losses, but on a distribution of expected credit losses for each bank–date pair to reflect that credit losses are uncertain. At each of these two dates, the respective $n\ by\ N$ matrix of simulated credit losses $\left(\boldsymbol{p}_t = \begin{bmatrix} p_{1,1,t} & \cdots & p_{1,N,t} \\ \vdots & \ddots & \vdots \\ p_{n,1,t} & \cdots & p_{n,N,t} \end{bmatrix}\right)$ is generated as follows:

$$p_{i,j,t} = \sum_{s=1}^{nl} PD_{i,j,t}^{s} \times LGD_{i,t}^{s} \times EAD_{i,t-1}^{s}\text{, with i} \in \{1,2,\dots,n\},$$

where $p_{i,j,t}$ is the simulated total expected credit loss of bank i for the j^{th} draw (out of N draws) of the realization of the random variable at date $t \in \{1,2\}$; nl is the number of sectors in the loan book; $PD_{i,j,t}^{s}$ is the j^{th} realization of the (random) probability of default for sector s referring to the period between dates t and $t-1$ for bank i; $LGD_{i,t}^{s}$ is the calibrated loss-given-default for sector s at date t, referring to bank i; and $EAD_{i,t-1}^{s}$ is the calibrated exposure at default for sector s at date $t-1$, referring to bank i.[11]

The stochastic component of credit losses stems uniquely from the distributions of probabilities of default, as **Figure 4** illustrates. LGD and EAD are calibrated for a given scenario. The distributions of probabilities of default (PDs) are generated based on satellite models that link the macrofinancial scenario with PDs.

In this process, the residuals of the satellite models (i.e., the components of the probabilities of default that are not explained by the models) are interpreted as shocks that create a gap between the realized PDs and their expected values.[12]

[11] Given the distributional approach to credit-loss modelling, EAD_t is adjusted to reflect the simulated expected losses as calculated based on the realized draws for PD_{t-1}.

[12] See Henry and Kok (2013); and Covas, Rump and Zakrajšek (2014) for a description of related approaches that use estimation residuals of satellite models to generate distributions of expected losses.

Figure 4: Schematic of the Generation of Credit-Loss Distributions (numeric values are for illustrative purposes only)

j = N	Exposure at default (EAD)	*	Probability of default (PD)	*	Loss-given default (LGD)	=	Total loss
Sector 1	$100		8.5%		70%		$5.95
							...
							...
							...
							...
							..
							...
					$p_{1,N,t}$	=	$29

j = 1	Exposure at default (EAD)	*	Probability of default (PD)	*	Loss-given default (LGD)	=	Total loss
Sector 1	$100		8%		70%		$5.6
Sector 2	$50		10%		90%		$4.5
Sector 3	$500		1%		20%		$1
...							
Sector $nl-2$	$60		2%		35%		$0.4
Sector $nl-1$	$90		3%		35%		$0.9
Sector nl	$120		3%		30%		$1.1
					$p_{1,1,t}$	=	$27

The impact of these credit shocks is mitigated, however, by the net income before credit losses and before amplification effects, such that the resulting retained earnings contribution before amplification effects is calculated as follows:

$$Retained\ earnings_{i,t} = (NetII_{i,t} - p_{i,t}) \times (1 - \tau_{i,t}) - Dividends_{i,t},$$

where $NetII_{i,t}$ is the sum of the net interest income and non-interest income minus non-interest expenses for bank i at date t, $\tau_{i,t}$ is the applicable tax rate, and $Dividends_{i,t}$ are the dividends distributed by bank i at date t.

MFRAF relies on two satellite models, which are briefly described in Appendix A, and expert judgment to calibrate banks' income under stress.

In addition, in top-down applications $Dividends_{i,t}$ may be constrained by the capital conservation buffer (CCB) as per OSFI's guidelines.[13] Thus, $Retained\ earnings_{i,t}$ may be subject to adjustments in top-down applications.

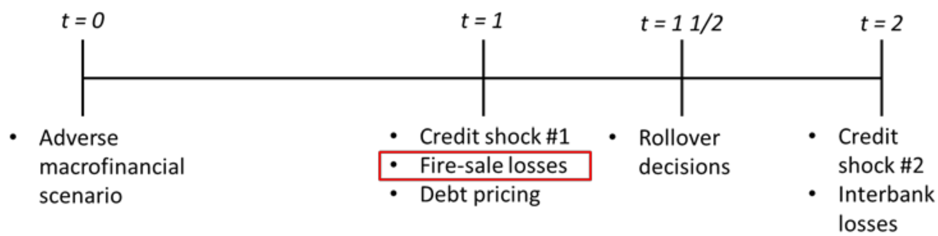
In addition to uncertainty about credit shocks, market participants are also likely to be uncertain about the banks' income-generating potential under the risk scenario. The errors of the bank-income satellite models can be used to produce a distribution of $NetII_{i,t}$ in much the

[13] See http://www.osfi-bsif.gc.ca/Eng/fi-if/rg-ro/gdn-ort/gl-ld/Pages/CAR_chpt1.aspx.

same way that the errors of the satellite models used to simulate probabilities of default under stress are used to produce a distribution of credit shocks.

3.2. Fire-sale module

Figure 5: Timeline—Fire-Sale Module



In this module,[14] the initial credit and market shocks are applied to the banks' initial solvency positions, leading to a deterioration of their leverage ratios. If this deterioration is limited such that the regulatory leverage constraint is satisfied for every bank, then the module concludes and there are no fire-sale losses.

Conversely, if at least one bank becomes leverage-constrained, the model assumes that de-leveraging occurs through securities sales that are used to repay maturing liabilities.[15] The resulting spiral of forced securities sales and MTM losses leads to fire-sale losses that affect all banks that hold similar securities. Formally, the module operates as follows:

1. Compute the leverage ratio for bank i after initial credit $\left(p_{i,j,1}\right)$ and market $\left(h_0\, X_{i,0}\right)$ shocks by applying a haircut (h_0) to the calibrated initial securities' holdings $\left(X_{\mathrm{i},0}\right)$ as shown in **Figure 5** for every draw j of the credit shock, and compare it with the regulatory leverage ratio:

$$\frac{E_i - p_{i,j,1} - h_0\, X_{i,0}}{M_{i,0} + Y_{i,0} + (1-h_0)\times X_{i,0}} \geq LR_{req}.$$

[14] The fire-sale module was developed by Bradley Howell while he was employed at the Bank of Canada.
[15] Note that securities sales alone would not lead to a change in the leverage ratio, as the decrease in assets from these sales would be compensated by an equal increase in cash holdings.

2. If the leverage constraint is satisfied for every bank, terminate the module.
3. If at least one bank is leverage-constrained, initiate the de-leveraging process:
   a. Randomly select one of the leverage-constrained banks;
   b. Calculate the loss for bank i from selling a prespecified lot size (LS) of each security type as a preliminary step to determining which securities sales would minimize the impact on solvency:
      i. Quantify sales that need to be effected to reach lot size *LS*, for each security type l, as a fraction of the post-market shock holdings:

$$\epsilon_{i,l} = \frac{LS}{current\ price_l \times (1 - h_{0,l}) \times X_{i,0,l}};$$

      ii. Calculate the potential future prices (i.e., the prices that would follow sales if these were carried out) for any given sale of a single security:

$$potential\ future\ prices_{i,l} = \rho_l \left(\frac{\sum_k [\alpha_{k,l} \times (1 - h_{0,l}) \times X_{k,0,l}]}{\sum_k [(1 - h_{0,l}) \times X_{k,0,l}]} + \epsilon_{i,l} \times \frac{(1 - h_{0,l}) \times X_{i,0,l}}{\sum_k [(1 - h_{0,l}) \times X_{k,0,l}]} \right),$$

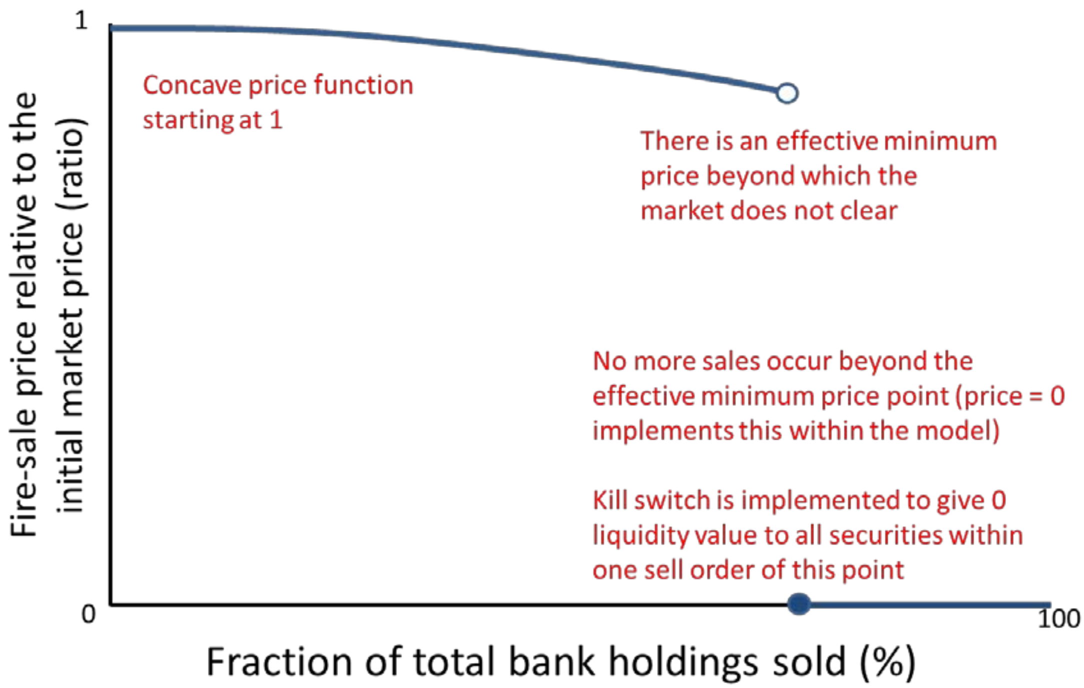
where $\alpha_{k,l}$ is the fraction of the holdings of security type l (after the initial market shock) that bank k has already sold in the previous round of fire sales, and ρ_l is the corresponding price-response function.

