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Rollover Risk, Liquidity and Macroprudential Regulation

I study rollover risk in wholesale funding markets when intermediaries hold liquidity *ex ante* and fire sales may occur *ex post*. Multiple equilibria exist in a global rollover game: intermediate liquidity holdings support equilibria with both positive and zero expected liquidation. A simple uniqueness refinement pins down the private liquidity choice, which balances the forgone expected return on investment with reduced fragility and costly liquidation. Due to fire sales, liquidity holdings are strategic substitutes. Intermediaries free ride on the holdings of other intermediaries, causing excessive liquidation. To internalize the systemic nature of liquidity, a macroprudential authority imposes liquidity buffers.

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Wholesale funding markets have received a great deal of attention since the financial crisis of 2007-09. Financial institutions that funded themselves with short-term debt provided by uninsured investors saw this funding dry up once the U.S. housing market became distressed. Huang and Ratnovski (2011) is an early paper that points to the dark side of wholesale funding. During the crisis, money market mutual funds also experienced large outflows from institutional investors (Schmidt, Timmermann, and Wermers 2016), and even secured short-term borrowing was a highly unstable source of funds (Martin, Skeie, and Thadden 2014). These events led to new regulation imposed on financial intermediaries, including

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Basel's Liquidity Coverage Ratio (LCR) (Basel Committee on Banking Supervision 2010a, b).

I study rollover risk in the wholesale funding market, where uninsured investors can withdraw (or refuse to roll over) funding from intermediaries at an interim date. Intermediaries choose their portfolio at the initial date, either holding liquid (safe and low-return) assets or making risky long-term investments. The return on the long-term investment is determined by aggregate economic conditions such as business cycle movements or house price shocks, which determine default rates. While promising a higher expected return, long-term investment is costly to liquidate at the interim date because of a lower and diminishing marginal product in alternative use (Shleifer and Vishny 1992, Kiyotaki and Moore 1997). This cost of liquidation is exacerbated by fire sales that occur when many intermediaries liquidate jointly (Allen and Gale 1994, Gromb and Vayanos 2002).

In the model, each investor receives a noisy private signal about the return on investment at the interim date, based on the global games approach in Morris and Shin (2003). Using the private information, each investor decides whether to roll over funding to the intermediary, where a "threshold investor" is indifferent. A low (high) realization of the investment return implies that many (few) investors receive unfavorable private signals. Hence, a small (large) proportion of investors roll over funding to the intermediary, which results in a large (small) amount of investment liquidated by the intermediary. However, holding precautionary liquidity allows the intermediary to drive a wedge between its withdrawals and the amount of liquidation required to serve these withdrawals.

The first result is that there are multiple equilibria in the global rollover game between investors when intermediaries can hold liquidity (Proposition 1). If an intermediary holds an intermediate or high amount of liquidity, there exists an equilibrium in which the threshold investor expects no liquidation (Lemma 1). If an intermediary holds a low or intermediate amount of liquidity, however, there exists an equilibrium in which the threshold investor expects liquidation (Lemma 2). The intermediate choice of liquidity supports multiple equilibria in the rollover subgame, even if the private information is fairly precise. This result, which hinges on the presence of precautionary liquidity holdings, contributes to a recent literature on multiplicity in global coordination games.[1]

To obtain intuition for the multiplicity result, consider the strategic incentives of investors to roll over funding. Despite the risk neutrality of all agents, the incomplete-information game with liquidity holdings yields intriguing strategic interactions, which differ across the cases with and without liquidation. First, intermediate liquidity holdings can support the equilibrium in which the threshold investor expects no

1. For example, Angeletos and Werning (2006) show that the aggregation of dispersed private information into a publicly observed market price, similar to Grossman and Stiglitz (1980), re-establishes multiplicity in a global game. See also Hellwig, Mukherji, and Tsyvinski (2006) for a market-based model of currency attacks with multiple equilibria. Angeletos, Hellwig, and Pavan (2006) examine how endogenous public information from a policy intervention generates multiple equilibria. Hellwig and Veldkamp (2009) analyze when the acquisition of information prior to coordination leads to multiple equilibria.

liquidation, resulting in a low threshold. At this point, the strategic incentives to roll over funding change from complementarity to substitutability. Second, intermediate liquidity holdings can also support the standard equilibrium in which the threshold investor expects some liquidation to occur. Global strategic complementarity in rollover decisions leads to coordination failure between investors, resulting in a high threshold. This equilibrium is obtained in Morris and Shin (2000), who abstract from liquidity holdings by the intermediary.[2]

To study the private incentives to hold liquidity, I use the simple and common uniqueness refinement of vanishing private noise. Consider first the benchmark of a single intermediary. Its choice of liquidity balances the opportunity cost of a higher expected return on investment with lower fragility and less costly liquidation (Proposition 2). If liquidation costs are large, the intermediary holds abundant liquidity and implements the equilibrium with zero expected liquidation. If liquidation costs are small, however, it holds scarce liquidity and implements the equilibrium with positive expected liquidation. Decreases in the liquidation cost or increases in the expected return on investment reduce liquidity. If the expected return on investment is sufficiently high, or the variance of the return sufficiently low, then the intermediary holds no liquidity at all.

The portfolio choice of the single intermediary is constrained efficient. A planner who takes the optimal rollover behavior of investors as given holds the same level of liquidity. As such, there is no role for a *microprudential* regulation of liquidity.

Consider now the case of two intermediaries. I focus on the interesting equilibrium in which positive liquidation by each intermediary is expected, which again occurs for a large liquidation cost. Fire sales occur in this equilibrium. Withdrawals by investors of the first intermediary induce liquidation and reduce the liquidation value of investment, which triggers larger liquidation by the second intermediary. As a result, the private choices of liquidity are strategic substitutes, whereby each intermediary free rides on the liquidity holdings of other intermediaries (Proposition 3). If one intermediary holds more liquidity, the liquidation cost of another intermediary is reduced as the effect of fire sales is less pronounced. Since holding liquidity is costly due to the forgone return on investment, the other intermediary reduces its liquidity holding. As a result, excessive liquidation occurs *ex post* that renders the private choice of liquidity as constrained inefficient *ex ante*.

This yields a role for a *macroprudential* regulation of liquidity (Proposition 4). A constrained planner internalizes the systemic nature of liquidity and is therefore interpreted as a macroprudential authority. Specifically, it takes into account that more liquidity held by one intermediary reduces the liquidation cost of other intermediaries

2. Rochet and Vives (2004) analyze the impact of balance-sheet variables, including liquidity, on the run threshold in the context of delegated management, which gives rise to a unique Bayesian equilibrium. By contrast, the rollover game I study may yield multiple equilibria. However, once the uniqueness refinement is applied, I replicate the effect of liquidity on the run threshold stated in their second proposition. Another difference is the analysis of two intermediaries that allows me to study the private and social incentives to hold liquidity from a systemic perspective. See also König (2015).

in the case of *ex post* fire sales, thereby also reducing the threshold below which a run on other intermediaries occurs. Therefore, the social choice of liquidity exceeds the private choice, so imposing a macroprudential liquidity buffer restores constrained efficiency. As a consequence, inefficient liquidation occurs only for the smallest possible range consistent with incomplete information.

Finally, could similar normative results be obtained in a model with capital choice?[3] I use a version of Gale (2010) described in Section 4.1, where capital absorbs losses in bad states. A potential benefit of capital is therefore to reduce the rollover risk of the intermediary. However, capital is costly, as motivated by an outside investment or consumption opportunity of shareholders who supply capital and are repaid in good states. Capital attracted by one intermediary has only an *indirect* effect on the threshold below which a run occurs on the other intermediary. In the limit of vanishing private noise, the link between capital choices across intermediaries even vanishes completely. This result contrasts with the *direct* effect of liquidity on the liquidation value and the run threshold of other intermediaries, which prevails in the limit of vanishing private noise. These results suggest that, in the context of fire sales, macroprudential regulation should target liquidity rather than capital.[4]

The most related papers are the global rollover games of Morris and Shin (2000) and Eisenbach (2013). Building on the seminal work of Carlsson and van Damme (1993) and Diamond and Dybvig (1983), Morris and Shin (2000) solve for the unique equilibrium in a bank run game, using global games techniques.[5] By contrast, I study the *ex ante* portfolio choice of intermediaries and show how precautionary liquidity can restore multiple equilibria under the sufficient condition proposed by Morris and Shin (2000).[6] Extending the analysis to multiple intermediaries and fire sales, I explore macroprudential regulation of liquidity. Eisenbach (2013) also analyzes an *ex post* coordination game in which investors roll over short-term debt. He studies the *ex ante* optimal maturity choice of funding to discipline a bank manager tempted by moral hazard and derives a two-sided inefficiency. In contrast, I study the optimal liquidity choice of intermediaries on the asset side.

This paper is related to the literature on pecuniary externalities and fire sales (Lorenzoni 2008, Korinek 2011). As in this literature, fire sales occur *ex post*, so private intermediaries do not take into account the full social value of liquidity holdings *ex ante*. As a result, macroprudential regulation is also desirable in this paper. Different from this literature, I study a Cournot-style game between intermediaries. This approach yields the result of free riding on another intermediary's liquidity holding, which is absent if agents are atomistic. Another difference is that I focus

3. I thank an anonymous referee for suggesting a comparison between capital and liquidity choice.

4. Other motives for capital are absent in this extension. For example, there is no agency conflict between the intermediary and its manager, which may require "skin in the game" as an incentive device.

5. Multiple equilibria occur in Diamond and Dybvig (1983) due to self-fulfilling beliefs. In this model of one-sided strategic complementarity, Goldstein and Pauzner (2005) extend global games tools to obtain a unique equilibrium.

6. Vives (2014) and Morris and Shin (2010) also study the role of liquidity in rollover games. However, they do not analyze the *ex ante* portfolio choice of intermediaries.

on rollover risk. Specifically, I show how this important feature of modern financial intermediaries is affected by *ex post* fire sales and *ex ante* liquidity choices.[7]

1. MODEL

I present a simple model of financial intermediation that builds on the model of Morris and Shin (2000) with a single intermediary. There are three main differences. First, the cost of liquidation is incurred at the interim date. Second, there are many intermediaries whose interim-date liquidation decisions impose fire sale externalities on each other. Third, intermediaries are allowed to hold liquidity. I use this set-up to revisit the issue of equilibrium uniqueness in a global rollover game and to analyze the liquidity choices at the initial date and their welfare properties.

