Heterogeneous Beliefs and Housing-Market Boom-Bust Cycles

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Abstract

This paper presents a business cycle model where different beliefs on the accuracy of news about future technological progress lead to heterogeneous expectations among households. The model shows that over-optimism of home-buyers causes housing-market boom-bust cycles if home-buyers are credit-constrained and savers do not share the over-optimism. If prices are sticky, the inflation rate and the nominal policy interest rate are low during booms and rising around the ends of booms, as typically observed in developed countries. The sensitivity of boom-bust cycles to monetary policy depends on the elasticity of labour supply. Higher availability of mortgage debt amplifies boom-bust cycles.

JEL Classification: E44, E52.

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1 Introduction

Strong booms in housing markets often end with significant drops in house prices. One of the explanations for these boom-bust cycles is over-optimism, while other explanations emphasize the roles of monetary policy and increased credit availability due to financial liberalization and innovation.[1] To investigate the roles of these factors in the formation of housing-market boom-bust cycles, this paper presents a business cycle model that succeeds in capturing the stylized features of housing-market boom-bust cycles observed in developed countries. Using this model, this paper investigates the conditions under which over-optimism of home-buyers causes housing-market boom-bust cycles, and analyzes the sensitivity of boom-bust cycles to monetary policy and the availability of mortgage debt determined by collateral constraints in the residential mortgage market.

The model is a version of so-called "news-shock" models where households receive noisy public signals (i.e., news) about future technological progress. A favourable signal generates over-optimism ex-post, if technological progress does not occur as signaled. The model also incorporates the residential mortgage market by introducing two types of households. One type of household is home-buyers, who finance housing investments through mortgage debt, and the other type is savers, who lend to home-buyers.[2] While it is standard in the literature to assume that all the households share homogeneous expectations by interpreting public signals identically, this model incorporates heterogeneous expectations between different types of households due to heterogeneous beliefs on the accuracy of public signals. The utility and production functions take standard forms in the model.

The model shows that, if household expectations are homogeneous, then over-optimism generated by a favourable signal does not cause an expectation-driven housing boom, since a rise in expected future consumption increases the real interest rate, which lowers house prices. However, this paper finds that over-optimism of home-buyers can cause simultaneous boom-

[1] For example, Taylor (2009) and Dokko, et al. (2009) discuss the cause of the recent housing-market boom-bust cycle in the U.S.

[2] This feature of the model follows Iacoviello (2005).

bust cycles in house prices and aggregate output, if home-buyers have borrowing constraints and savers do not share the over-optimism of home-buyers, believing that the signal is very noisy. This result is supported by suggestive evidence from U.S. data that heterogeneous expectations between young and old households, which are proxies for home-buyers and savers, respectively, have been highly correlated with house price growth rates. Moreover, the model shows that, when prices are sticky and the central bank follows a standard Taylor rule, the inflation rate and the policy interest rate decline during booms and rise at the end of booms, as typically observed during housing-market boom-bust cycles in developed countries.

The model explains these dynamics as follows. Over-optimism of home-buyers causes an expectation-driven housing boom, since home-buyers increase housing investments on expectations that future house prices will rise. While they need to increase mortgage debt to finance housing investments, credit supply from savers partially meets the rise in credit demand, since savers expect that the expectation-driven boom will not sustain, so that they increase savings for a future recession. Home-buyers cannot finance all the housing investments through mortgage debt, however, since they can borrow only up to the collateral value of their housing and are required to make downpayments. As a result, home-buyers increase labour supply to raise internal funds, which increases aggregate output. The rise in labour supply and savings reduces effective real wages and the real interest rate. A resulting drop in the marginal cost of production lowers the inflation rate, which leads the central bank to cut its policy rate. When the optimistic expectations of home-buyers are not realized ex-post, a housing bust occurs, and savings and labour supply decline. As a consequence, the inflation rate rises, inducing a monetary policy tightening.