The price response to a sale is expressed as the ratio of the price after the sale to the price registered immediately after the initial market shock. The higher the fraction of total bank holdings of this particular security that has been sold, the larger the price impact is, up to a critical threshold.

This threshold is defined with respect to a price below which the market does not clear. In other words, below this minimum price (ratio) the lack of market depth makes it impossible to liquidate any lot of the security. For example, the market price can decrease significantly enough to make sellers decide to delay the liquidation of the security in expectation of higher prices in the future. This type of security is not eligible for sale if this threshold is reached.

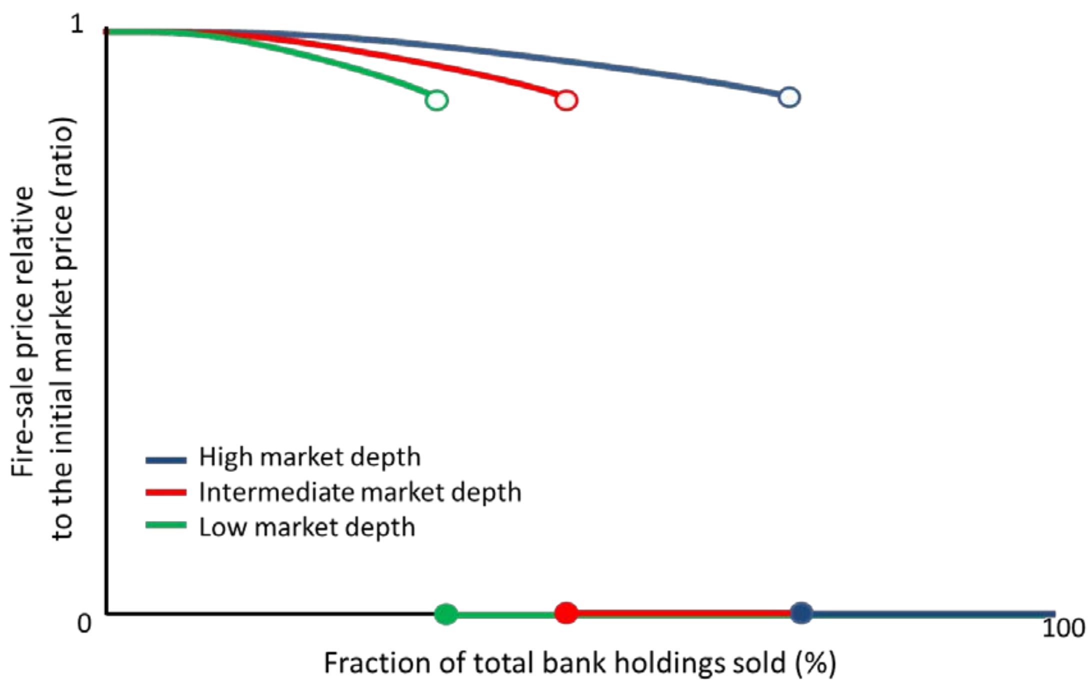
Naturally, the properties of this function vary by security, the extent to which other players (e.g., buy-side investors) are affected by the given stress scenario and the precise nature of the stress scenario. **Figure 6** and **Figure 7** illustrate graphically the properties of this function.[16]

Figure 6: Example of a Price-Response Function



[16] The calibration of the price-response function for each type of security is risk-scenario-specific and, in addition to the banks considered in MFRAF, takes into account the various types of market participants that hold the securities. These other participants are not modelled in MFRAF per se, but the calibration, which is based on financial market expertise, takes into account how these other participants would be affected by the risk scenario and what their reaction would be (e.g., whether they would also be selling the securities considered, thus adding to the downward price pressures, or whether they would keep or even increase their holdings of these securities). For an alternative approach, see Cifuentes, Ferrucci and Shin (2005).

Figure 7: Illustrative Price-Response Function for Different Levels of Market Depth

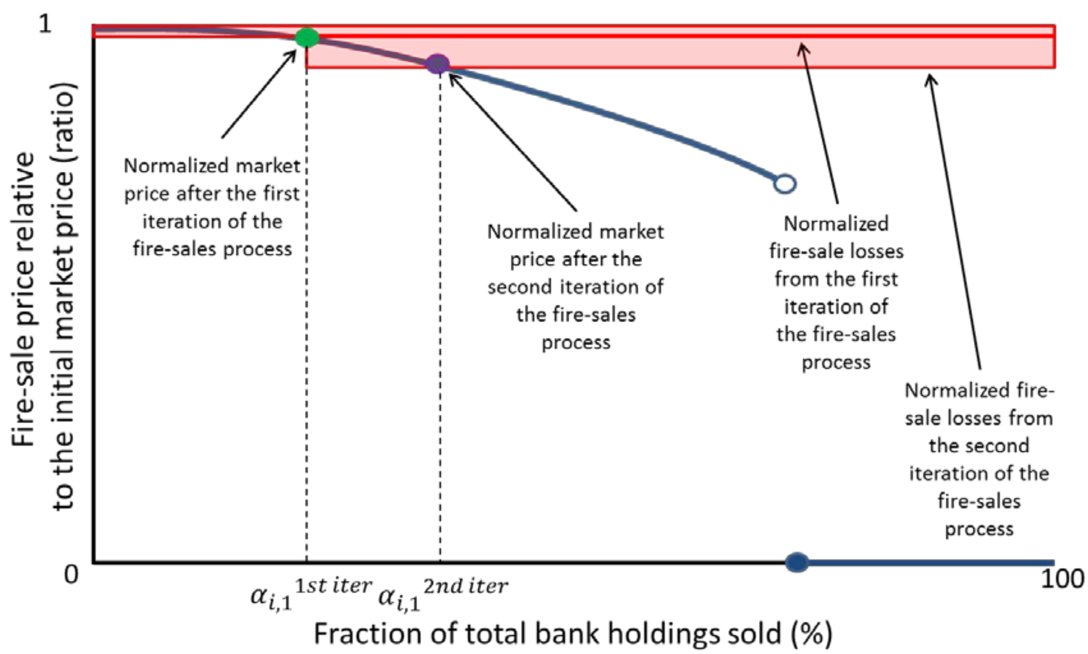


iii. Calculate MTM losses for bank *i* for the potential sale of each security type:

$$potential\ MTM_{i,l} = (current\ prices_l - potential\ future\ prices_{i,l})\times(1-\alpha_{i,l})\times(1-h_{0,l})\times X_{i,0,l}.$$

Figure 8 illustrates the functioning of the price-response curves and the assessment of the potential MTM losses for the case of one bank and one security.

Figure 8: Iterative Quantification of Fire-Sale Losses



c. Execute the sale that minimizes losses at iteration k.[17] The proceeds of this sale are given by:

$$sales^k{}_{i,l} = future\ price_{i,l} \times \epsilon_{i,l} \times (1 - h_{0,l}) \times X_{i,0,l}.$$

However, this sale causes a downward pressure on the prices of the securities types sold, which, through MTM losses, further deteriorates other banks' leverage ratios:

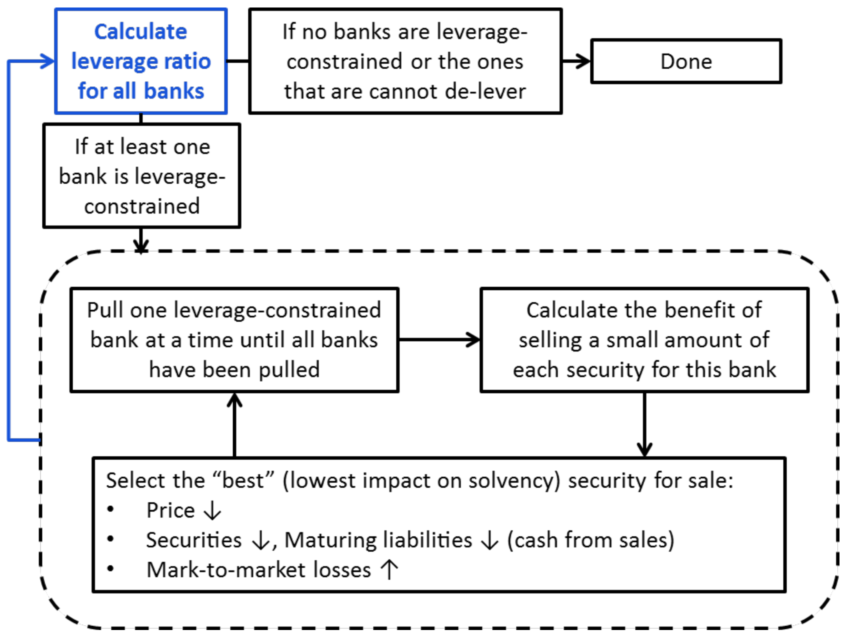
$$LR_{i,j} = \frac{E_i - p_{i,j,1} - h_0\, X_{i,0} - \sum_l MTM_{i,l}}{M_{i,0} + Y_{i,0} + (1 - h_0) \times X_{i,0} - \sum_l MTM_{i,l} - \sum_k \sum_l sales^k{}_{i,l}}.$$

4. Repeat steps 1 to 3 until no bank is leverage-constrained or banks cannot take actions to further de-lever.[18] Compute MTM losses to be reflected on banks' CET1 ratios in subsequent modules.

[17] Note that the recursive nature of the sales process can give rise to a final outcome where different types of securities are sold, even though in each iteration only one type of security is chosen.
[18] An internal flag is raised if there are still leverage-constrained banks that cannot de-lever.

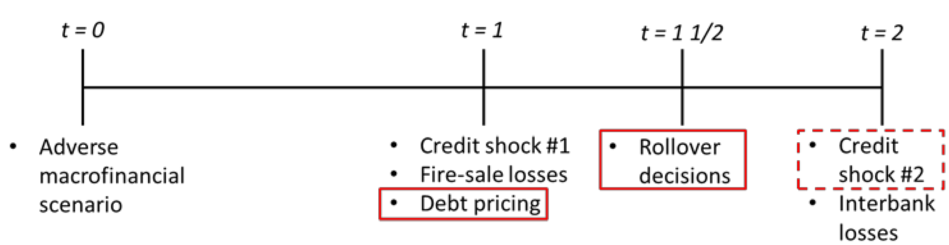
Figure 9 provides a schematic description of the previous steps.