1.1 Agents and preferences

The economy extends over three dates, $t \in \{0, 1, 2\}$, and there is a single good for consumption and investment. A finite number of intermediaries $N \in \{1, 2\}$ raise funds from a unit continuum of risk-neutral uninsured investors $i \in [0, 1]$. Investors consume at the final date and receive a payoff $\pi_i = c$. An individual intermediary has the index $n = 1, ..., N$.

1.2 Investment

Intermediaries simultaneously choose their portfolio at the initial date. They can hold a liquid asset y_n, such as central bank reserves and government bonds, which yield a unit safe return at the subsequent date. They can also invest by originating loans to the real economy at the initial date, which constitutes a constant returns-to-scale technology, that yield a risky payoff r at the final date. The portfolio choice of intermediaries is publicly observed at the interim date.

Investors are endowed with two units of the good at the initial date. Apart from claims on the intermediary, investors can hold liquidity but direct investment is infeasible due to inferior skills in monitoring or loan collection.[8] The intermediary is funded purely with debt (equity funding is studied in Section 4.1). Because of free entry, an intermediary maximizes the expected utility of its investors (Allen and Gale 2007). The intermediary offers a contract that promises a unit repayment at the interim date, and an equal share of the asset value at the final date (Dasgupta

7. Fire sales can also affect portfolio diversification. Wagner (2009) studies the effect of *ex post* fire sales on the *ex ante* diversification choice of banks, where privately optimal diversification choices can be either excessively high or low. Wagner (2011) studies the diversification–diversity trade-off in *ex ante* portfolio choices. Since joint liquidation is costly *ex post*, investors have an incentive to hold diverse portfolios *ex ante*.

8. Instead of providing liquidity insurance for risk-averse investors (Diamond and Dybvig 1983), financial intermediation occurs in this paper because of an intermediary's superior monitoring or loan collection skills.

2004, Goldstein 2005, Shapiro and Skeie 2015). Since the participation constraint of investors is satisfied, each intermediary attracts one unit of funding at the initial date if $N = 2$, or the single intermediary attracts both units of funding if $N = 1$.

1.3 Information

The following information structure is common knowledge. Following Carlsson and van Damme (1993), there is incomplete information about the return on risky investment. Investors share a common prior about the profitability of risky investment:

$$r \sim \mathcal{N}\left(\bar{r}, \alpha^{-1}\right), \tag{1}$$

where the expected return on investment is superior to liquidity, $\bar{r} > 1$, and $\alpha \in (0, \infty)$ measures the precision (inverse variance) of public information.[9] Each investor receives a private signal at the interim date (Morris and Shin 2003):

$$x_i = r + \epsilon_i,\ \epsilon_i \sim \mathcal{N}\left(0, \gamma^{-1}\right), \tag{2}$$

where the idiosyncratic noise ϵ_i is identically and independently distributed as well as independent of the investment return, and $\gamma \in (0, \infty)$ measures the precision of private information.

1.4 Costly liquidation

Loans are costly to liquidate at the interim date and its liquidation value is $\psi \in (0, 1]$. This value is determined endogenously from a downward-sloping demand for liquidated investment, as in Eisenbach (2013). For example, assets are relocated to a less-productive sector (Shleifer and Vishny 1992, Kiyotaki and Moore 1997). In Shleifer and Vishny (1992), liquidation values are depressed after an industry-specific shock, since distress sales take place to unlevered industry outsiders who value industry-specific assets less. Since the marginal product of liquidated assets in alternative use is continuous, positive, and diminishing, the liquidation value is symmetric, increases in either intermediary's liquidation proceeds, and is convex in the liquidation proceeds of a given intermediary $l_n \geq 0$:

$$\psi(L) = (1 + \chi L)^{-1}, \tag{3}$$

where $\chi > 0$ is a coefficient and $L \equiv \frac{1}{2}\sum_{n=1}^{2} l_n$ are the average proceeds from liquidation.[10] Note that $\frac{d\psi}{dl_1} = \frac{d\psi}{dl_2} = -\psi^2 \frac{\chi}{2} < 0$ and $\frac{d^2\psi}{dl_1^2} = \frac{d^2\psi}{dl_2^2} = \frac{d^2\psi}{dl_1 dl_2} = \psi^3 \frac{\chi^2}{2} > 0$.

9. This prior may be induced by a public signal: $\bar{r} = r + \eta$, where the noise $\eta \sim \mathcal{N}(0, \alpha^{-1})$ is independent of the return. Furthermore, the aggregate noise is independent of each of the idiosyncratic noise terms ϵ_i.

10. This specification satisfies an "invariance property." That is, the liquidation value is independent of whether two banks liquidate half of the total amount of liquidated assets in the case of $N = 2$ or one large bank liquidates all of it in the case of $N = 1$. I thank an anonymous referee for pointing out this property.

Fires sales occur when intermediaries jointly liquidate investment. Limited participation in asset markets can lead to cash-in-the-market pricing and therefore underpricing of assets (Allen and Gale 1994). In the interpretation of financial investment, financial arbitrageurs cannot pick up assets in fire sales, since they are constrained by losses and outflows themselves (Gromb and Vayanos 2002). Where loans are secured on real estate, for example, foreclosures generate negative spillovers for the owners of nearby property. Quantifying this effect for housing in Massachusetts, Campbell, Giglio, and Pathak (2011) find discounts due to forced sales after bank foreclosures of up to 27%.

Upon receiving the private information x_i, investors decide whether to roll over funding to intermediaries at the interim date. Let $w_n \in [0, 1]$ be the proportion of investors who withdraw. In the case of high withdrawals, $w_n > y_n$, an intermediary requires liquidation proceeds $l_n \equiv w_n - y_n \in [0, 1 - y_n]$, resulting in a liquidation volume of $\frac{l_n}{\psi} > l_n$. Since liquidity reduces the liquidation cost by driving a wedge between withdrawals and liquidation, banks may hold liquidity.

1.5 Payoffs

The payoffs to investors depend on whether liquidation occurs. First, if many investors withdraw from intermediary n, all liquidity is exhausted and some liquidation occurs. The final-date payoffs to an investor who rolls over funding is

$$c_n = \frac{1 - y_n - \frac{l_n}{\psi}}{1 - w_n} r = \frac{1 - w_n - \chi l_n L}{1 - w_n} r. \tag{4}$$

An investor's incentive to roll over funding is affected by the rollover behavior of other investors. For a single intermediary, more withdrawals from other investors increase costly liquidation and therefore reduce the final-date consumption, $\frac{dc_n}{dw_n} = -\chi \frac{l_n(2-w_n-y_n)}{(1-w_n)^2} r < 0$. This establishes strategic complementarity in the withdrawal decisions of investors. In the case of two intermediaries, more withdrawals by investors of the other intermediary (subject to positive liquidation $l_{-n} > 0$) also reduce the liquidation value and thus final-date consumption, $\frac{dc_n}{dw_{-n}} = -\chi \frac{l_n}{2(1-w_n)} r < 0$. This establishes strategic complementarity in the withdrawal decisions of investors across intermediaries.

Second, if withdrawals are small relative to the intermediary's liquidity, then no liquidation occurs and the payoff to an investor who rolls over comprises an equal share of final-date assets:

$$c_n = \frac{y_n - w_n + (1 - y_n)r}{1 - w_n}. \tag{5}$$

The withdrawal decisions of investors at the other intermediary have no impact because no liquidation occurs. The withdrawal decision of investors at the same intermediary affect final-date consumption according to $\frac{dc_n}{dw_n} = \frac{(1-y_n)(r-1)}{(1-w_n)^2}$, so these

TABLE 1

TIMELINE OF THE MODEL

Initial date ($t = 0$)	Interim date ($t = 1$)	Final date ($t = 2$)
1. Endowed investors fund intermediaries 2. Intermediaries simultaneously choose liquidity y_n	1. Private information x_i about return on investment 2. Investors simultaneously decide whether to roll over 3. Intermediaries liquidate some investment, l_n	1. Investment matures and return is publicly observed 2. Remaining investors withdraw 3. Consumption

withdrawal decisions are strategic complements if and only if $r < 1$. The following timeline summarizes the model.

1.6 Solving for the equilibrium

Working backwards, I start by analyzing Bayesian equilibria in the incomplete-information rollover game at the interim date, which is a proper subgame. An investor's strategy is a plan of action for each private signal x_i. For any portfolio choice (y_1, y_2), a profile of strategies is a Bayesian equilibrium in the subgame if the actions of each investor's strategy maximize the expected utility conditional on the private information x_i, taking as given the strategies followed by all other investors. I focus on threshold strategies, whereby an investor rolls over if and only if the private information is sufficiently good relative to an intermediary-specific threshold that depends on the portfolio choices of intermediaries:

$$x_i \geq x_n^*(y_n, y_{-n}). \tag{6}$$

At the initial date, the liquidity holdings (y_1^*, y_2^*) are a Nash equilibrium in the complete-information portfolio choice game if the choice y_n^* maximizes the objective function of each intermediary, subject to the withdrawal thresholds $x_n^*(y_n, y_{-n})$ and a given liquidity held by the other intermediary, y_{-n}^*.

2. EQUILIBRIUM

Analyzing first the case of a single intermediary ($N = 1$), I show that introducing liquidity can restore multiple equilibria despite the standard global game refinement of slightly noisy but precise private information. In order to study the optimal *ex ante* portfolio choice, I provide a stronger condition necessary and sufficient for uniqueness. The privately optimal liquidity choice is characterized and constrained efficient, so there is no role for *microprudential* regulation of liquidity. This contrasts with the case of multiple intermediaries ($N = 2$) studied in Section 3, where the private choices of intermediaries are constrained inefficient, yielding a role for *macroprudential* regulation.