This result implies that the availability of credit plays a crucial role in the formation of expectation-driven housing booms. It is a natural consequence that housing-market boom-bust cycles become sensitive to monetary policy and the degree of collateral constraints in the residential mortgage market, since these factors affect the real interest rate and the borrowing capacity of home-buyers. Policy experiments demonstrate that the sensitivity of boom-bust cycles to monetary policy depends on the trade-off between inflation stabilization through policy commitment and destabilization of the nominal interest rate due to active policy responses to inflation fluctuations. If the benefit from the former factor dominates the cost from the latter factor, then more rigid inflation-targeting rules stabilize boom-bust cycles, but otherwise they amplify boom-bust cycles. The model indicates that the balance between the two factors depends on the elasticity of labour supply. Also, the model shows that less stringent collateral constraints in the residential mortgage market amplify boom-bust cycles by letting overly optimistic home-buyers increase housing investments through higher leverage.

This paper adds to the news-shock literature that analyzes expectation-driven business cycles. See Beaudry and Portier (2004, 2007), Jaimovich and Rebelo (2009), Christiano, et al. (2008), and Schmitt-Grohé and Uribe (2008), for example. While it is standard in the literature to assume homogeneous household expectations, this paper analyzes the implications of heterogeneous household expectations generated by disagreement on the accuracy of public signals about future fundamentals.

In this regard, this paper is related to Piazzesi and Schneider (2008) and Fostel and Geanakoplos (2009). Piazzesi and Schneider quantify the effects of heterogeneous household expectations in an overlapping generations model by using household survey data directly, and Fostel and Geanakoplos theoretically analyze asset-market boom-bust cycles due to heterogeneous prior beliefs among investors. Also, this paper is related to the behavioural finance literature that analyzes positive price-volume correlations in asset markets due to heterogeneous beliefs among investors.[3]

Finally, the monetary policy analysis in this paper contributes to the literature on stabilization effects of monetary policy during asset bubbles. See Bernanke and Gertler (1999, 2001), Cecchetti, et al (2000), Gilchrist and Leahy (2002), Dupor (2005), and Christiano, et

[3]See Hong and Stein (2007) for a survey of this literature.

al. (2008), for example. This paper is especially related to the latter three papers, which analyze monetary policy effects on endogenous non-fundamental dynamics of asset prices and output due to unfulfilled expectations.

The rest of the paper is organized as follows. Section 2 summarizes the stylized features of housing-market boom-bust cycles in developed countries and the correlation between heterogeneous household expectations and the real house price growth rate in U.S. data. Section 3 describes the model. Section 4 analyzes the model with flexible prices. Section 5 analyzes the model with sticky prices. Section 6 concludes.

2 Empirical motivation

2.1 Stylized features of housing-market boom-bust cycles in developed countries

Figure 1 shows average dynamics of macroeconomic variables during housing-market boom-bust cycles in developed countries between 1970s and 1990s. Since the levels of the variables are not necessarily stationary across countries or time periods, the variables in each boom-bust episode are normalized to have zero mean over the 40-quarter time window around the peak quarter of the boom, and then each panel in the figure shows the median of the normalized variables in each quarter of the time window.[4] See Table 1 for the peak quarters of the booms identified by Ahearne, et al. (2005) and also Appendix A for data details.

Figure 1 indicates that the growth rates of aggregate output and hours-worked have tended to be high during housing booms and low after the ends of booms, while CPI inflation rates and short-term nominal interest rates have tended to be low during booms and rising around the ends of booms. This result is consistent with the findings of Bordo and Jeanne (2002) and Ahearne, et al. (2005) using similar methodologies, as well as the event studies

[4] More specifically, to construct each panel in the figure, normalize each variable for each boom-bust episode by subtracting the average of the variable over the time window around the peak of the boom from the variable. Then, pool normalized values across boom-bust cycles and derive the median of the pooled values for each quarter of the time window. In each panel, the time window shown is centered around the peak quarters of the booms.