Figure 9: Schematic of the Fire-Sale Module



3.3. Liquidity-risk module

Figure 10: Timeline — Liquidity-Risk Module



After the initial credit and market shocks and the ensuing fire-sale losses, the bank's solvency position (which is defined with respect to a default threshold in MFRAF, such as the 4.5 per cent CET1 minimum capital ratio) deteriorates.

This deterioration leads to an increase in the probability of default, thus causing an increase in the cost of newly issued unsecured wholesale funding. In turn, the increase in the cost of funding that is not passed onto bank clients further deteriorates the bank's solvency position. [19] Solvency and funding costs are therefore intrinsically linked. Thus, a good understanding of the determinants of a bank's access to funding (and of their interaction with solvency concerns) is crucial to quantifying the impact of the stress scenario on the resilience of the banking system. This is the purpose of the liquidity-risk module.

As **Figure 2** shows, a fraction of a bank's liabilities matures over the course of the exercise horizon (D_S). We assume that the bank attempts to access the unsecured wholesale funding market at $t = 1$ to raise the funding required to replace these liabilities.[20] This approach allows us to assess whether the bank is indeed able to access the unsecured funding market and under what price conditions. The model-implied required rate on the newly issued funding is then used to compute the additional cost of funding attributed to the feedback effects between solvency and liquidity risks.

In the liquidity-risk module, we assume that banks obtain funding through a short-term debt contract at $t = 1$. Given its short-term nature, this contract exposes banks to rollover risk. That is, a bank is exposed to the risk that a fraction of its creditors decide to withdraw at $t = 1\,1/2$ instead of $t = 2$. If the fraction of creditors withdrawing is large enough, the bank may be forced to liquidate assets, which may occur at fire-sale prices. Thus, the ability of the bank to honour its obligations with its creditors may depend on the actions taken by the remaining creditors.[21]

Naturally, if the bank remained solvent at $t = 2$ regardless of the fraction of creditors withdrawing at $t = 1\,1/2$, each individual creditor would not have any incentive to withdraw. Conversely, if the bank was unable to honour its obligations with its creditors at $t = 2$ even if no creditor decided to withdraw at the interim date, each individual creditor would not have

[19] The increase in the cost of funding is defined as the spread between the model-implied cost of funding and a reference rate consistent with the simulated values for the net interest margin. MFRAF assumes a constant pass-through rate. More details are provided in **Table 2**.
[20] The calibration of these maturing liabilities is based on cash outflows reported by banks for liabilities with similar characteristics to those described in this module.
[21] For the sake of simplicity, the loss-given-default experienced by a creditor who decides to roll over should the bank experience a run is assumed to be constant and calibrated based on expert judgment.

any incentive to provide funding. A more complex case arises when the early liquidation costs stemming from withdrawals determine whether or not banks are able to remain solvent at $t = 2$. Formally, this problem is analyzed by modelling the interaction of a bank with its creditors as a coordination game with strategic complementarities (see Appendix B).

In this coordination game, the bank offers investors an unsecured debt contract that can be redeemed at $t = 1\,1/2$ or at $t = 2$ without penalty. The rollover decision, however, is delegated to professional fund managers, who receive a noisy private signal with respect to the credit shock that will affect the bank at $t = 2$. Based on this signal, each fund manager decides whether or not to withdraw the funding provided. As the formal model[22] in Appendix B shows, the bank becomes unable to honour its obligations at $t = 2$ if the credit shock exceeds a certain threshold, called the illiquidity threshold. Importantly, this threshold depends on the following factors:

- The early liquidation costs resulting from fund managers' decisions to withdraw and the associated need for banks to liquidate assets to offset these withdrawals. These depend on the assets available for sale and the corresponding market liquidity conditions.[23] The higher these costs are, the less likely it is that the bank will be able to access unsecured funding.
- The bank's solvency prospects. These depend on the bank's initial solvency position, the magnitude of the credit and market shocks, and their profitability. The better these prospects are, the more likely it is that the bank will be able to access unsecured funding.
- The degree of fund managers' conservativism. In the model, fund managers are motivated by their own remuneration prospects, which depend on making the correct choice. That is, if they decide to continue lending to the bank and the bank remains solvent, they benefit from inflows. However, if they withdraw and the bank remains

[22] The formal model closely follows Ahnert et al. (2016) and Rochet and Vives (2004).

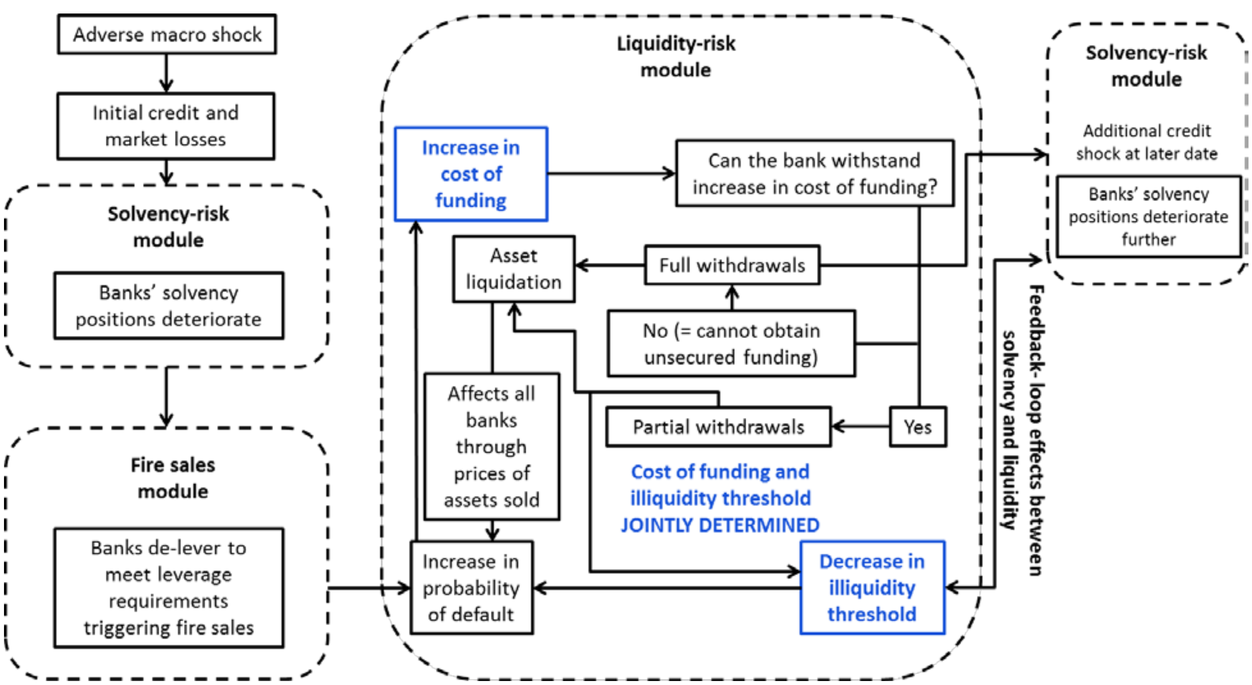
[23] The initial market liquidity conditions and price impact of sales follow from the preceding fire-sale module. However, for tractability reasons, prices are assumed to return to the levels implied immediately after the fire-sale module once the early liquidation costs are calculated for the assets sold by the bank to offset the withdrawals.

solvent, they are penalized with outflows (see Rochet and Vives 2004). The more conservative fund managers are about this trade-off, the less likely it is that the bank is able to access unsecured funding.

In addition to the factors listed above, the illiquidity threshold also depends on the face value of debt (or, alternatively, the cost of funding) offered by banks to investors at $t = 1$. Similarly, the conditions under which fund managers will roll over the bank's debt influence the face value of debt that banks offer to investors in the first place. Thus, the illiquidity threshold and the face value of debt are jointly determined in equilibrium.

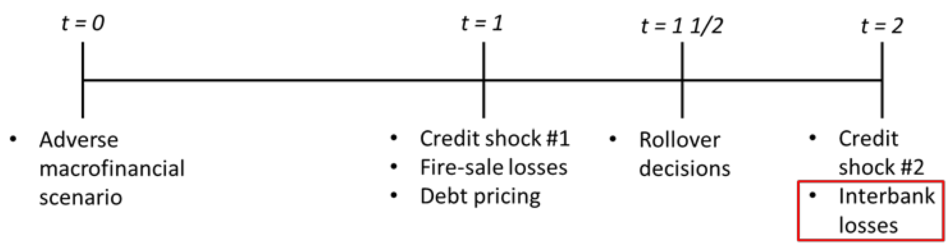
If such determination is not possible (i.e., there is no solution to the equations specifying the illiquidity threshold and the face value of debt), the bank cannot access unsecured funding and must liquidate assets to repay the initial maturing liabilities. Furthermore, the factors listed above affect the pricing conditions similar to the way in which they affect access to unsecured funding. **Figure 11** summarizes the liquidity-risk module.

Figure 11: Schematic of the Liquidity-Risk Module



3.4. Interbank module

Figure 12: Timeline—Interbank Module



As shown in **Figure 12**, the interbank module is the last component of MFRAF and captures the counterparty losses that D-SIBs would experience as a result of their claims on other D-SIBs should the latter default.

Some banks may become unable to fulfil their obligations to other D-SIBs due to credit, fire-sale and liquidity losses that lead to the deterioration of their solvency positions. If a D-SIB defaults, its interbank creditors suffer a loss, which may in turn make them unable to fulfil their own interbank obligations. Therefore, these interbank losses may depend not only on the solvency of banks' immediate counterparties, but also on the solvency of the counterparties of their own counterparties.

MFRAF computes these losses by using the clearing payment vector approach developed by Eisenberg and Noe (2001).[24] The clearing vector approach assumes that interbank claims[25] are junior to all other non-equity claims. We divide the stylized balance sheet into assets and liabilities according to whether these originate inside (internal assets/liabilities) or outside (external assets/liabilities) the network of D-SIBs (**Figure 13**). External liabilities are liabilities to non-D-SIBs, such as retail deposits, while internal liabilities are the internal assets of the banks' D-SIB counterparties.