2.1 Rollover Subgame

Consider the equilibrium withdrawal behavior of investors at the interim date. Each investor uses the private information x_i to form a posterior about the return on investment, $R_i \equiv \mathbb{E}[r|x_i]$, and the proportion of withdrawing investors, $W_i \equiv w|x_i$, both of which are derived in Appendix A. If withdrawals are so large that investment is fully liquidated, then withdrawing is the dominant action. Otherwise, the "threshold investor" is indifferent between withdrawing and rolling over upon receiving the threshold signal $x_i = x^*$:

$$\mathbb{E}[\pi_i|x_i = x^*] \equiv 1, \tag{7}$$

where the left-hand side is the expected payoff from rolling over conditional on receiving the threshold signal x^* and the right-hand side is the payoff from withdrawing. Because of the one-to-one mapping between the posterior mean R_i and the private signal x_i (see Appendix A.1), equation (7) defines a fundamental threshold R^*, which is more convenient to use than the threshold signal x^*. To achieve scale invariance, y_n denotes the amount of liquidity *per unit of funding* in this section.

Zero expected liquidation by threshold investor. If the threshold investor expects zero liquidation, $W^* = W(R^*) \leq y$, then the indifference condition yields

$$(1 - y)(R^* - 1) = 0. \tag{8}$$

If there is maturity transformation, $y < 1$, then the fundamental threshold is $R^* \equiv 1$.[11] Recall that strategic complementarity (substitutability) in rollover decisions arises for an investment return below (above) unity, which yields the stated unique fundamental threshold.

For this threshold to be consistent with the supposed zero liquidation expected by the threshold investor, liquidity must exceed a lower bound:

$$y \geq y_L \equiv \Phi\left(\sqrt{\delta}[1 - \bar{r}]\right) < \frac{1}{2}, \tag{9}$$

where the posterior about the proportion of investors who withdraw from the intermediary, derived in Appendix A.1, is used and $\delta \equiv \frac{\alpha^2(\alpha+\gamma)}{\gamma(\alpha+2\gamma)}$ collects precision parameters. Lemma 1 summarizes.

LEMMA 1. ***Zero expected liquidation.*** *In the single-intermediary case with sufficient liquidity, $y \in [y_L, 1)$, there exists a threshold equilibrium in which the threshold investor expects zero liquidation, $W^* \leq y$. For any finite precision of private information, $\gamma \in (0, \infty)$, this equilibrium prescribes an investor to roll over if and only if $R_i \geq R^* = 1$.*

Positive expected liquidation by threshold investor. For a single intermediary, the impact of liquidation of risky investment on its equilibrium value is fully

11. If there is no maturity transformation, $y = 1$ or "narrow banking," then the rollover decision of investors is irrelevant and the asset value of the intermediary is always unity. Without loss of generality, $R^* \to -\infty$ in this case.

internalized. If the threshold investor expects positive liquidation, $W^{**} = W(R^{**}) > y$, the indifference condition yields

$$R^{**} = \frac{1 - W(R^{**})}{1 - W(R^{**}) - \chi\,[W(R^{**}) - y]^2} > 1, \tag{10}$$

which defines R^{**} implicitly. As in Morris and Shin (2003), uniqueness of the fundamental threshold R^{**} requires a sufficiently precise private signal $\gamma \in (\underline{\gamma}, \infty)$.[12] Since the threshold investor expects positive liquidation, the rollover decisions of investors are strategic complements because of costly liquidation, which pushes the fundamental threshold above the level in the case of no expected liquidation, $R^{**} > 1 = R^*$.

For this fundamental threshold to be consistent with the supposed positive liquidation expected by the threshold investor, liquidity must be below an upper bound:

$$y < y_H \equiv \Phi\left(\sqrt{\delta}[R^{**} - \bar{r}]\right). \tag{11}$$

Thus, the ranking of bounds on liquidity is $y_H > y_L$, justifying the subscripts. Lemma 2 summarizes.

LEMMA 2. ***Positive expected liquidation.*** *In the single-intermediary case with $y \in [0, y_H)$, there exists a threshold equilibrium in which the threshold investor expects some liquidation, $W^{**} > y$. If private information is sufficiently precise, $\gamma \in (\underline{\gamma}, \infty)$, then this equilibrium prescribes an investor to roll over if and only if $R_i \geq R^{**} > 1$, as implicitly and uniquely defined by equation (10).*

Taken together, Lemmas 1 and 2 characterize the optimal behavior of investors in the rollover subgame at the interim date, as summarized by Proposition 1.

PROPOSITION 1. ***Liquidity choice and multiple Bayesian equilibria in rollover subgame.*** *Consider the rollover subgame at the interim date with a single intermediary, $N = 1$, and sufficiently high but finite precision of private information, $\gamma \in (\underline{\gamma}, \infty)$. The number of Bayesian equilibria in threshold strategies is determined by the intermediary's liquidity choice at the initial date:*

- *If the liquidity holdings of the intermediary satisfy $y \geq y_H$, then the unique Bayesian equilibrium is the equilibrium with zero liquidation expected by the threshold investor.*
- *If the liquidity holdings of the intermediary satisfy $y < y_L$, then the unique Bayesian equilibrium is the equilibrium with positive expected liquidation.*
- *However, for an intermediate amount of liquidity, $y \in [y_L, y_H)$, both equilibria exist.*

12. More precisely, uniqueness requires the slope of the left-hand side of equation (10) to exceed the slope of the right-hand side, $1 > \sqrt{\delta}\phi(\cdot)\chi \frac{(W-y)^2+2(1-W)(W-y)}{(1-W-\chi(W-y)^2)^2} > 0$, where $\phi(\cdot) \leq \frac{1}{\sqrt{2\pi}}$ is the probability distribution function of the standard normal distribution. Thus, the precision of private information must be sufficiently high, $\gamma > \underline{\gamma}$.

The standard condition in global games—high but finite precision of private information—does not guarantee uniqueness in the current rollover subgame once intermediaries are allowed to hold liquidity *ex ante*. Apart from the usual equilibrium with positive expected liquidation when the intermediary holds little liquidity, there also exists an equilibrium with zero expected liquidation that is supported by sufficient liquidity holdings. In short, depending on the liquidity choice of an intermediary, different equilibria exist in the subgame.

Proposition 1 nests the unique Bayesian equilibrium in Morris and Shin (2000) as special case. In the absence of liquidity, $y = 0 < y_L$, the threshold investor always expects positive liquidation. There are some investors who receive adverse signals and do not roll over, even if the realized return on investment is high. Thus, the equilibrium with zero expected liquidation cannot be sustained, such that the equilibrium with positive expected liquidation is the unique equilibrium.

Actual withdrawals. A consequence of Proposition 1 is that some intermediate realizations of the investment return are consistent with both equilibria. To develop this point, I determine the amount of actual liquidation in each equilibrium, which depends on the realized return on investment.

No liquidation occurs if the liquidity holdings suffice to serve actual withdrawals, $w|r \leq y$, which requires a sufficiently high realized return on investment for a given *ex ante* liquidity choice. The private signal conditional on the return is distributed as $x_i|r \sim \mathcal{N}(r, \gamma^{-1})$, so the proportion of investors who withdraw is $\Phi(\sqrt{\gamma}[\bar{x} - r])$ for any signal threshold $\bar{x}$. Hence, the lower bound on the realized return on investment, for any signal threshold $\bar{x}$, is

$$r \geq \bar{x} + \frac{\Phi^{-1}(y)}{\sqrt{\gamma}}, \tag{12}$$

where $\Phi^{-1}(\cdot)$ is the inverse of the cumulative probability function of the standard Gaussian.

First, consider the equilibrium with zero expected liquidation. From the posterior distribution of the return on investment (Appendix A.1), the signal threshold becomes $x^* \equiv 1 - \frac{\alpha}{\gamma}(\bar{r} - 1) < 1$. Therefore, zero actual liquidation occurs if the realized investment return is sufficiently high:

$$r \geq r_L \equiv 1 - \frac{\alpha}{\gamma}(\bar{r} - 1) - \frac{\Phi^{-1}(y)}{\sqrt{\gamma}}.$$

Intuitively, more liquidity allows the intermediary to serve more withdrawals without liquidating investment. More withdrawals occur for lower realized investment returns, so the lower bound on the investment return decreases in the liquidity holding, $\frac{\partial r_L}{\partial y} < 0$.

Second, consider the equilibrium with positive expected liquidation. The lower bound on the investment return is determined analogously and now depends on the fundamental threshold R^{**}. Zero actual liquidation again occurs if the realized return on investment is sufficiently high:

$$r \geq r_H \equiv R^{**} - \frac{\alpha}{\gamma}(\bar{r} - R^{**}) - \frac{\Phi^{-1}(y)}{\sqrt{\gamma}}.$$

For the same reasons, the lower bound on the investment return decreases in the liquidity holding, $\frac{\partial r_H}{\partial y} < 0$. Since investors roll over less frequently in this equilibrium ($R^{**} > 1$), a higher realized return on investment is required to ensure zero actual liquidation, $r_H > r_L$, justifying the subscripts.

Corollary 1 summarizes and expresses the multiplicity result of Proposition 1 in terms of actual liquidation at the interim date. In sum, the *ex ante* liquidity holding of intermediaries determines whether the threshold investor expects liquidation to occur, while the realized return on investment determines, for a given level of liquidity, whether liquidation actually occurs.

COROLLARY 1. *For an intermediate level of liquidity, $y \in [y_L, y_H)$, any intermediate return on investment $r \in [r_L, r_H)$ is consistent with both actual liquidation (in the equilibrium with positive expected liquidation) and no liquidation (in the equilibrium with zero expected liquidation).*

2.2 Uniqueness Refinement

For an analysis of the private and social incentives to hold liquidity at the initial date to be meaningful, the equilibrium in the rollover subgame at the interim date must be unique. In this section, I provide a simple condition that is necessary and sufficient for uniqueness in this subgame.

Uniqueness requires the range of intermediate liquidity holdings $[y_L, y_H)$ to vanish. Since $R^{**} > 1$, $\delta \to 0$ is both a necessary and sufficient condition for $y_L - y_H \to 0$. Therefore, the precision of public information relative to private information must vanish, $\frac{\alpha}{\gamma} \to 0$. For a given precision of public information, this condition is ensured by vanishing private noise, $\gamma \to \infty$. This condition is quite commonly used in global games applications but it is stronger than what is needed in a stand-alone global game without liquidity choice (e.g., Morris and Shin 2000, 2003).