by Borio and Lowe (2002).[5] Confirming the robustness of this result, Figures 2 and 3 show that similar results hold for the pre-1985 and the post-1985 subsample periods, respectively.[6]

In addition, Figure 4 shows the average dynamics of annual employment growth rates for young and old workers during housing-market boom-bust cycles in developed countries between 1970s and 1990s.[7] Even though age-specific actual hours-worked data are ideal for this paper, they are not available across countries in the OECD database. The figure shows that employment of young workers (under 44 years old) has tended to grow strongly during booms and weakly after booms, while employment of old workers (over 44 years old) has tended to grow more strongly after booms than before the ends of booms, except for the period around the ends of booms. The subsample data in the figure shows that this feature of the dynamics is mainly due to the post-1985 boom-bust cycles.

This paper presents a business cycle model to show that over-optimism of home-buyers causes housing-market boom-bust cycles accompanied by the stylized features shown in Figure 1, if home-buyers are credit-constrained and savers, who supply mortgage loans to home-buyers, do not share the over-optimism. Also, the model generates the stylized heterogeneous dynamics of labour supply indicated by the panels for 1970-2000 and 1985-2000 in Figure 4, given that young and old households are taken as proxies for home-buyers and savers, (i.e., those who have positive net financial assets), respectively.

[5] See Figure 11 in Bordo and Jeanne (2002) for CPI inflation rates and Charts 3.1-3.3 in Ahearne, et al. (2005) for nominal policy interest rates, CPI inflation rates, and real GDP growth rates. See Section 4 of Borio and Lowe (2002) for their analysis. While Detken and Smets (2004) also find counter-cyclical fluctuations in nominal policy interest rates during past boom-bust cycles in asset markets, including both equity and real estates, they emphasize rising inflation during booms. This paper discusses the cause of rising inflation near the ends of booms in Section 5.4.

[6] The median and the quartiles of short-term nominal interest rates fluctuate more in Figures 2 and 3 than in Figure 1. These large fluctuations in subsample periods are out of the range between 1st and 3rd quartiles when data are pooled over the entire sample period. The significant decline in short-term nominal interest rates after housing booms in Figure 3 reflects that many developed countries experienced a permanent decline in the mean level of short-term interest rates after housing booms around 1990.

[7] The method for constructing the figure is the same as for Figures 1-3. The width of the time window is 10 years.

2.2 Heterogeneous household expectations and the real house price growth rate in U.S. data

Figure 5 shows suggestive evidence from U.S. data that motivates this paper to analyze a correlation between house prices and heterogeneous household expectations. The figure compares the nationwide real house price growth rate with the difference in the Index of Consumer Expectations (ICE) between young (under 44 years old) and old (over 45 years old) households, which are taken as proxies for home-buyers and savers, respectively.[8] The figure indicates that the real house price growth rate has tended to be high when young households are more optimistic about future economic conditions than old households. The correlation coefficient between the two variables in the figure is 0.29 over the entire sample period.

The ICE consists of responses to three survey questions. One of them is about the expected future financial condition for the respondent herself, while the other two are about expected future financial and employment conditions in the economy. As this paper analyzes household expectations about aggregate economic conditions, Figure 5 includes the differences between young and old households in the two questions about future aggregate economic conditions, that is, expected financial conditions in the economy during the next 12 months (labeled "BUS12") and expected employment conditions in the economy during the next 5 years (labeled "BUS5"). The two variables move similarly to the ICE, even though the BUS5 was more volatile than the ICE and the BUS12 in the late 1990s.[9]

To confirm the correlation between heterogeneous household expectations and house price growth rates more formally, Table 2 shows the result of the regression of the real house price growth rate on household expectations (the difference in the ICE between young and old households, and the average level of the ICE across all age cohorts), real GDP growth rates,

[8]The Index of Consumer Expectations is provided by the Reuters/University of Michigan Surveys of Consumers. This index summarizes household expectations of future economic conditions, excluding consumer confidence in current economic conditions. See Appendix A for more data details.