[24] Gauthier, He and Souissi (2010) provide an intuitive example of how the clearing payment vector works.
[25] These include deposits, loans, bankers' acceptances, reverse repurchase agreements, debt holdings and over-the-counter derivatives. See **Table 1** for details.

Figure 13: Stylized Balance Sheet for Bank *i* Based on the Origin of Assets and Liabilities

Assets	Liabilities and shareholders' equity
EA_i – *External Assets*	EL_i – *External Liabilities*
	IL_i – *Internal Liabilities*
IA_i – *Internal Assets*	E_i – *Equity*

Let us denote the interbank exposures matrix by Π, where π_{ij} is the nominal exposure of bank i to bank j at $t = 2$, and $i, j \in \{1, \ldots, n\}$.[26] Then, internal assets and liabilities are defined, respectively, as

$$IA_i = \sum_{j=1}^{n} \pi_{ij},$$

$$IL_j = \sum_{i=1}^{n} \pi_{ij}$$

and the matrix of normalized interbank exposures is given by

$$\widetilde{\Pi} = \begin{bmatrix} 0 & \ldots & \frac{\pi_{1n}}{IL_n} \\ \ldots & \ldots & \ldots \\ \frac{\pi_{n1}}{IL_1} & \ldots & 0 \end{bmatrix}.$$

Thus, the value of equity is calculated as the sum of external assets after losses and incoming payments minus outgoing payments:

$$EA_{after\ credit, fire-sale\ and\ liquidity\ losses, i} + \sum_{j=1}^{n} \tilde{\pi}_{ij} IL_j - EL_i - IL_i.$$

Assuming limited liability and proportional repayment, the total payments made by banks are determined by the following *clearing payment vector*:[27]

[26] An example of this type of exposure is an interbank loan made by bank i to bank j. By convention, $\pi_{ii} = 0$.

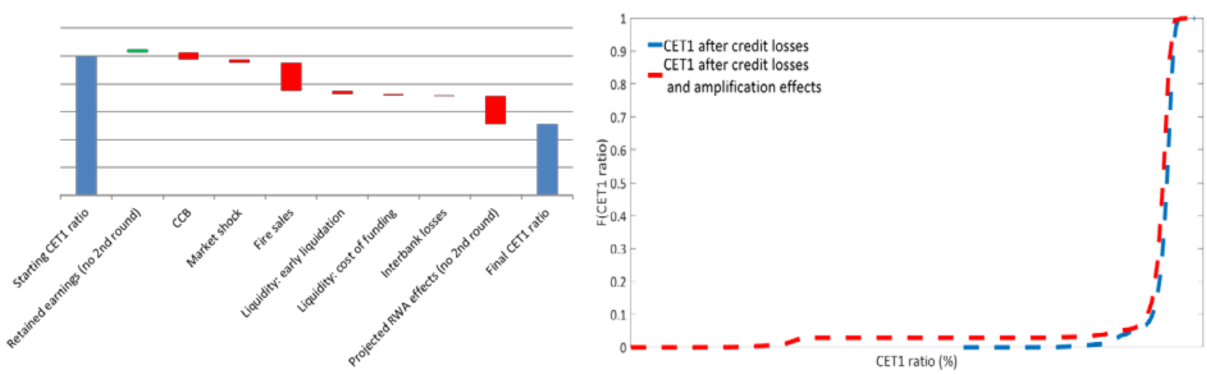
$$\wp_i^\star = min\left\{IL_i, max\left\{0, EA_{after\ credit, fire\ sale\ and\ liquidity\ losses, i} + \sum_{j=1}^{n} \tilde{\pi}_{ij}\wp_j^\star - EL_i\right\}\right\}.$$

The losses bank i imposes on other D-SIBs equal the difference between the IL_i and $\wp_i^\star$.

3.5. System-wide distributions of losses

MFRAF's modular structure allows us to decompose the drivers of the evolution of banks' capital positions, as illustrated in **Figure 14**. The figure shows the summary of the attribution analysis through the means of the distributions of the respective effects (left) and the cumulative distribution of the CET1 ratio after both first- and second-round losses are factored in (right).[28] This decomposition can be done at both an aggregate and individual bank level. Note that the retained earnings contribution before amplification effects does not include MTM losses calculated in the fire-sales module and changes to risk-weighted assets (RWA), which are calibrated using a satellite model.

Figure 14: Illustrative Attribution Analysis (numeric values for illustrative purposes only)



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[27] Eisenberg and Noe (2001) prove the existence and uniqueness of this payment vector under certain conditions.

[28] Note that since the model considers a distribution for credit shocks, which are amplified by the second-round effects, MFRAF produces a distribution for CET1 ratios after both first- and second-round losses are reflected on banks' balance sheets. These amplification effects are noticeable in the tail of the distribution. That is, for the most extreme draws of the credit shocks, the CET1 ratio is substantially lower after the second-round effects are factored in compared with its values when only first-round losses are taken into account.

4. Data Requirements

Table 1 lists the data and parameter requirements. Note that the data sources differ, depending on the type of model application (top-down or OSFI–Bank of Canada MST "hybrid" application).

Table 1: Data and Parameter Requirements

Data description	Frequency	Source[29]		Format	Start date
		Top-down	Hybrid		
Illiquid assets	Monthly	Calibrated based on the Balance Sheet (M4) regulatory return		$ millions	1981
Total assets				$ millions	1981
Liquid assets		Net Cumulative Cash Flow Report (NCCF)		$ millions	2015[30]
Maturing liabilities				$ millions	2015
Net interest income	Quarterly	NIM satellite model	Banks' bottom-up submissions	$ millions	1994
Non-interest income		NINT satellite model		$ millions	1994
Non-interest expenses		Calibrated based on the Income Statement (P3) regulatory return		$ millions	1983
Tax rates				per cent	1983
Dividends				$ millions	1993
Exposures at default (EADs)		Calibrated based on Non Mortgage Loans (A2), Mortgage Loans (E2) and Monthly Average Return of Assets and Liabilities (L4) regulatory returns	Banks' bottom-up submissions	$ millions	1993

[29] As indicated in the introduction, MFRAF can be used as both a hybrid model in the OSFI–Bank of Canada MST and as a top-down model for internal applications. The source of the data used as input varies, depending on the application considered.

[30] This date refers to the current template of the Net Cumulative Cash Flow return. The template and revision history can be found at http://www.osfi-bsif.gc.ca/eng/fi-if/rtn-rlv/fr-rf/dti-id/Pages/NCCF.aspx.

Probabilities of default (PDs)	Quarterly	Corporate PD Model and HRAM[31]	Banks' bottom-up submissions	per cent	n/a
RWA	Quarterly	RWA satellite model[32]	Banks' bottom-up submissions	$ millions	2013
Initial CET1 capital ratio	Quarterly	Basel Capital Adequacy (BCAR(BA)) regulatory return	Banks' bottom-up submissions	per cent (ratio)	2013
Interbank matrix	Monthly	Interbank and Major Exposures (EB/ET) regulatory return		$ millions	2012

5. Calibration

Table 2 provides a high-level description of the elements of the model with calibration requirements, which vary with how the model is applied. Some parameters are calibrated based on banks' internal model estimates in "hybrid" applications.

Table 2: Calibration Guidelines

Elements requiring calibration		High-level description of elements to consider for calibration
Module	**Element**	
	LGD	Loss-given-defaults (LGDs) must be calibrated by Bank staff for top-down applications of MFRAF. This calibration can be informed by various sources of information (e.g., LGDs considered by other central banks in their stress tests, realized loss rates on banks' loan portfolios in countries that experienced severe stresses, LGDs reported by banks in MST exercises, etc.). Sensitivity analyses are typically conducted to assess the sensitivity of results to the calibration of the LGDs.
	PD	Starting-point PDs: the calibration of PDs for top-down MFRAF applications requires judgment on starting-point PDs, since these are not readily available

[31] See Bruneau and Djoudad (forthcoming) and Peterson and Roberts (2016), respectively.

[32] See footnote 9.

		from supervisory data. Values reported by banks in MST exercises are usually combined with expert judgment on the evolution of PDs regarding the time elapsed between reporting for the latest MST and the top-down application.
	Non-interest expenses	Calibrated based on the Income Statement (P3) regulatory return. For top-down applications, the calibration of non-interest expenses is based on efficiency ratios (per cent).
	Tax rates	
	Dividends	
Fire-sale module	Price-response functions	Price-response functions are characterized by two elements: market depth and the maximum price decline, both of which are affected by the stress scenario considered. Market depth is, to some extent, a structural feature of an asset market (e.g., the Government of Canada bond market is much deeper than the corporate bond market) and thus is somewhat invariant to stress scenarios. However, it is important for the calibration to take into consideration how the various players in this market would be affected by the stress scenario considered in order to reflect how market depth could be affected. The calibration of the parameters of these functions relies on expert judgment of staff from the Bank's Financial Markets Department.
	Minimum leverage ratio	The regulatory leverage ratio defined by OSFI[33] relies on measures of capital and exposures that are different from those in MFRAF. Consequently, the model accommodates the different capital and exposure measures by determining a buffer above the regulatory minimum that banks hold at the beginning of the exercise. If the capital decline due to losses, in terms of CET1, exceeds this buffer, then the bank becomes leverage-constrained. More precisely, $$MFRAF\_cal.levLimit = \frac{CET1\ Capital - Buffer}{Total\ Assets - Buffer},$$ where $$OSFI\ levLimit = \frac{OSFI\ levLimit\ Capital\ Measure}{OSFI\ levLimit\ Exposure\ Measure} = \frac{All - in\ Tier\ 1\ Capital}{\begin{gathered} On - Balance - Sheet\ Exposures + \\ Derivative\ Exposures + \\ Security\ Financing\ Transaction\ Exposures + \\ Off\ Balance\ Sheet\ Exposures \end{gathered}};$$ $$Buffer = \frac{OSFI\ levLimit\ Capital\ Measure - OSFI\ levLimit \times OSFI\ levLimit\ Exposure\ Measure}{1 - OSFI\ levLimit}.$$

[33] See http://www.osfi-bsif.gc.ca/eng/fi-if/rg-ro/gdn-ort/gl-ld/Pages/lr.aspx#cal.