As a consequence, the bounds on the liquidity holding converge, while the bounds on the realized return on investment required for no actual liquidation converge to the fundamental thresholds:

$$\begin{aligned}
y_L &\to \frac{1}{2} \leftarrow y_H, \\
r_L &\to R^* = 1, \\
r_H &\to R^{**} \to \frac{1}{1 - 2\chi\left(\frac{1}{2} - y\right)^2} > 1,
\end{aligned} \tag{13}$$

where the sufficient condition of $\chi < 2$ is imposed to ensure a positive denominator of R^{**} for any possible choice of liquidity, $y \in [0, \frac{1}{2})$. Corollary 2 summarizes.

COROLLARY 2. ***Unique equilibrium in each rollover subgame.*** *Consider the rollover subgame at the interim date with a single intermediary, $N = 1$. If fundamental uncertainty vanishes, $\gamma \to \infty$, then there exists a unique Bayesian equilibrium in threshold strategies:*

- *If the intermediary holds abundant liquidity, $y \geq \frac{1}{2}$, the equilibrium with zero expected liquidation occurs and the fundamental threshold is $R^* = 1$. Actual liquidation occurs if and only if the realized return on investment is low, $r \leq r_L \to 1$.*
- *If the intermediary holds scarce liquidity, $y < \frac{1}{2}$, the equilibrium with positive expected liquidation occurs and the fundamental threshold is $R^{**} \to [1 - 2\chi(\frac{1}{2} - y)^2]^{-1} > 1$. Actual liquidation occurs if and only if the realized return on investment is low, $r \leq r_H \to R^{**}$.*

The fundamental threshold of the equilibrium with positive expected liquidation, R^{**}, decreases in the liquidity holding of the intermediary at a diminishing rate (shown in Figure B1) and increases in the liquidation cost coefficient, and is invariant to the expected return on investment:

$$\frac{dR^{**}}{dy} = -4\chi\left(\frac{1}{2} - y\right)(R^{**})^2 < 0, \tag{14}$$

$$\frac{d^2R^{**}}{dy^2} = 4\chi\left[(R^{**})^2 - 2R^{**}\left(\frac{1}{2} - y\right)\frac{dR^{**}}{dy}\right] > 0, \tag{15}$$

$$\frac{dR^{**}}{d\chi} = 2(R^{**})^2\left(\frac{1}{2} - y\right)^2 > 0, \tag{16}$$

$$\frac{dR^{**}}{d\bar{r}} = 0. \tag{17}$$

To ensure $R^{**} \leq \bar{r}$, which bounds the probability of a run, I impose the sufficient condition of a lower bound on the expected return on investment, $\bar{r} \geq \bar{r}_1 \equiv \frac{2}{2-\chi}$.

2.3 Liquidity Choice

The simple refinement establishes a unique link between an intermediary's liquidity choice and the withdrawal threshold of investors in the rollover subgame. This allows us to study the intermediary's optimal liquidity choice at the initial date and derive consequences for individual fragility.

The liquidity choice has both marginal and discrete impact. The marginal impact is present only in the equilibrium with positive expected liquidation, where more liquidity reduces the fundamental threshold. The discrete impact is the selection of the equilibrium in the rollover subgame as summarized by Corollary 2. Therefore, I derive the optimal liquidity levels in cases of abundant and scarce liquidity in turn and compare the objective function globally.

As derived in Appendix A.2, the objective function of the intermediary is the expected utility of an investor and comprises the unit return on liquidity and the proceeds from investment. Investment is liquidated if its realized return falls short of some threshold R, which occurs with probability $F(R) = \Phi(\sqrt{\alpha}[R - \bar{r}])$. Otherwise, investment is continued, where the expected investment return conditional on continuation is $\mathbb{E}[r|r > R] = \bar{r} + \frac{f(R)}{\sqrt{\alpha}(1-F(R))} > \bar{r}$ and density $f(R) = \phi(\sqrt{\alpha}[R - \bar{r}])$. Taken together, the expected utility of an investor is

$$EU(y) = y + (1-y)\left[\frac{F(R)}{1+\chi(1-y)} + (1 - F(R))\,\mathbb{E}[r|r > R]\right]. \tag{18}$$

What are the effects of changes in liquidity holdings or the fundamental threshold on the expected utility? As derived in Appendix A.2, the direct marginal cost of liquidity from an *ex ante* perspective is the forgone net return on investment plus the saved liquidation costs:

$$\frac{\partial EU}{\partial y} = 1 - \mathbb{E}[r, r \geq R] - \frac{F(R)}{[1+\chi(1-y)]^2}. \tag{19}$$

For the problem to be nontrivial, there must be a direct marginal cost of liquidity, $\frac{\partial EU}{\partial y} < 0$. One can show that $\bar{r} \geq \bar{r}_2 \equiv 2 - \frac{1}{(1+\chi)^2}$ is a sufficient condition, which I assume henceforth.

Liquidity may also have an indirect effect via the fundamental threshold, whereby a lower threshold increases expected utility, since it results in a smaller area of runs on the intermediary:

$$\frac{\partial EU}{\partial R} = -(1-y)\sqrt{\alpha} f(R)\left(R - \frac{1}{1+\chi(1-y)}\right) < 0. \tag{20}$$

The effect of liquidity on this threshold depends on its magnitude. First, abundant liquidity implements the equilibrium with zero expected liquidation in the rollover subgame. The fundamental threshold $R^* = 1$ is independent of liquidity, so there is no marginal benefit of liquidity in this subgame. Since the marginal cost of liquidity is positive, the optimal amount of liquidity to implement this equilibrium is the lower bound $y^* = \frac{1}{2}$ (narrow banking, $y = 1$, is never optimal) and yields:

$$EU^* \equiv EU(y^*) = 1 + \frac{1}{2}\left[(\bar{r} - 1)(1 - F[1]) + \frac{f[1]}{\sqrt{\alpha}}\right] > 1. \tag{21}$$

Second, scarce liquidity holdings implement the equilibrium with positive expected liquidation, R^{**}. In contrast to the equilibrium with zero expected liquidation, the marginal benefit of liquidity is strictly positive ($\frac{\partial R^{**}}{\partial y} < 0$). Therefore, holding more liquidity, within the feasible bounds of $y < \frac{1}{2}$, allows the intermediary to serve a larger proportion of withdrawing investors without costly liquidation of investment. As a result, both the amount of coordination failure between investors and the fundamental threshold decrease, which indirectly increases the expected utility:

$$\frac{\partial EU}{\partial R^{**}} \frac{\partial R^{**}}{\partial y} > 0.$$

Taking the direct and indirect effects of liquidity holdings together, the intermediary's optimal amount of liquidity to implement the equilibrium with positive expected liquidation solves

$$y^{**} \equiv \arg \max_{y \in [0, \frac{1}{2})} EU(y) \text{ s.t. } R^{**} = R^{**}(y).$$

If an interior solution $y^{**} \in (0, \frac{1}{2})$ exists, it balances the indirect benefits of liquidity via reducing the fundamental threshold with the direct cost of liquidity in terms of forgone net investment return:

$$\overbrace{-\frac{\partial EU}{\partial y}}^{MC} = \overbrace{\frac{\partial EU}{\partial R^{**}} \frac{\partial R^{**}}{\partial y}}^{MB}. \tag{22}$$

The associated level of expected utility is

$$EU^{**} \equiv EU(y^{**}) = y^{**} + (1 - y^{**}) \left[\frac{F(R^{**})}{1 + \chi(1 - y^{**})} + (1 - F(R^{**}))\bar{r} + \frac{f(R^{**})}{\sqrt{\alpha}} \right]. \tag{23}$$

We are now ready to characterize the *ex ante* liquidity choice of the intermediary. Given the optimal liquidity choices that implement the equilibrium with positive expected liquidation, y^{**}, and the equilibrium with zero expected liquidation, y^{*}, respectively, the intermediary compares the expected utility, so $y^{global} \equiv \arg\max\{EU^{*}, EU^{**}\}$. It holds scarce liquidity to implement the equilibrium with positive expected liquidation in the rollover subgame if and only if $EU^{**} \geq EU^{*}$.

PROPOSITION 2. ***Optimal liquidity choice.*** *Consider a single intermediary, $N = 1$, and vanishing fundamental uncertainty, $\gamma \to \infty$. There exists a unique liquidation cost level $\bar{\chi} > 0$ such that:*

- *for $\chi > \bar{\chi}$, the intermediary implements the equilibrium with zero expected liquidation, $y^{global} = y^{*} = \frac{1}{2}$;*

- *for $\chi \leq \bar{\chi}$, the intermediary implements the equilibrium with positive expected liquidation, $y^{global} = y^{**} \in [0, \frac{1}{2})$. If $\bar{r} < \bar{r}_3$, the solution is interior and implicitly defined by equation (22). The liquidity holding increases in the liquidation cost coefficient and decreases in the expected return on investment:*

$$\frac{dy^{**}}{d\chi} > 0, \tag{24}$$

$$\frac{dy^{**}}{d\bar{r}} < 0. \tag{25}$$

However, if $\bar{r} \geq \bar{r}_3$, then the intermediary holds no liquidity, $y^{**} = 0$. Moreover, there exists a threshold level of the precision of public information, $\bar{\alpha} \in (0, \infty)$, such that the intermediary holds no liquidity, $y^{global} = y^{**} = 0$, for any $\alpha > \bar{\alpha}$.

PROOF. See Appendix A.3, where the bound $\bar{r}_3$ is stated. ■

The liquidity choice of the intermediary trades off the forgone expected return on investment with the gain in terms of reducing the fundamental threshold below which costly liquidation occurs. The intermediary optimally implements the equilibrium with positive expected liquidation if the liquidation cost, and therefore the benefit of liquidity, is low relative to the opportunity cost of liquidity. Accordingly, either a higher expected return on investment or a lower liquidation cost reduces the optimal holdings of liquidity. Furthermore, if the opportunity cost of liquidity is particularly large relative to the cost of liquidation, the intermediary optimally holds no liquidity. The lower bound on the expected return on investment, $\bar{r}_3$, is defined by a zero marginal benefit of liquidity if none is held:

$$\left.\frac{dEU}{dy}\right|_{y=0,\ \bar{r}=\bar{r}_3} \equiv 0. \tag{26}$$

No liquidity is also held if the public information is very precise, because then most realizations of the investment return will be close to its mean $\bar{r} > 1$, where no liquidation occurs. Since the benefit of liquidity is small relative to its cost in this case, the intermediary optimally holds no liquidity.

Next, I briefly study whether there is a role for government intervention. Throughout the paper, I adopt the notion of constrained efficiency. That is, the social planner takes the optimal behavior of investors as given but is allowed to choose the liquidity holdings of intermediaries.

LEMMA 3. ***Constrained efficiency.*** *The intermediary's liquidity choice is constrained efficient.*

The case of a single intermediary is simple. The intermediary and the planner have the same objective function, the *ex ante* expected utility of investors, and face the same constraints, especially the incomplete information of investors about the return on investment and the associated rollover behavior of investors. Given that they also share the same choice variable, the liquidity holdings, they must make the same choices. In other words, the single intermediary internalizes the impact of its liquidity choice on liquidation volume and the liquidation value of investment, so the private allocation is constraint efficient and there is no role for a microprudential regulation of liquidity. Next, I study how the presence of other intermediaries, and the impact of a given intermediary's liquidity choice on other intermediaries and their investors, will change this benchmark result.

3. MACROPRUDENTIAL LIQUIDITY REGULATION

In this section, I study multiple intermediaries ($N = 2$) and show that the liquidity choices of intermediaries are constrained inefficient, which implies a role for *macroprudential* liquidity regulation. Since part of the analysis overlaps with the previous section, I focus on the case where the existence of multiple intermediaries matters for the private and social incentives to hold liquidity. That is, I focus on the equilibrium with positive expected liquidation in the rollover subgame, which again occurs for a sufficiently low liquidation cost coefficient, $\chi < \tilde{\chi}$, similar to the single-intermediary case. (The equilibrium with zero expected liquidation generalizes, $R_n^* = 1$ for $\gamma \to \infty$ and $y_n^* = \frac{1}{2}$.)

3.1 Rollover Subgame

In the case of two intermediaries, $N = 2$, we have the symmetric expression $L = \frac{l_1+l_2}{2}$. Suppose that the threshold investor at intermediary n expects positive liquidation at either intermediary, $W_n^{***} > y_n$ and $W_{n,-n}^{***} > y_{-n}$, where the proportion of investors who withdraw from the other intermediary, $W_{n,-n}^{***} = W_{n,-n}(R_n^{***}, R_{-n}^{***})$, is also derived in Appendix A.1. (If the threshold investor were to expect $W_{n,-n}^{***} \leq y_{-n}$, then there is no effect of the presence of the other intermediary on the fundamental threshold.) Hence, the indifference condition of the threshold investor at intermediary n yields the following best response correspondence at the interim date:

$$R_n^{***}(R_{-n}^{***}; y_n, y_{-n}) = \frac{1 - W_n^{***}}{1 - W_n^{***} - \frac{\chi}{2}\left(W_n^{***} - y_n\right)\left(W_n^{***} - y_n + W_{n,-n}^{***} - y_{-n}\right)}. \tag{27}$$

As private noise vanishes, both fundamental thresholds converge to the same function that only depends on liquidity holdings. This result arises from the symmetry of the average volume of liquidation across intermediaries and taking the limit of vanishing noise. Lemma 4 summarizes.

LEMMA 4. *Consider the case of multiple intermediaries, $N = 2$, and vanishing fundamental uncertainty, $\gamma \to \infty$. If liquidity is scarce, $y_1, y_2 \in [0, \frac{1}{2})$, there exists a* *unique equilibrium in which the threshold investor expects positive liquidation by intermediaries. It is characterized by the fundamental threshold:*

$$R_n^{***}(y_n, y_{-n}) = \frac{1}{1 - \chi\left(\frac{1}{2} - y_n\right)(1 - y_n - y_{-n})} > 1. \tag{28}$$

PROOF. See Appendix A.4. ■

This threshold is corresponds to the previously derived threshold R^{**}. Because of the scale invariance of the liquidation value and a constant endowment, we obtain $R_n^{***} = R^{**}$ for a symmetric liquidity holding, $y_1 = y_2 \equiv y$.

Next, the impact of liquidity is explored further. The marginal benefit of liquidity is again strictly positive in the equilibrium with positive expected liquidation, where more liquidity reduces the fundamental threshold:

$$\frac{dR_n^{***}}{dy_n} = -\chi\left(\frac{3}{2} - y_n - y_{-n}\right)(R^{***})^2 < 0. \tag{29}$$

Moreover, in the case of two intermediaries, liquidity also has a *systemic* nature. More liquidity held by another intermediary also reduces the fundamental threshold of a given intermediary:

$$\frac{dR_{-n}^{***}}{dy_n} = -\chi\left(\frac{1}{2} - y_{-n}\right)(R^{***})^2 < 0. \tag{30}$$

3.2 Private Choice of Liquidity

Turning to the initial date, I solve for a symmetric equilibrium in the portfolio choice game. Each intermediary chooses its liquidity holding by taking the equilibrium in rollover subgame $R^{***}(y_n, y_{-n})$ into account and another intermediary's liquidity holding y_{-n} as given:

$$y_n^{***}(y_{-n}) \equiv \arg \max_{y_n \in [0, \frac{1}{2})} EU_n(y_n, y_{-n}) \text{ s.t. } R_n^{***} = R^{***}(y_n, y_{-n}). \tag{31}$$

If an interior solution exists, $y^{***} > 0$, the best response correspondence $y_n^{***}(y_{-n})$ is defined by

$$-\frac{\partial EU_n}{\partial y_n} = \frac{\partial EU_n}{\partial R^{***}} \frac{dR_n^{***}(y_n, y_{-n})}{dy_n}, \tag{32}$$

where the private choice of liquidity again balances the private marginal benefit of liquidity (avoiding costly liquidation and thus reducing the fundamental threshold) with the private marginal cost (related to the forgone investment return). The previous conditions imposed to ensure a positive marginal cost of liquidity are still sufficient in the case of two intermediaries, so $\frac{\partial EU_n}{\partial y_n} < 0$.

Proposition 3. ***Private liquidity choice.*** *Consider the case of multiple intermediaries, $N = 2$, and vanishing fundamental uncertainty, $\gamma \to \infty$. There exists a best response function $y_n^{***}(y_{-n})$ for each intermediary. If $\bar{r} \geq \bar{r}(y_{-n})$, then $y_n^{***} = 0$, else $y_n^{***} \in (0, \frac{1}{2})$ is implicitly and uniquely defined by equation (32). For the interior solution, liquidity holdings are strategic substitutes:*

$$\frac{dy_n^{***}}{dy_{-n}} < 0. \tag{33}$$

Proof. See Appendix A.5. ■

The best response function, a unique solution y_n^{***} for any given y_{-n}, arises from the global concavity of the expected utility function in liquidity holdings. Similar to the single-intermediary case, there exists a unique threshold of the expected return on investment above which no liquidity is held, since its opportunity cost is too high relative to the benefit of liquidity. In the case of two intermediaries, this threshold depends on the liquidity holding of the other intermediary, $\bar{r}(y_{-n})$.

The private choices of liquidity are strategic substitutes. Greater liquidity holdings by another intermediary reduce the liquidation cost of a given intermediary. The fire sale effect is less pronounced since the marginal product of liquidated investment in alternative use is higher. Since holding liquidity is costly (forgone investment return), the given intermediary optimally reduces its liquidity holding. That is, each intermediary free rides on the liquidity held by other intermediaries.

The best response function is identical across intermediaries. Since there exists a unique crossing of best response functions, the private choice of liquidity is identical across intermediaries, $y_n^{***} \equiv y^{***}$, and is implicitly given by $\frac{dEU_n}{dy_n}(y^{***}, y^{***}) = 0$. The fundamental threshold accordingly is $R^{***}(y^{***}, y^{***})$. In this symmetric equilibrium, no liquidity is held, $y^{***} = 0$, if and only if the expected return on investment is sufficiently high, where this lower bound is again determined by a zero marginal benefit of liquidity when none is held:

$$\bar{r} \geq \bar{r}_4 \equiv \frac{1 + \frac{f\left(\frac{2}{2-\chi}\right)}{\sqrt{\alpha}} - \frac{1+\frac{\chi}{2}}{1+\chi} F\left(\frac{2}{2-\chi}\right) + \frac{9\chi^2}{4(1+\chi)}\left(\frac{2}{2-\chi}\right)^3 \sqrt{\alpha} f\left(\frac{2}{2-\chi}\right)}{1 - F\left(\frac{2}{2-\chi}\right)}. \tag{34}$$

3.3 Social Choice of Liquidity

A constrained planner chooses the liquidity holdings of both intermediaries at the initial date, taking the optimal rollover decision of investors at the interim date into account. In contrast to an intermediary, the planner internalizes the benefit of one intermediary's liquidity holding on another intermediary. Since the constrained planner captures these systemic effects of liquidity, my preferred interpretation is that of a macroprudential authority. The social choice of liquidity that implements the equilibrium with positive expected liquidation, $y_k^{SP} \in [0, \frac{1}{2})$, maximizes social welfare:

$$\left(y_1^{SP}, y_2^{SP}\right) \equiv \arg\max_{y_1, y_2} SWF \equiv EU_1 + EU_2 \text{ s.t. } R_k^{***} = R^{***}(y_1, y_2). \tag{35}$$

If an interior solution exists, it is characterized by the first-order condition:

$$\overbrace{-\frac{\partial EU_n}{\partial y_n}}^{SMC} = \overbrace{\frac{\partial EU_n}{\partial R_n}\frac{\partial R_n}{\partial y_n} + \frac{\partial EU_{-n}}{\partial y_n} + \frac{\partial EU_{-n}}{\partial R_{-n}}\frac{\partial R_{-n}}{\partial y_n}}^{SMB}. \tag{36}$$

The optimization problem is fully symmetric. Both first-order conditions yield the same condition, given by equation (36). Therefore, the social choice of liquidity is $y^{SP} \equiv y_1^{SP} = y_2^{SP}$.

The constrained planner balances the social marginal cost with the social marginal benefit. The private and social marginal costs of liquidity coincide because the opportunity cost of liquidity is unchanged ($SMC = MC$). By contrast, the social marginal benefit of liquidity exceeds the private marginal benefit ($SMB > MB$). Apart from the beneficial effect of liquidity via fewer runs on the intermediary ($\frac{\partial R_n^{***}}{\partial y_n} < 0$), which is identical to the private marginal benefit, the planner also considers the beneficial effect on the investors of the other intermediary. There are two benefits. First, more liquidity allows an intermediary to serve more withdrawing investors and therefore mitigates the effect of fire sales. Therefore, the marginal product of liquidated investment in alternative use is higher, which directly raises the expected utility of investors at the other intermediary ($\frac{\partial EU_{-n}}{\partial y_n} > 0$). Second, more liquidity reduces the coordination failure between investors across intermediaries and therefore reduces the fundamental threshold at the other intermediary ($\frac{\partial R_{-n}^{***}}{\partial y_n} < 0$).