	Lot size	The size of the lot of securities sold in each step of the fire-sale module must be small enough to have a limited impact on prices.
Liquidity module	Maturing liabilities	Unsecured wholesale funding subject to rollover. The calibration of the various funding sources under stress reflects the characteristics of the funding source (i.e., more or less stable) and the features of the stress scenario (e.g., in a scenario where housing markets are severely affected, mortgage-backed securities would be proportionally more affected).
	Bank debt investors' outside option (*rF*)	The outside option of bank debt investors reflects the non-default component of the opportunity cost of investing in bank debt. This cost includes the risk-free rate and the additional compensation to hold bank debt-like claims if these were risk-less. Since bank debt is less liquid than Government of Canada bonds, a liquidity premium is a natural choice for this non-default component.
	Bank debt reference rate (*rRef*)	Calibrated rate consistent with banks' income projections with respect to interest expenses on maturing liabilities. The ex post effect on CET1 resulting from the endogenous cost of funding is automatically taken into account by the liquidity-risk module.
	Degree of conservativeness of fund managers (γ)	Rochet and Vives (2004) argue that the decisions of these managers are governed by their compensation. If an entity to which they decided to extend funding goes bankrupt, a manager's relative compensation from rolling over is negative, $-c < 0$. Otherwise, the relative compensation is positive, $b > 0$. The conservativeness ratio $\gamma = \frac{c}{c+b}$ summarizes these payoff parameters. Chevalier and Ellison (1997) estimate the sensitivity of net flows to excess returns in the following year. Using their estimates (see **Figure 1** and **Figure 2**) for the sensitivity of net flows to positive (negative) excess returns to calibrate (c), $\gamma's$ suggested calibration lies between 0.2 and 0.3.
	Default threshold	CET1 ratio below which unsecured wholesale investors experience losses. The regulatory minimum of 4.5 per cent is used as a benchmark.
	Pass-through rate	Rate of increase in cost of funding passed through to bank clients.
	LGD on bank debt	In line with a typical assumption for the valuation of credit default swaps, an LGD of 60 per cent is recommended.
Network module	Interbank exposures	The interbank module assumes proportional priority in liquidation. All classes of interbank exposures (other than cross holdings of equity) are treated as unsecured claims.

6. Simulations

This section presents a set of simulations to illustrate the main mechanisms at play in the new liquidity-risk module. These illustrate the impact of the liquidity-risk losses and their decomposition for a set of simulated scenarios. Each scenario is generated from a calibration of baseline losses based on data referring to the global financial crisis period and then varying the severity of credit/market losses and market liquidity conditions jointly with the level of maturing liabilities.[34]

Note that some of the data used to calibrate the model in this section differ from those listed in Section 4 and Section 5, given that the sources of information used in a typical application for the reference period are only partially available.

The model is calibrated such that the unsecured debt contracts are offered to wholesale investors in 2008Q4 ($t = 1$ in MFRAF), rollover decisions occur in 2009Q2 $(t = 1\ 1/2)$ and the credit shock crystallizes in 2009Q4 ($t = 2$).

[34] Note that since MFRAF's modules are interdependent, the validation of a specific module is invariably a joint test of that particular module and the preceding ones. Thus, a simplified version of MFRAF, consisting of the solvency and liquidity-risk modules, is used to run the simulations. The lack of data on interbank exposures before the third quarter of 2012 also precludes a comparable analysis of interbank network losses.

6.1. Data sources

Table 3 (analogous to Table 1 in typical applications) displays the sources of the data used to calibrate the model.

Table 3: Data Sources—2007–09 Financial Crisis

Parameters	Data source
$\boldsymbol{R}$,[35] $\boldsymbol{I}, \boldsymbol{M}, \boldsymbol{E_0}$	Banks' annual reports
$\boldsymbol{D_S}$	Banks' annual reports and judgment
$\boldsymbol{p_2}$	Panel vector error-correction model (VECM, modified version of Bruneau and Djoudad (forthcoming)) that links macroeconomic variables to provisions for credit losses
$\boldsymbol{r}$[36]	Bank of Canada website

6.2. Calibration strategy

Even though the rate that banks offer is determined endogenously, the investors' outside option must be calibrated.[37] The impact of the materialization of stress has two effects on the cost of funding: an increase in the liquidity premium and an increase in the risk premium.

To account for the increase in the liquidity premium observed during times of stress (see, for example, Longstaff 2004; Vayanos 2004; Brunnermeier 2009; Musto, Nini and Schwarz 2014), we add the Canada Mortgage Bond (CMB)–Government of Canada spread at the end of 2008 to the 3-month overnight index swap (OIS) rate at the end of 2008 in the calibration of the investors' outside option (see **Figure 15**).

[35] In the implementation of the module, all equations are written in terms of capital, which precludes the need to calibrate the precise rate of return on assets.

[36] The calibration also requires a reference funding rate in the absence of stress in order to compute the impact of the liquidity-solvency feedback effects on CET1 ratios.

[37] Even though investors are assumed to be patient and risk-neutral, this assumption may not hold in practice. Therefore, the calibration is not perfectly consistent with the theoretical model.

Figure 15: Canada Mortgage Bond-Government of Canada spread—Proxy for Liquidity Premium (basis points)[38]

Source: Bloomberg.

The main drivers of the solvency-liquidity feedback effects are the initial solvency and liquidity conditions and the distribution of credit shocks that materialize in the fourth quarter of 2009. The distributions of these shocks are generated using the residuals of a modified version of the model in Bruneau and Djoudad (forthcoming) that links macrofinancial variables to charges for impairment and were centered at the values for provisions for credit losses reported in banks' 2009 annual reports.

Market participants are likely to be uncertain about banks' income-generating potential as well as credit shocks. To embed this source of uncertainty into MFRAF, the errors of the bank-income satellite models are used to produce a distribution for net income in much the same way that the errors of the VECM are used to produce a distribution of credit shocks.

As mentioned earlier, since assets do not mature at the end of the last period in MFRAF, the probability of insolvency is calculated with reference to a default threshold. This threshold is set at 4.5 per cent of RWA, in line with the post-crisis regulatory minimum for the CET1 ratio.

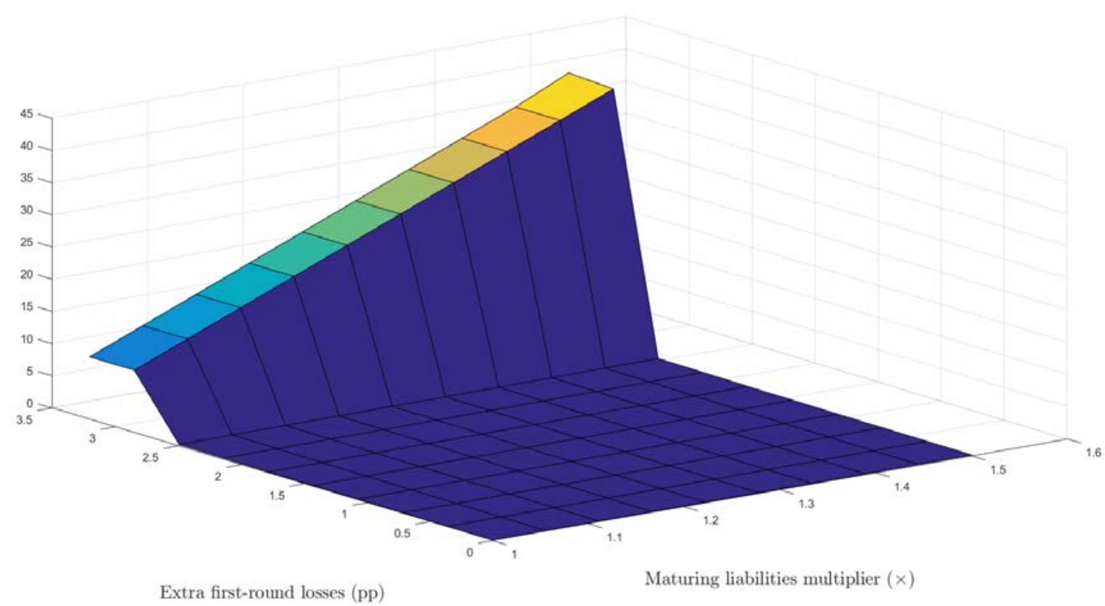
[38] Proxy chosen following Fontaine, Selody and Wilkins (2009).

6.3. Results

The results for the baseline-losses calibration show that the magnitude of the feedback effects is economically small, which is consistent with the overall impact of the financial crisis on the Canadian banking system (see, for example, Arjani and Paulin 2013). Although baseline losses are small, a comparative statics exercise is useful to illustrate the main mechanisms at play.

As explained in Section 3.3, the initial solvency positions of banks and the amount of liabilities that mature over the course of the stress horizon play an important role in determining the liquidity-risk losses that follow from the scenario. For this reason, in this section, we run the model for different combinations of additional first-round losses, expressed in percentage points[39] of RWAs and multiples of the baseline maturing liabilities.[40] **Figure 16** presents liquidity-risk losses, expressed as a percentage of the total assets of D-SIBs, as a function of the two key variables.

Figure 16: Liquidity-Risk Losses as a Percentage of Total Initial Risk-Weighted Assets of the Big Six Banks



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[39] That is, if the mean of the baseline distribution of first-round losses is 3 per cent of RWA, 0.5 indicates that the simulated distribution has a mean of 3.5 per cent of RWA.

[40] That is, 1.5x indicates that the maturing liabilities used in the simulation are 150 per cent of the ones used in the baseline.

The effect of increases in maturing liabilities on total liquidity-risk losses depends strongly on banks' solvency. This is because, as first-round losses become larger, banks are less likely to be able to access the unsecured funding market to obtain funding. If they cannot access this funding, banks are forced to liquidate assets at fire-sale prices to repay maturing liabilities (**Figure 17**).

Fire-sale losses are particularly large when banks' proceeds from liquidating securities are not sufficient to repay maturing liabilities and they are forced to liquidate illiquid assets, leading to large losses. Moreover, even for those banks still able to access the funding market, asset liquidation becomes more costly, given that the sales from liquidated banks introduced a downward pressure in prices.

Overall, these two effects explain the steep ascent of the percentage of total losses attributed to fire sales shown in **Figure 18**. Finally, the combination of costly liquidation of assets to meet funding withdrawals and higher solvency risk leads to increases in the rate required by bank debt investors to provide funds to banks, further increasing liquidity-risk losses.