Proposition 4. ***Social liquidity choice.*** *Consider the case of multiple intermediaries, $N = 2$, and vanishing fundamental uncertainty, $\gamma \to \infty$. There exists a unique social liquidity choice. If the expected return on investment is high, $\bar{r} \geq \bar{r}_5 > \bar{r}_4$, then the constrained planner holds no liquidity, $y^{SP} = 0$. Otherwise, a symmetric solution exists and is implicitly characterized by equation (36). In this interior solution, the socially optimal liquidity holding exceeds the private optimum:*

$$y^{SP} > y^{***}. \tag{37}$$

As a result, the fundamental threshold is lower than under the private liquidity choice:

$$R^{SP} < R^{***}. \tag{38}$$

Proof. See Appendix A.6, which also contains the definition of $\bar{r}_5$. ■

The macroprudential authority holds more liquidity on both the extensive and intensive margin. On the extensive margin, it holds zero liquidity for a smaller range of parameters than a private intermediary. That is, the lower bound on the expected return on investment required for the planner to hold no liquidity is higher, $\bar{r}_5 > \bar{r}_4$. Intuitively, the macroprudential authority is more reluctant to hold zero liquidity since it considers the systemic benefits of liquidity, not only the private benefits. If the expected return on investment takes an intermediate value $\bar{r} \in [\bar{r}_4, \bar{r}_5)$, then the private intermediary optimally holds zero liquidity, $y^{***} = 0$, while the macroprudential authority holds a positive amount of liquidity, $y^{SP} > 0$.

On the intensive margin, the macroprudential authority holds more liquidity than private intermediaries. That is, $y^{SP} > y^{***}$ for $\bar{r} < \bar{r}_4$, where the difference between the social and private choices of liquidity, $y^{SP} - y^{***}$, is interpreted as a *macroprudential liquidity buffer*. A macroprudential authority takes the *systemic* nature of liquidity into account. As a result, the social choice of liquidity exceeds the private choice, because the former internalizes the social marginal costs of liquidation due to fire sales (that is, the social marginal benefit of liquidity). Therefore, the macroprudential authority mitigates the coordination failure between investors across intermediaries and reduces the range of *ex post* inefficient liquidation, $R^{SP} < R^{***}$.

4. DISCUSSION

This section has three purposes. First, I contrast the results to a model in which intermediaries subject to rollover risk choose capital, not liquidity. Second, I discuss some model assumptions and their impact on the results. Third, I relate my findings to the LCR proposed under Basel III regulation and implemented in several jurisdictions.

4.1 A Model with Bank Capital Choice

Consider a version of the model with capital choice in the spirit of Gale (2010), where the role of capital is loss absorption in bad states. There is a separate class of risk-neutral investors called shareholders. They are endowed at the initial date and are willing to supply capital to the intermediary. Shareholders have an outside investment or consumption opportunity at $t = 0$ that returns $\rho \geq \bar{r}$, which makes capital costly. The endowment or mass of shareholders is sufficiently large to meet any demand from intermediaries. The intermediary balances this exogenous cost of capital with a potential endogenous benefit of reducing its rollover risk, which enhances financial stability.

To contrast with the liquidity choice studied before, I set $y_n = 0$. Therefore, liquidation equals withdrawals, $w_n = l_n$, and the unique equilibrium in the rollover subgame features positive expected liquidation. Let R_n^{**} be the fundamental threshold below which a run occurs (to be determined below). Turning to the capital choice, let $e_{0n} \geq 0$ be the amount of capital intermediary n attracts at $t = 0$. For simplicity, I consider repayment (dividends) of e_{2n} at $t = 2$ if and only if no run occurs. Therefore, the participation constraint of shareholders is $\rho\, e_{0n} \leq e_{2n}[1 - F(R_n^{**})]$, where $1 - F(R_n^{**})$ is the probability of no run on intermediary n. As Gale (2010) shows, the problem of the intermediary under free entry can be stated as maximizing the expected utility of investors subject to nonnegative profits of the intermediary and participation by shareholders.

The intermediary raises one unit of wholesale deposits and e_{0n} of capital from shareholders at $t = 0$ to make risky investments. Therefore, the final-date consumption level is

$$c_n = \frac{1 + e_{0n} - \frac{w_n}{\psi}}{1 - w_n} r - e_{2n}, \tag{39}$$

where the liquidation value is $\psi = (1 + \frac{\chi}{2}[w_1 + w_2])^{-1}$. The resulting threshold is

$$R_n^{**} = \frac{(1 + e_{2n})(1 - W_n^{**})}{1 + e_{0n} - W_n^{**}\left[1 + \frac{\chi}{2}\left[W_n^{**} + W_{n,-n}^{**}\right]\right]} \to \frac{1 + e_{2n}}{1 - \frac{\chi}{2} + 2e_{0n}}, \tag{40}$$

for vanishing private noise, $\gamma \to \infty$. In this limit, the threshold increases in the cost of liquidation and the dividend payments, while it decreases in the amount of capital raised by the intermediary.

Capital holdings of one intermediary have only an *indirect* effect on the fundamental threshold of the other intermediary. This effect vanishes as private noise vanishes, so the threshold R_n^{**} is independent of the capital choice of intermediary $-n$. In other words, there is no scope for macroprudential regulation of capital by the intermediary in this model. This result on capital choice contrasts with the previous result on liquidity choice. Liquidity has a *direct* effect on the run threshold of either intermediary that is also preserved in the limit of vanishing private noise. Liquidity drives a wedge between withdrawals and liquidation of investment, reducing the liquidation by one intermediary and increasing the liquidation value of another intermediary.

4.2 Model Assumptions and Extensions

I have analyzed the welfare implications in the case of two intermediaries. This can be generalized to any finite number of intermediaries without losing either the strategic substitutability in private choices of liquidity or their welfare properties. While the strategic aspect of liquidity choices vanishes in the limiting case of a continuum of intermediaries, the welfare result prevails.

Similar to Dasgupta (2004) and Shapiro and Skeie (2015), I study an exogenous debt contract whereby an investor receives a unit payment upon not rolling over. I do not attempt to endogenize the debt contract, but this could be achieved, for example, by assuming idiosyncratic liquidity risk that creates demand for liquidity, as in Diamond and Dybvig (1983). However, I endogenize the portfolio choice of intermediaries, which improves upon Morris and Shin (2000), who explicitly abstract from both the optimal contract design and the *ex ante* portfolio choice of intermediaries.

The strategic interaction between investors arises from costly liquidation of investment due to diminishing marginal product in alternative use. Similarly, it arises since the promised interim-date payment exceeds the liquidation value of investment in Goldstein and Pauzner (2005). The current set-up allows an analysis not only of the *ex ante* portfolio choice of intermediaries subject to rollover risk, but also of the systemic nature of liquidity in the case of multiple intermediaries and fire sales.

Investors deposit with a single intermediary for simplicity. However, one could allow for a diversification of intermediaries with whom investors deposit. For concreteness, consider the case in which a proportion ω of investors deposit with both intermediaries. Then, the difference between the objective functions of the intermediary and the macroprudential authority shrinks but the welfare result prevails. While the objective functions coincide in the extreme case of $\omega = 1$, this scenario is unlikely, because some investors are unsophisticated or face high costs of diversifying their deposits.

4.3 Relation to Regulatory Debate

I have developed a model of financial intermediation with rollover risk on the liability side and optimal choice of liquidity on the asset side. Since assets may jointly be liquidated in a fire sale, the intermediaries' choices of liquidity are strategic substitutes, so each intermediary free rides on the liquidity holdings of other intermediaries. This set-up results in constraint inefficient private liquidity choices and creates a role for macroprudential regulation. A constrained planner, preferably interpreted as a macroprudential authority, internalizes the systemic nature of liquidity. On the intensive margin, it requires intermediaries to hold more liquidity than under *laissez faire*. On the extensive margin, the macroprudential authority holds positive liquidity for a larger range of parameters than a private intermediary. This macroprudential liquidity buffer improves welfare, especially whenever fire sales are severe, such as during a financial crisis.

There are different options for a macroprudential authority to require intermediaries to hold larger amounts of liquidity. Two of these are currently discussed and implemented within the endorsed Basel III framework. The first option is to require intermediaries to hold a short-term liquidity buffer against unforeseen liquidity outflows. Translated into the model set-up, such an LCR rule requires intermediaries to hold more liquidity in the initial period, that is *before* shocks are realized (and investors receive their private signal about the solvency of the intermediary). According to Basel Committee on Banking Supervision (2013), the LCR does this by ensuring that banks have an adequate stock of unencumbered high-quality liquid assets (HQLA) that can be converted easily and immediately in private markets into cash to meet their liquidity needs for a 30–calendar day liquidity stress scenario (item 16). Additional liquidity holdings required from intermediaries by a macroprudential authority can therefore be interpreted as the LCR. Both in the model and according to Basel Committee on Banking Supervision (2013), an intermediary can draw down from its pile of HQLA during a period of financial stress (item 17).

This leads to the question of the optimal implementation of the LCR. One option is to request each intermediary to hold additional liquidity to offset the effect of liquidity free riding. Another option is to create a systemic liquidity fund that all intermediaries contribute to. While another intermediary's liquidity can be used to dampen the effects of a fire sale, investors who withdraw can only be served with the liquidity held by an intermediary. Therefore, another intermediary's liquidity is only a partial substitute. This suggests that the preferable way to implement a macroprudential liquidity buffer is in the form of increased liquidity requirements for individual intermediaries.

5. CONCLUSION

This paper studies rollover risk in the wholesale funding market when intermediaries can hold liquidity *ex ante* and are subject to fire sales *ex post*. I show that the presence of liquidity, which drives a wedge between the amount of withdrawals and the liquidation volume, restores multiple equilibria even if a global game refinement is used. Apart from the usual equilibrium with liquidation, coordination failure, and a high fundamental threshold, an equilibrium with a low threshold exists for sufficiently high levels of liquidity. Liquidity holdings serve withdrawing investors and therefore support an equilibrium in which the threshold investor expects no liquidation to occur. The simple refinement of vanishing private noise is necessary and sufficient for uniquely pinning down the rollover decisions of investors. It also allows us to characterize the privately optimal liquidity choice, which balances the forgone higher return on investment with fewer liquidation costs.