Figure 17: Number of Excluded Banks Unable to Access the Unsecured Funding Market

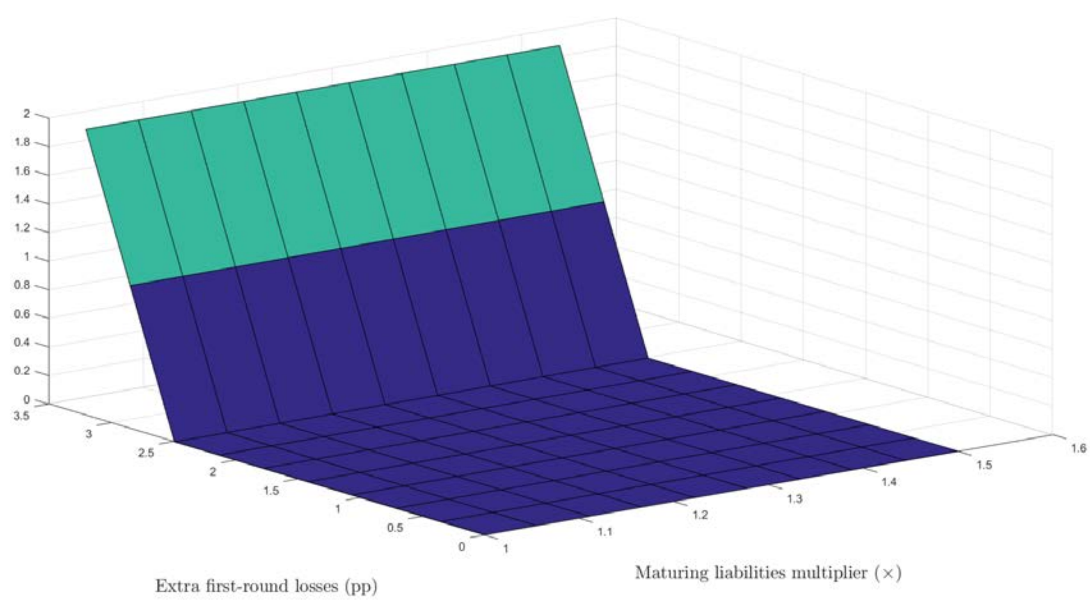
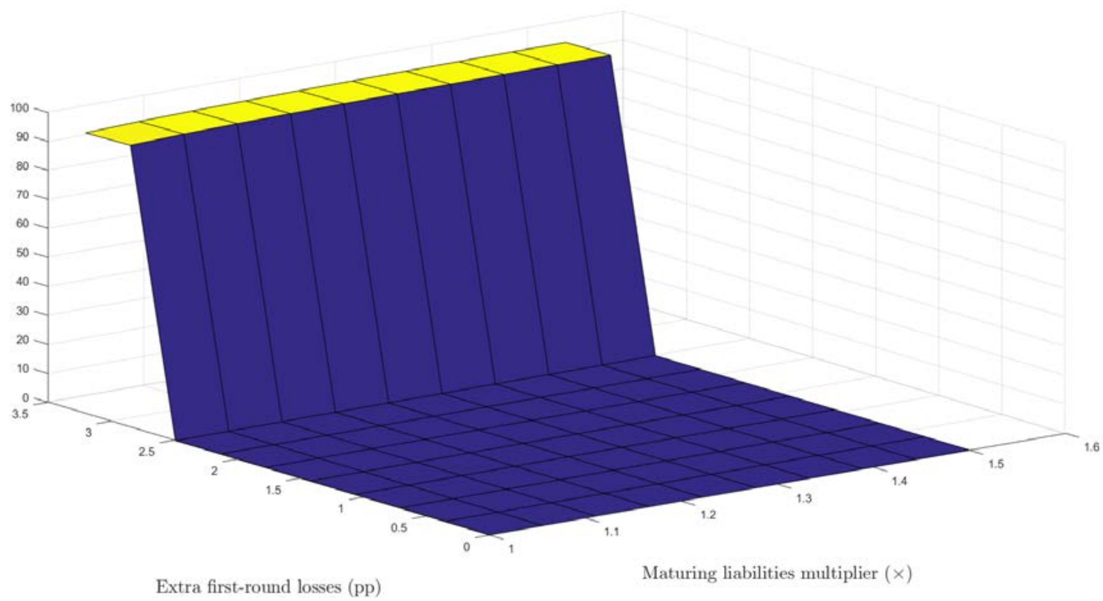
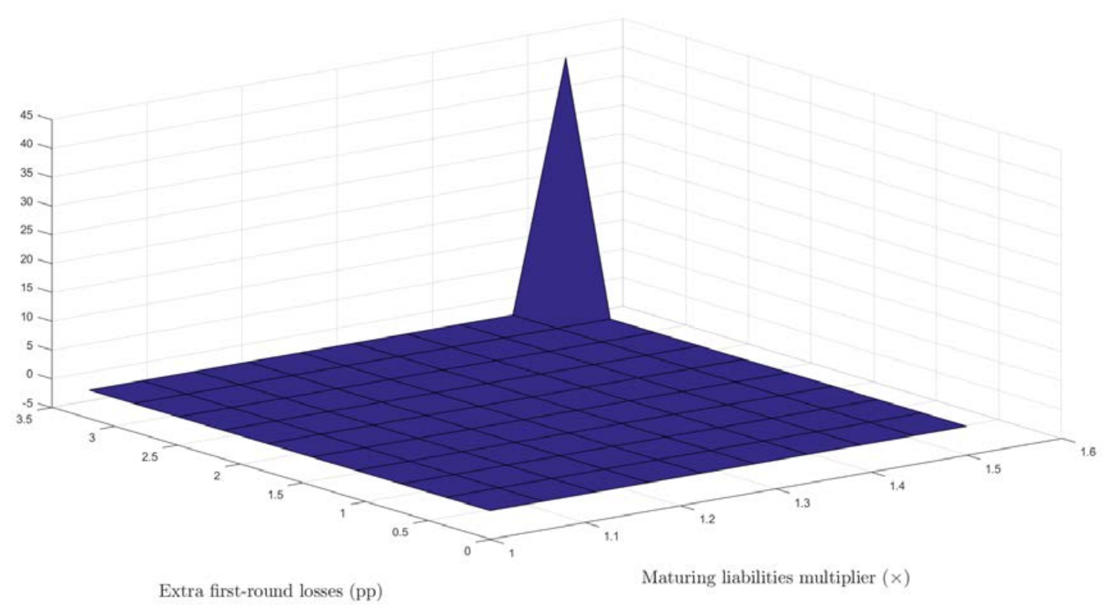


Figure 18: Percentage of Liquidity-Risk Losses Attributed to Fire Sales



It is also instructive to understand to what extent liquidated banks impose additional losses on the remaining banks. To this end, the simulations were repeated, but the mechanism through which the fire sales of each bank affect prices of securities sold by others was artificially shut down.

Figure 19: Difference in Liquidity-Risk Losses Brought about by Contagious Fire Sales (basis points), Market-Depth Baseline Calibration



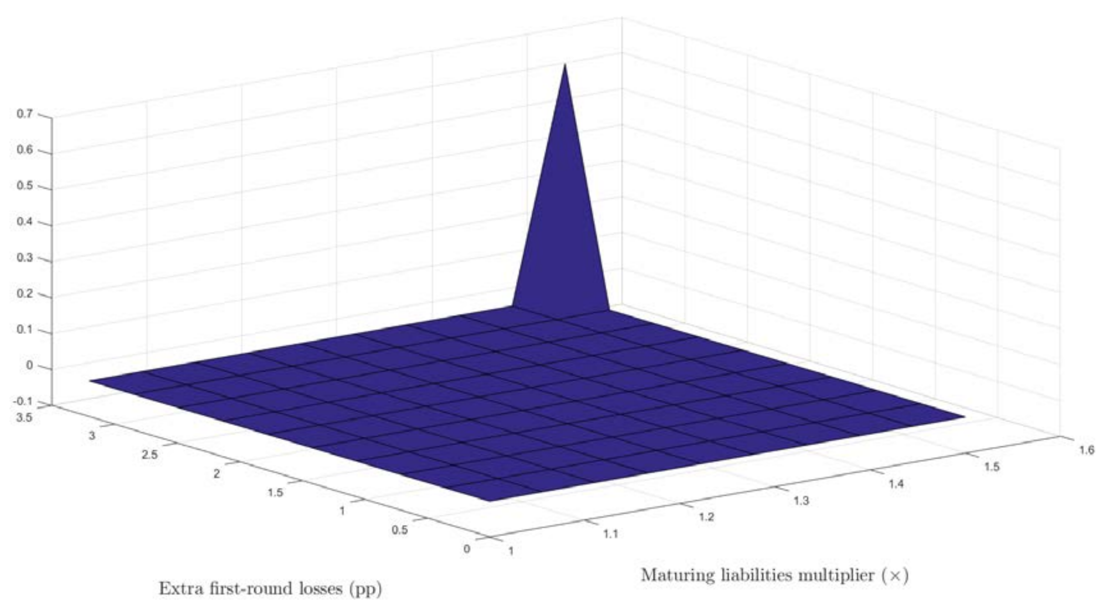
As **Figure 19** shows, the difference in liquidity-risk losses as a fraction of total assets brought about by contagious fire sales is relatively limited and is restricted to a specific region.

All else being equal, as the maturing liabilities multiplier increases, so does the dollar value of the equilibrium withdrawals, which leads banks to sell a higher fraction of their assets to meet those withdrawals. Since the number of banks excluded from the unsecured funding market and the intensity of their asset liquidation both increase non-linearly with additional first-round losses, the market liquidity conditions faced by banks also deteriorate. Thus, for high values of the combination of extra first-round losses and the maturing liabilities multiplier, we observe a positive difference in losses brought about by contagious fire sales.

The absence of large differences is explained by the fact that the number of liquidated banks does not hinge on whether contagious fire sales are present in the simulations.

Not surprisingly, as **Figure 20** shows, the fraction of total losses attributed to fire sales also does not vary substantially with contagion, since the percentage in the baseline calibration is already very high.

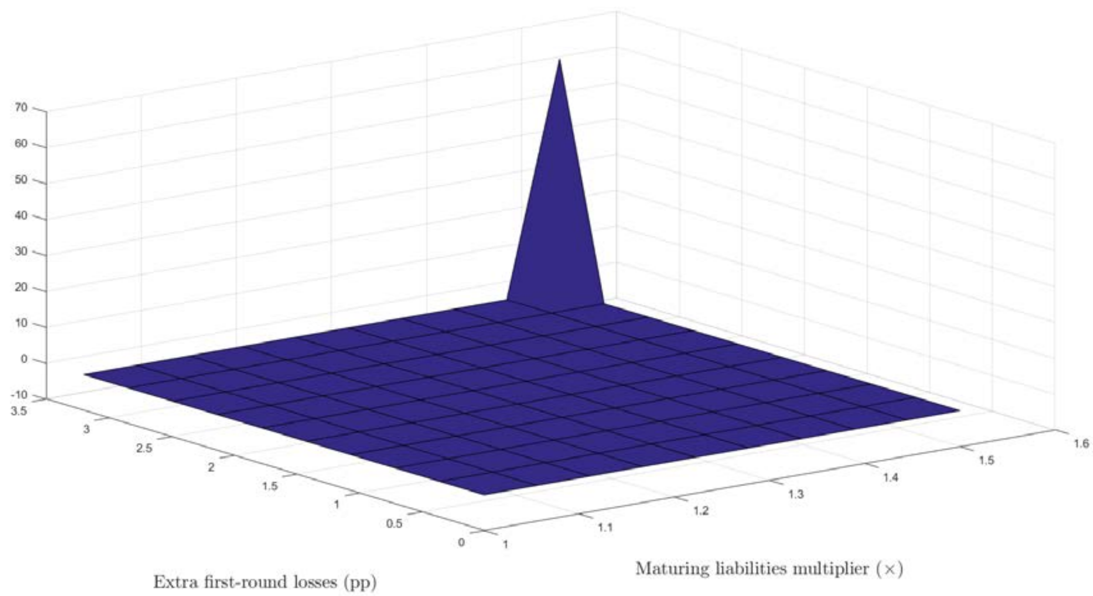
Figure 20: Difference in the Fraction of Fire-Sale Losses Attributed to Contagion (basis points)



Fire-sale losses also depend on market liquidity conditions. To illustrate this effect, we considered a lower market-depth calibration.

All else being equal, the impact on liquidity losses is non-linear and is concentrated at the combination of the highest extra first-round losses and the highest maturing liabilities multiplier (**Figure 21**). The absence of stark changes can be explained by the fact that the reduction in market depth does not lead to a change in the number of banks excluded from the unsecured funding market.

Figure 21: Difference in Liquidity-Risk Losses Brought about by a Reduction in Market Depth (basis points)



7. Conclusion

Banks are exposed to multiple sources of risk that interact in a highly non-linear manner. Consequently, tools aimed at assessing how banks would be affected by stress conditions need to account for these second-round effects to provide useful insights for policy-makers. This technical report describes how the recent enhancements to MFRAF contribute to this ultimate goal. These enhancements consist of an explicit modelling of fire-sale dynamics and feedback-loop effects between solvency and liquidity risks captured through both the pricing and asset-liquidation channels.

As illustrated in the fire-sale and liquidity-risk modules, individual behaviours that can seem desirable from the perspective of an individual bank (e.g., asset liquidations) can ultimately lead to additional losses at other institutions. For this reason, future developments of MFRAF will likely focus on modelling the behaviour of banks under multiple regulatory and market constraints. Understanding how banks behave under stress may not only help to better understand stress transmission channels but may also contribute to the design of policies directed at mitigating systemic risk.

References

Ahnert, T., K. Anand, P. Gai and J. Chapman. 2016. “Asset Encumbrance, Bank Funding and Financial Fragility.” Bank of Canada Staff Working Paper No. 2016-16.

Anand, K., G. Bédard-Pagé and V. Traclet. 2014. “Stress Testing the Canadian Banking System: A System-Wide Approach.” Bank of Canada *Financial System Review* (June): 61–68.

Anand, K., C. Gauthier and M. Souissi. 2015. “Quantifying Contagion Risk in Funding Markets: A Model-Based Stress-Testing Approach.” Bank of Canada Staff Working Paper No. 2015-32.

Angbazo, L. 1997. “Commercial Bank Net Interest Margins, Default Risk, Interest-Rate Risk and Off-Balance Sheet Banking,” *Journal of Banking and Finance* 21 (1): 55–87.

Arjani, N. and G. Paulin. 2013. “Lessons from the Financial Crisis: Bank Performance and Regulatory Reform.” Bank of Canada Staff Discussion Paper No. 2013-4.

Bank for International Settlements (BIS), Financial Stability Board and International Monetary Fund. 2009. “Guidance to Assess the Systemic Importance of Financial Institutions, Markets and Instruments: Initial Considerations.”

Basel Committee on Banking Supervision (BCBS). 2004. "Principles for the Management and Supervision of Interest Rate Risk." Bank for International Settlements.

———. 2009. "Principles for Sound Stress-Testing Practices and Supervision." Bank for International Settlements (May).

Bidder, R., R. Giacomini and A. McKenna. 2016. "Stress Testing with Misspecified Models." Federal Reserve Bank of San Francisco Working Paper 2016-26.

Breuer T. and I. Csiszar. 2013. "Model Risk Plausibility Constraints." *Journal of Banking and Finance* 37 (5): 1552–1559.

Bruneau, G. and R. Djoudad. Forthcoming. "Corporate Probability of Default Model."

Brunnermeier, M. K. 2009. "Deciphering the Liquidity and Credit Crunch 2007–2008." *Journal of Economic Perspectives* 23 (1): 77–100.

Carlsson, H. and E. van Damme. 1993. "Global Games and Equilibrium Selection," *Econometrica* 61 (5): 989–1018.

Chevalier, J. and G. Ellison. 1997. "Risk Taking by Mutual Funds as a Response to Incentives." *Journal of Political Economy* 105 (6): 1167–1200.

Cifuentes, R., G. Ferrucci and H. S. Shin. 2005. "Liquidity Risk and Contagion." *European Economic Journal* 3 (2–3): 556–566.

Covas, F. B., B. Rump and E. Zakrajšek. 2014. "Stress-Testing US Bank Holding Companies: A Dynamic Panel Quantile Regression Approach." *International Journal of Forecasting* 30 (3): 691–713.

Dees, S., J. Henry and R. Martin (eds). 2017. *STAMP€: Stress-Test Analytics for Macroprudential Purposes in the Euro Area.* Frankfurt: ECB.

Eisenberg, L. and T. Noe. 2001. "Systemic Risk in Financial Systems." *Management Science* 47 (2): 236–249.

Fontaine, J. S., J. Selody and C. Wilkins. 2009. "Improving the Resilience of Core Funding Markets." Bank of Canada *Financial System Review* (December): 41–46.

Gauthier, C., Z. He and M. Souissi. 2010. "Understanding Systemic Risk: The Trade-Offs Between Capital, Short-Term Funding and Liquid Asset Holdings." Bank of Canada Staff Working Paper No. 2010-29.

Gauthier, C., M. Souissi and X. Liu. 2014. "Introducing Funding Liquidity Risk in a Macro Stress-Testing Framework." *International Journal of Central Banking* 10 (4): 105–141.

Henry, J. and C. Kok (eds). 2013. "A Macro Stress Testing Framework for Assessing Systemic Risk in the Banking Sector." ECB Occasional Paper No 152.

Ho, T. S. Y. and A. Saunders. 1981. "The Determinants of Bank Interest Margins: Theory and Empirical Evidence." *Journal of Financial and Quantitative Analysis* 16 (4): 581—600.

International Monetary Fund (IMF). 2014. "Canada: Financial Sector Assessment Program; Stress Testing—Technical Note." IMF Country Report No. 14/69.

Longstaff, F. A. 2004. "The Flight-to-Liquidity Premium in US Treasury Bond Prices." *Journal of Business* 77 (3): 511–526.

Musto, D., G. Nini and K. Schwarz. 2014. "Notes on Bonds: Liquidity at All Costs in the Great Recession." Working paper, Wharton School of the University of Pennsylvania.

Peterson, B. and T. Roberts. 2016. "Household Risk Assessment Model." Bank of Canada Technical Report No. 106.

Rochet, J.-C. and X. Vives. 2004. "Coordination Failures and the Lender of Last Resort: Was Bagehot Right After All?" *Journal of the European Economic Association* 2 (6): 1116–1147.

Vayanos, D. 2004. "Flight to Quality, Flight to Liquidity, and the Pricing of Risk." NBER Working Paper 10327.

Appendix A: Bank Income Models

Banks' income in MFRAF is decomposed into net interest, non-interest non-trading, and trading income. We use expert judgment to calibrate trading income and two satellite models to simulate banks' net interest income and non-interest non-trading income in a manner that is consistent with the macrofinancial scenario.

Net interest income

We obtain the net interest income[41] by multiplying banks' projected interest-earning assets by the net interest margins simulated using the Net Interest Margin (NIM) model.

In the NIM, as in Ho and Saunders (1981), banks are treated as risk-averse dealers in the credit market acting as intermediaries between borrowers and lenders. In this setting, banks use money market operations to smooth the asymmetric arrival of demands for and supplies of funds. Since this intermediation activity exposes banks to both refinancing and credit risks, they impose a margin to compensate for these risks. Following Angbazo (1997), the model includes a proxy for credit risk and the real GDP growth rate to account for movements in credit risk and macroeconomic factors in the cyclical fluctuations of net interest margins. The asset growth rate is calibrated in line with the macrofinancial scenario being analyzed.

The model is based on the following error-correction specification:

$$\Delta N_t = constant + \alpha(N_{t-1} + \beta_1 ST_{t-1} + \beta_2 LT_{t-1}) + \gamma_1 CR_{t-1} + \gamma_2 GDP_{t-1} + \gamma_3 \Delta N_{t-1} + \epsilon_t,$$

where

- ϵ_t: error term;
- α: long-term adjustment coefficient;
- Δ: first difference operator;

[41] An alternative yet related approach to non-interest non-trading modelling can be found in Dees, Henry and Martin (2017, Ch. 5).