In a simple set-up with two intermediaries, I explore fire sales, whereby one intermediary's liquidation volume increases the liquidation costs of another intermediary. The positive implication of fire sales is that liquidity holdings are partial substitutes. Intermediaries free ride on each other's liquidity holdings, causing excessive liquidation of productive investment. The normative implication of fire sales is that intermediaries hold insufficient liquidity relative to a constrained planner, leading to a higher incidence of *ex post* inefficient liquidation. Since such a planner internalizes the systemic nature of liquidity, it is best interpreted as a macroprudential authority.

More broadly, I offer a natural laboratory for studying macroprudential policies in a microfounded setting. Other elements relevant to the conduct of macroprudential regulation are omitted in this framework, such as the interaction between capital and liquidity requirements, portfolio diversification, or penalties on early withdrawals. These are exciting avenues for subsequent research.

APPENDIX A

A.1 Posterior Distributions

Return. The posterior distribution of the return on investment is also normal. The posterior precision is the sum of the prior precisions and the precision of the private information. The mean is a weighted average of the prior and the private signal with the respective precisions as weights:

$$r|x_i \sim \mathcal{N}\left(\frac{\alpha\bar{r} + \gamma x_i}{\alpha + \gamma}, \frac{1}{\alpha + \gamma}\right). \tag{A1}$$

The ratio of the precision of the prior (public signal) relative to the private signal, $\frac{\alpha}{\gamma}$, determines the extent to which the posterior mean depends on the private signal. The more precise the private signal is relative to the prior, the more the posterior is determined by the private signal. As private noise vanishes, $\gamma \to \infty$, the posterior mean converges to the private signal.

Proportion of investors who withdraw. Using a law of large numbers, the posterior proportion of investors who withdraw from intermediary n is

$$W_{in} = \Phi\left(\sqrt{\delta}\left[R_n^* - \bar{r}\right] + \sqrt{\frac{\gamma(\alpha + \gamma)}{\alpha + 2\gamma}}\left[R_n^* - R_i\right]\right), \tag{A2}$$

where $\Phi(\cdot)$ is the cumulative distribution function of the standard normal and $\delta \equiv \frac{\alpha^2(\alpha+\gamma)}{\gamma(\alpha+2\gamma)}$ summarizes precision parameters. Therefore, the threshold investor has the following posterior:

$$W_n^* \equiv W_i|_{x_i = x^*} = \Phi(z_{1n}), \tag{A3}$$

$$z_{1n} \equiv \sqrt{\delta}\left[R_n^* - \bar{r}\right]. \tag{A4}$$

If private noise vanishes, $\gamma \to \infty$, then $\delta \to 0$ and $W_n^* \to \frac{1}{2}$. For multiple banks, the posterior proportion of investors who withdraw from another intermediary is

$$W_{i,n,-n} = \Phi\left(\sqrt{\delta}\left[R_{-n}^* - \bar{r}\right] + \sqrt{\frac{\gamma(\alpha + \gamma)}{\alpha + 2\gamma}}\left[R_{-n}^* - R_{in}\right]\right). \tag{A5}$$

Therefore, the threshold investor has the following posterior:

$$\left(W_{n,-n}\right)^* \equiv W_{i,n,-n}\Big|_{x_i = x_n^*} = \Phi(z_{2n}), \tag{A6}$$

$$z_{2n} \equiv \sqrt{\delta}\left[R_{-n}^* - \bar{r}\right] + \sqrt{\delta}\frac{\gamma}{\alpha}\left[R_{-n}^* - R_n^*\right]. \tag{A7}$$

A.2 Derivation of Expected Utility

Consider the case of a single intermediary, $n = 1$. As private noise vanishes, $\gamma \to \infty$, the realized proportion of investors who withdraw from the intermediary at the interim date is

$$w^*(r) = \Phi\left(\frac{\alpha}{\sqrt{\gamma}}[R - \bar{r}] + \sqrt{\gamma}[R - r]\right) \to \begin{cases} 0 & r > R \\ \frac{1}{2} \text{ if } r = R, \\ 1 & r < R \end{cases} \tag{A8}$$

for any fundamental threshold R. Therefore, there is no (full) liquidation of investment if the return on investment is above (below) the threshold R. Investors receive an equal share of the asset value, where the liquidation value of investment simplifies to $\psi(L) = \frac{1}{1+\chi(1-y)}$, so we have

$$\begin{aligned} EU(y) &= \int_{-\infty}^{R} \left[y + \frac{1-y}{1+\chi(1-y)}\right]\phi(\sqrt{\alpha}[r - \bar{r}])dr \\ &\quad + \int_{R}^{\infty} [y + (1-y)r]\phi(\sqrt{\alpha}[r - \bar{r}])dr \\ &= y + (1-y)\left[\frac{F(R)}{1+\chi(1-y)} + (1 - F(R))\,\mathbb{E}[r|r > R]\right], \end{aligned} \tag{A9}$$

where $\mathbb{E}[r|r > R] = \bar{r} + \frac{f(R)}{\sqrt{\alpha}(1-F(R))}$ and $\mathbb{E}[r, r > R] = (1 - F(R))\bar{r} + \frac{f(R)}{\sqrt{\alpha}}$. The partial derivatives of the expected utility are

$$\frac{\partial EU}{\partial y} = 1 - \mathbb{E}[r, r > R] - \frac{F(R)}{[1+\chi(1-y)]^2}, \tag{A10}$$

$$\frac{\partial^2 EU}{\partial y^2} = -\frac{2\chi F(R)}{[1+\chi(1-y)]^3} < 0, \tag{A11}$$

$$\frac{\partial EU}{\partial R} = -(1-y)\sqrt{\alpha}f(R)\left(R - \frac{1}{1+\chi(1-y)}\right) < 0, \tag{A12}$$

$$\frac{\partial EU}{\partial y \partial R} = \sqrt{\alpha}f(R)\left(R - \frac{1}{[1+\chi(1-y)]^2}\right) > 0, \tag{A13}$$

$$\frac{\partial^2 EU}{\partial R^2} = -(1-y)\sqrt{\alpha}f(R)\left[1 + \alpha\left(R - \frac{1}{1+\chi(1-y)}\right)(\bar{r} - R)\right] < 0. \tag{A14}$$

A.3 Proof of Proposition 2

This proof is in four steps. First, y^{**} is the global maximum subject to the intermediary implementing the equilibrium with positive expected liquidation. The second-order total derivative is strictly negative:

$$\frac{d^2EU}{dy^2} = \frac{\partial^2 EU}{\partial y^2} + 2\frac{\partial^2 EU}{\partial y \partial R}\frac{\partial R}{\partial y} + \frac{\partial^2 EU}{\partial R^2}\left(\frac{\partial R}{\partial y}\right)^2 + \frac{\partial EU}{\partial R}\frac{\partial^2 R}{\partial y^2} < 0, \tag{A15}$$

where the signs arise from (i) the results for how the fundamental threshold depends on the liquidity holding stated in the main text, and (ii) the previously stated partial derivatives.

Second, I show that there exists a unique threshold level of the liquidation cost parameter, $\bar{\chi} \in (0, \infty)$, at which the intermediary switches from implementing the equilibrium with zero expected liquidation in the rollover subgame to the equilibrium with positive expected liquidation, where $EU^* \equiv EU^{**}(\bar{\chi})$. Observe that $EU^* = EU(y^* = \frac{1}{2})$ is independent of χ, while $EU^{**} = EU(y^{**})$ strictly decreases in it. By the envelope theorem, we have

$$\frac{dEU^{**}}{d\chi} = \frac{\partial EU^{**}}{\partial \chi} = -\frac{(1-y^{**})^2 F(R^{**})}{[1+\chi(1-y)]^2} < 0. \tag{A16}$$

At the lower boundary, $\chi \to 0$, we have $R^{**} \to 1$ and thus $y^{**} \to 0$. Hence, $EU^{**} \to F(1) + [1 - F(1)]\bar{r} + \frac{f(1)}{\sqrt{\alpha}} > EU^*$, so $y^{global} = y^{**} = 0$. At the upper bound, $\chi \to \infty$, the positive denominator of R^{**} requires $y^{**} \to \frac{1}{2}$. Since $\frac{\partial EU}{\partial R} < 0$, we have that $EU^{**} \leq \overline{EU} \equiv EU(R=1) = \frac{1}{2}(1 + [1 - F(1)]\bar{r} + \frac{f(1)}{\sqrt{\alpha}}) < EU^*$, so $y^{global} = y^* = \frac{1}{2}$. By continuity, there exists a threshold $\bar{\chi} \in (0, \infty)$ that equalizes the expected utilities and, by strict monotonicity, this threshold is unique.