- N_t: aggregate net interest margin for D-SIBs, measured as the ratio of aggregate net interest income over aggregate interest-earning assets;
- ST_t: 3-month Government of Canada T-bill yield (proxy for rate of return on liabilities);
- LT_t: 3-year to 5-year Government of Canada bond rate (proxy for rate of return on assets);
- CR_t: proxy for credit risk (e.g., difference between the 5-year corporate and government rates);
- GDP_t: real gross domestic product growth rate.

Non-interest non-trading income

We simulate non-interest non-trading income[42] by multiplying banks' forecast total assets[43] by the non-interest non-trading margin simulated using the non-interest non-trading (NINT) model.

Considerations of the nature of banks' activities that generate non-interest non-trading income guide the modelling approach, given the lack of literature on banks' non-interest non-trading income.[44] The model includes the growth rate of real GDP as one of the explanatory variables, for two reasons: (i) a significant portion of service charges and fees is based on business volume, which we expect to correlate with economic growth; and (ii) banks expanded their non-traditional activities to benefit from corporate clients' shift toward market financing and from the reallocation of bank deposits to a variety of capital-market wealth products in household portfolios, both of which are likely driven by the economic development–financial deepening conjunction.

[42] An alternative yet related approach to non-interest non-trading modelling can be found in Dees, Henry and Martin (2017, Ch. 7).
[43] Asset-growth forecasts are calibrated in line with the adverse macrofinancial scenario analyzed.
[44] These include securitization, wealth management, loan fees, etc.

The model also includes proxies for both short- and long-term rates to be consistent with the fact that certain non-interest income can be highly sensitive to market interest rates (see BCBS 2004).

The model is based on the following error-correction specification:

$$\Delta NINT_t = constant + \alpha(NINT_{t-1} - \gamma_1 GDP_{t-1} - \gamma_2 ST_{t-1} - \gamma_3 LT_{t-1}) + \sum_{i=1}^{2} \beta_i \Delta NINT_{t-i} + \boldsymbol{X}_t \boldsymbol{\delta} + \boldsymbol{Z}_t \boldsymbol{\theta} + \epsilon_t \,,$$

where

- NINT_t: non-interest non-trading income divided by asset size, multiplied by 100;
- ST_t: 3-month Government of Canada T-bill yield;
- LT_t: 3-year to 5-year Government of Canada bond rate;
- $\boldsymbol{X}_t$: includes contemporaneous quarterly changes in real GDP and the short-term interest rate;
- $\boldsymbol{Z}_t$: includes contemporaneous quarterly growth rate in the TSX (proxy for asset price) and the level of the US Michigan Consumer Confidence Index (proxy for investor sentiment).

Appendix B: Debt Pricing and the Rollover Model

This model is an extension of Rochet and Vives (2004) and closely follows Ahnert et al. (2016). Consider a stylized financial system populated by a banker and a continuum of wholesale investors of measure one. All agents are risk-neutral. All events occur over three dates $t \in \{1, 1\ 1/2\ , 2\}$. The banker consumes at $t = 2$, while investors are indifferent between consuming at $t = 1\ 1/2$ or $t = 2$. All investors have a monetary unit endowment at $t = 1$ and access to a safe storage technology that yields $r > 1$ at $t = 2$ per monetary unit invested at $t = 1$.

At $t = 1$, the banker combines its own funds, $E_0\ (equity)$, with funding, $D_0 \equiv 1$, obtained from investors to finance a combination of illiquid and liquid assets denoted by I and M, respectively. Illiquid asset returns materialize at $t = 2$ and are equal to $R > r$. However, if these assets are liquidated prematurely, the banker obtains only a fraction ψ of R, with $\psi R < r$. Moreover, illiquid assets are exposed to a credit shock at $t = 2$, which reduces the value of the banker's illiquid assets by an amount S. The shock has a continuous probability density function $f(S)$ and a continuous cumulative distribution function $F(S)$.

Investors receive an unsecured debt claim in return for the funding D_0 they provide to the banker at $t = 1$, which can be withdrawn without penalty at $t = 1\ 1/2$ or rolled over until $t = 2$. The rollover decision is delegated to a group of professional fund managers, indexed by $fm \in [0,1]$. These managers face a relative cost of rolling over equal to $\gamma \in (0,1)$. Rochet and Vives (2004) argue that the decisions of these managers are governed by their compensation. If an entity to which they decided to extend funding goes bankrupt, a manager's relative compensation from rolling over is negative, $-c < 0$. Otherwise, the relative compensation is positive, $b > 0$. The conservativeness ratio $\gamma = \frac{c}{c+b}$ summarizes these payoff parameters. The higher γ is, the more conservative fund managers are, and the less likely debt is rolled over. The face value of debt, $D \leq R$, is set at $t = 1$ and is independent of the withdrawal date. **Figure 22** and **Figure 23** show, respectively, the bank's balance sheet at $t = 1$ after financing and investment and at $t = 2$ after the shock S has materialized, assuming all debt is rolled over at $t = 1\ 1/2$.

Figure 22: Bank's Balance Sheet at *t=1*

Assets	Liabilities and shareholders' equity
I	$D_0 \equiv 1$
M	E_0

Figure 23: Bank's Balance Sheet at *t=2* Assuming All Debt Is Rolled Over

Assets	Liabilities and shareholders' equity
$IR - S$	D
Mr	$E_2 \equiv \max\{0, IR - S + Mr - D\}$

If a fraction $l \in [0,1]$ of fund managers decide not to roll over the debt claims at $t = 1\ 1/2$, the banker liquidates $max\left\{\frac{lD-M}{\psi R}, 0\right\}$ of its illiquid assets to withstand the withdrawals.[45] Given the partial liquidation and the credit shock affecting the illiquid assets, the value of all assets at the final date amounts to $R\left[(1 + E_0 - M) - max\left\{\frac{lD-M}{\psi R}, 0\right\}\right] + max\{M - lD, 0\}r - S$. Consequently, bankruptcy occurs at $t = 2$ if

$$R(1 + E_0 - M) - max\left\{(lD - M)r, \frac{lD - M}{\psi}\right\} - S < (1 - l)D.$$

When the credit shock S is common knowledge, the rollover game exhibits multiple equilibria. If $l = 1$, bankruptcy is prevented whenever the shock is smaller than a lower bound $\underline{S} \equiv R(1 + E_0 - M) - max\left\{(D - M)r, \frac{D-M}{\psi}\right\}$. Similarly, if $l = 0$, the bank is insolvent whenever $S > \overline{S} \equiv R(1 + E_0 - M) + Mr - D$.

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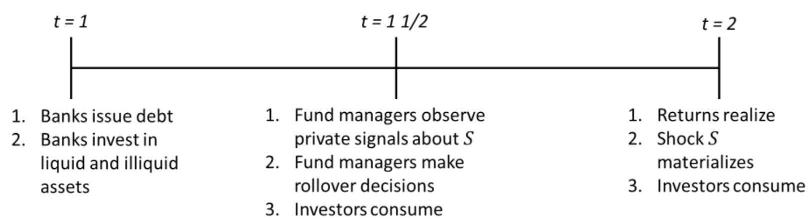
[45] Note that since premature liquidation of one unit of the illiquid asset yields only ψR, the banker needs to liquidate $\max\left\{\frac{lD-M}{\psi R}, 0\right\}$ of the illiquid asset to serve the withdrawals.

In line with the global games literature pioneered by Carlsson and van Damme (1993), we assume that there is incomplete information about the credit shock S. Fund managers do not observe it before it is realized at t = 2, but they receive a noisy signal x_{fm} at $t = 1\ 1/2$. Formally, $x_{fm} \equiv S + \epsilon_{fm}$, where ϵ_{fm} is an idiosyncratic noise term drawn from a continuous distribution H with support $[-\epsilon, \epsilon]$, for $\epsilon > 0$. The idiosyncratic noise is independent of the shock and independently distributed across fund managers.

Figure 24 summarizes the timeline of the debt pricing and rollover model.

Figure 24: Timeline of the Simple Model



Equilibrium

The model is solved by backward induction. First, we characterize fund managers' rollover decisions at $t = 1\ 1/2$, given the cost of funding. Then, we determine the face value of debt, D, such that the investors' participation constraint in the rollover game binds. The following proposition characterizes fund managers' rollover decisions.

Proposition (Proposition 1 of Ahnert et al. 2016) In the limit of vanishing private noise, $\epsilon \rightarrow 0$,[46] there exists a unique equilibrium in the threshold strategies characterized by a signal threshold, x^*, and a shock threshold, $S^*(D) \equiv R(1+E_0-M) - max\left\{\frac{\gamma D-M}{\psi}, (\gamma D-M)r\right\} - (1-\gamma)D \in \left(\underline{S}, \overline{S}\right)$, such that fund managers roll over debt if and only if $S \leq S^*$.

Proof: The proof follows directly from the proof in Ahnert et al. (2016), once liquidity holdings M are allowed for.

The feedback effects from liquidity to solvency are evidenced in the expression for $S^{\star}$**.** The higher the liquidity of banks' assets (i.e., higher M or ψ), the lower the early liquidation costs that reduce banks' long-term capital positions. However, a more-liquid asset portfolio is also less profitable, which is detrimental for banks' capital positions. Therefore, whenever there is a mismatch between expected withdrawals and liquidity holdings, the banks' solvency is adversely affected.

At $t = 1$, to maximize its expected equity value, the banker sets the smallest face value of unsecured debt, D, consistent with the participation constraint of investors:[47]

$$r = F\left(S^*(D^{\star})\right)D^{\star} + \psi \int_{S^*(D^{\star})}^{S^*_{max}} [S^*_{max} - S]dF(S),$$

where $S^*_{max} \equiv R(1+E_0-M) - max\left\{\frac{\gamma D-M}{\psi}, (\gamma D-M\,)r\right\}$.

The existence and uniqueness of $(D^{\star})$ are not guaranteed. This implies that whenever a solution for $D^{\star}$ cannot be found within the admissible interval for $D^{\star}$, the bank cannot place the desired amount of unsecured debt (i.e., there is a market freeze).

[46] In practice, this assumption corresponds to fund managers having quite precise knowledge of the realization of the credit shock.

[47] Recall that for the sake of simplicity, in the integration of this model in MFRAF, the loss-given-default, the second term in the right-hand side of the indifference equation above, is assumed to be constant and is calibrated based on expert judgment.