Third, I derive the comparative statics of the liquidity holdings in the case of the interior solution y^{**}. Using the implicit function theorem, and $\frac{d^2EU}{dy^2} < 0$, the stated inequalities obtain if $\frac{d^2EU}{dyd\chi} > 0$ and $\frac{d^2EU}{dyd\bar{r}} < 0$, where, for $a \in \{\bar{r}, \chi\}$:

$$\frac{d^2EU}{dyda} = \frac{\partial^2 EU}{\partial y \partial a} + \frac{\partial^2 EU}{\partial y \partial R}\frac{dR}{da} + \left(\frac{\partial^2 EU}{\partial R \partial a} + \frac{\partial^2 EU}{\partial R^2}\frac{\partial R}{\partial a}\right)\frac{dR}{dy} + \frac{\partial EU}{\partial R}\frac{d^2 R}{dyda}. \tag{A17}$$

In turn, these inequalities follow from our previous results and the following partial derivatives:

$$\frac{\partial^2 EU}{\partial y \partial \chi} = \frac{2(1-y)F(R)}{[1+\chi(1-y)]^3} > 0, \tag{A18}$$

$$\frac{\partial^2 EU}{\partial R \partial \chi} = -\frac{(1-y)^2 f(R)\sqrt{\alpha}}{[1+\chi(1-y)]^2} < 0, \tag{A19}$$

$$\frac{d^2 R^{**}}{dy d\chi} = -\frac{4\left(\frac{1}{2}-y\right)\left[1+2\chi\left(\frac{1}{2}-y\right)^2\right]}{\left[1-2\chi\left(\frac{1}{2}-y\right)^2\right]^3} < 0, \tag{A20}$$

$$\frac{\partial^2 EU}{\partial y \partial \bar{r}} = -[1-F(R)] - \sqrt{\alpha} f(R)\left[R - \frac{1}{[1+\chi(1-y)]^2}\right] < 0, \tag{A21}$$

$$\frac{\partial^2 EU}{\partial R \partial \bar{r}} = (1-y)\sqrt{\alpha} f(R)(\bar{r}-R)\left[R - \frac{1}{1+\chi(1-y)}\right] > 0, \tag{A22}$$

$$\frac{d^2 R^{**}}{dy d\bar{r}} = 0. \tag{A23}$$

Fourth, when does the intermediary hold no liquidity, $y^{**} = 0$? On the one hand, public information may be very precise such that much weight is put on realized returns on investment close to its mean $\bar{r}$, and little on those realizations where liquidation occurs. Consider $\alpha \to \infty$. Then, $f(1) \to 0 \leftarrow F(1)$, so $EU^* \to \frac{1+\bar{r}}{2}$. Likewise, $f(R^{**}) \to 0 \leftarrow F(R^{**})$, so $EU^{**} \to y + (1-y)\bar{r} > EU^*$ for any $y < \frac{1}{2}$. Therefore, $y^{global} = y^{**} = 0$. By continuity, there exists a $\bar{\alpha} < \infty$ such that $y^{global} = y^{**} = 0$ for any $\alpha > \bar{\alpha}$.

On the other hand, when is the marginal value of liquidity negative even at zero holding? Formally, we require that

$$\left.\frac{dEU}{dy}\right|_{R^{**},\, y=0} < 0. \tag{A24}$$

Using $R^{**}(y=0) = \frac{2}{2-\chi}$, this inequality yields a lower bound on the expected return on investment relative to the cost of liquidation:

$$\bar{r} > \bar{r}_3 \equiv 1 + \frac{\chi(2+\chi)F\left(\frac{2}{2-\chi}\right)}{1-F\left(\frac{2}{2-\chi}\right)} + \frac{f\left(\frac{2}{2-\chi}\right)}{\sqrt{\alpha}\left[1-F\left(\frac{2}{2-\chi}\right)\right]}\left(\alpha\frac{24\chi^2}{(1+\chi)(2-\chi)^3} - 1\right). \tag{A25}$$

No liquidity is held if the cost of liquidation vanishes, because $\bar{r}_3 \to 1 - \frac{f(1)}{\sqrt{\alpha}[1-F(1)]} < 1$ for $\chi \to 0$.

A.4 Proof of Lemma 4

First, $W_n^{***} \to \frac{1}{2}$ as fundamental uncertainty vanishes, $\gamma \to \infty$. Next, note that cumulative distribution function of the standard normal is bounded within [0, 1] as R_{-n} diverges. One can show by contradiction that $W_{n,-n}^{***} \to \frac{1}{2}$ in any symmetric equilibrium. Taken together, this implies the expression for R_n^{***} stated in the main text.

A.5 Private Liquidity Choice

This proof mostly mirrors previous proofs, especially Appendix A.3, so only a brief outline follows.

Since $\chi < \tilde{\chi}$, the global optimum is $y^{global} = y^{**}$. The signs of all the second-order partial derivatives are unchanged (see below), so the second-order derivative is still unambiguously negative for any given liquidity held by the other intermediary, $\frac{d^2EU_n}{dy_n^2} < 0,\ \forall y_{-n}$. Therefore, at most one solution exists. An interior solution exists if and only if $\bar{r} < \bar{r}(y_{-n})$, where this upper bound on the expected return on investment now depends on the liquidity holding of the other intermediary. As before, this bound is determined by

$$\left.\frac{dEU_n}{dy_n}\right|_{R_n^{***},\, y_n=0,\, \bar{r}} \equiv 0. \tag{A26}$$

Solving for the expected return, we use $R_n \equiv R_n^{***}(y_n = 0) = [1 - \frac{\chi}{2}(1 - y_{-n})]^{-1}$ to obtain:

$$\bar{r}(y_{-n}) \equiv \frac{1 + \frac{f(R_n)}{\sqrt{\alpha}}\left[1 + \alpha\chi R_n^2(\frac{3}{2} - y_{-n})\Big(R_n - (1 + \frac{\chi}{2}(2 - y_{-n}))\Big)\right] - \frac{1+\frac{\chi}{2}(1-y_{-n})}{1+\frac{\chi}{2}(2-y_{-n})}F(R_n)}{1 - F(R_n)}. \tag{A27}$$

Finally, to establish that liquidity holdings are strategic substitutes, the implicit function theorem is used. Since $\frac{d^2EU_n}{dy_n^2} < 0$ and $\frac{d^2EU_n}{dy_n dy_{-n}} < 0$, the stated result follows. These signs arise from total differentiation:

$$\frac{d^2EU_n}{dy_n^2} = \frac{\partial^2 EU_n}{\partial y_n^2} + 2\frac{\partial^2 EU_n}{\partial y_n \partial R}\frac{dR}{dy_n} + \frac{\partial^2 EU_n}{\partial R^2}\left(\frac{dR_n}{dy_n}\right)^2 + \frac{\partial EU_n}{\partial R}\frac{\partial^2 R}{\partial y_n^2}, \tag{A28}$$

$$\begin{aligned}\frac{d^2EU_n}{dy_n dy_{-n}} = {} & \frac{\partial^2 EU_n}{\partial y_n \partial y_{-n}} + \frac{\partial^2 EU_n}{\partial y_n \partial R}\frac{dR}{dy_{-n}} + \frac{dR}{dy_n}\left(\frac{\partial^2 EU_n}{\partial R \partial y_{-n}} + \frac{\partial^2 EU_n}{\partial R^2}\frac{dR_n}{dy_{-n}}\right) \\ & + \frac{\partial EU_n}{\partial R}\frac{\partial^2 R}{\partial y_n \partial y_{-n}},\end{aligned} \tag{A29}$$

since the second-order partial derivatives stated below are signed as follows:

$$\frac{dR}{dy_{-n}} = -\chi(R^{***})^2\left(\frac{1}{2} - y_n\right) < 0, \tag{A30}$$

$$\frac{d^2R}{dy_n dy_{-n}} = \chi\left[(R^{***})^2 - 2R^{***}\left(\frac{1}{2} - y_{-n}\right)\frac{dR}{dy_{-n}}\right] > 0, \tag{A31}$$

$$\frac{\partial^2 EU_n}{\partial y_n \partial y_{-n}} = -\frac{\chi F(R)}{2\left[1 + \frac{\chi}{2}(2 - y_n - y_{-n})\right]^2}\left(1 + \frac{\chi}{2}\left[y_n - y_{-n}\right]\right) < 0, \tag{A32}$$

$$\frac{\partial^2 EU_n}{\partial R_n \partial y_{-n}} = \frac{\chi(1 - y_n)f(R)\sqrt{\alpha}}{2\left[1 + \frac{\chi}{2}(2 - y_n - y_{-n})\right]^2} > 0. \tag{A33}$$

A.6 Social Liquidity Choice

The total derivatives of the social welfare function with respect to the liquidity holdings are

$$\begin{aligned}\frac{dSWF}{dy_n} &= \frac{dEU_n}{dy_n} + \frac{\partial EU_{-n}}{\partial y_n} + \frac{\partial EU_{-n}}{\partial R_{-n}}\frac{dR_{-n}}{dy_n} > \frac{dEU_n}{dy_n},\\ \frac{d^2SWF}{dy_n^2} &= \frac{d^2EU_n}{dy_n^2} + \frac{\partial^2 EU_{-n}}{\partial y_n^2} + \frac{\partial EU_{-n}}{\partial R_{-n}}\frac{d^2R_{-n}}{dy_n^2}\\ &\quad + \left(2\frac{\partial^2 EU_{-n}}{\partial R_{-n}\partial y_n} + \frac{\partial^2 EU_{-n}}{\partial R_{-n}^2}\frac{dR_{-n}}{dy_n}\right)\frac{dR_{-n}}{dy_n} < 0,\end{aligned}$$

so the social welfare function is globally concave in the liquidity holding of each bank. Therefore, there exists a unique solution y^{SP}. If $\bar{r} \geq \bar{r}_5$, then $y^{SP} = 0$, else the solution is interior. The lower bound on the expected return on investment again solves $\left.\frac{dSWF}{dy_n}\right|_{y_1=0=y_2,\bar{r}=\bar{r}_5} \equiv 0$, which yields

$$\bar{r}_5 \equiv \bar{r}_4 + \frac{\frac{\chi}{2(1+\chi)}F\left(\frac{2}{2-\chi}\right) + \frac{6\chi^2}{(1+\chi)(2-\chi)^3}\sqrt{\alpha}f\left(\frac{2}{2-\chi}\right)}{1 - F\left(\frac{2}{2-\chi}\right)} > \bar{r}_4. \tag{A34}$$

Since $\left.\frac{dEU_n}{dy_n}\right|_{y_n=y^{***},\, y_n=y^{***}} \equiv 0$, we have $y^{SP} > y^{***}$ by global concavity. Hence, $R^{SP} < R^{***}$.

APPENDIX B: FIGURE

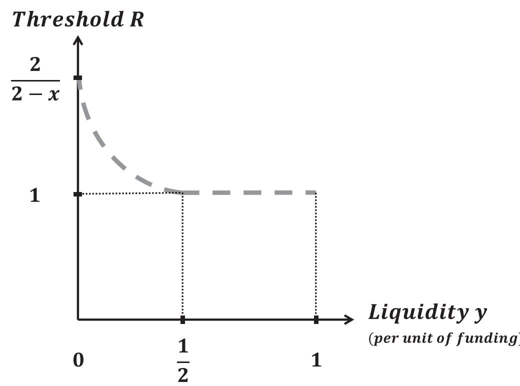

FIG. B1. The Rollover Threshold as a Function of an Intermediary's Liquidity Holding (Per Unit of Funding) in the Case of Vanishing Private Noise, $\gamma \to \infty$.

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