[9]Over the entire sample period in the figure, the correlation coefficient between the real house price growth rate and the difference in BUS12 is 0.23 and the correlation coefficient between the real house price growth rate and the difference in BUS5 is 0.26.

ex-post real interest rates, and lagged dependent variables. The coefficients are estimated by OLS, assuming that the differences in household expectations between young and old households are orthogonal to unobserved house price growth shocks. Here this paper uses OLS as the first approach for confirming the correlation between heterogeneous household expectations and house price growth rates, since it is difficult to identify appropriate instrument variables for the regression.

The regression analysis confirms the implication of Figure 5. Table 2 shows that the coefficient of the contemporaneous difference in the ICE between young and old households is significantly positive at 1% significance level. Also, the t test cannot reject the null hypothesis that the sum of the coefficients of the current and lagged differences in the ICE between young and old households is 0. These results are consistent with the model, indicating that house prices grow more strongly when young households are more optimistic about future economic conditions than old households, but that the expectation-driven house price growth is corrected later. In the model, the correction of house prices occurs when optimism of home-buyers turns out to be wrong ex-post. While Table 2 shows the regression result with the lag length of 3 quarters, similar results hold even when the lag length is varied from 2 quarter to 8 quarters.[10]

The results shown in Table 2 do not significantly change even if the ICE is replaced by BUS12 for both the average level for all age cohorts and the difference between young and old households. Also, if the ICE is replaced by BUS5, the coefficient of the current difference in the BUS5 between young and old households remain positive, even though it becomes insignificant at 10% significance level.

[10]The coefficient of the contemporaneous difference in household expectations is significantly positive at 5% or 1% significance level and the t test cannot reject the null hypothesis that the sum of the coefficients of the current and lagged differences in the ICE between young and old households is 0.

3 The model

The model incorporates two types of households who take and provide mortgage loans, respectively, as well as borrowing constraints such that households can borrow only up to the collateral value of housing. There are also monopolistic producers of intermediate inputs, competitive producers of final goods, and a central bank.

3.1 Firms

There is a continuum of households who consume final goods. Final goods are produced by a CES function of intermediate inputs:

$$y_t = \left[\int_0^1 (y_{j,t})^{\frac{\theta-1}{\theta}} dj \right]^{\frac{\theta}{\theta-1}}, \tag{1}$$

where y_t is the amount of final goods produced and $y_{j,t}$ is the amount of intermediate inputs of variety j. Each variety of intermediate inputs is produced by technology represented by a standard Cobb-Douglas function:

$$y_{j,t} = (k_{j,t})^{\alpha} (A_t l_{j,t})^{1-\alpha}, \tag{2}$$

where $k_{j,t}$ is rented capital stock, A_t is labour augmenting technology, $l_{j,t}$ is employed hours-worked, and α ($\in (0,1)$) is the capital share in production.

Final-good producers take prices as given, earning zero profit. Cost minimization by final-good producers implies that the demand function for each variety of intermediate inputs is:

$$y_{j,t} = \left(\frac{P_{j,t}}{P_t} \right)^{-\theta} y_t, \tag{3}$$

where $P_{j,t}$ is the nominal price for the variety j of intermediate inputs and P_t is the aggregate price index defined by:

$$P_t = \left[\int_0^1 (P_{j,t})^{1-\theta} dj \right]^{\frac{1}{1-\theta}}. \tag{4}$$

The index P_t is the nominal unit production cost of final goods when final-good producers minimize the production cost. Since the final-good market is competitive, P_t becomes the nominal price of final goods.

Each variety of intermediate inputs is produced by a monopolistic producer. Each monopolistic producer can only infrequently adjust the price of one's product with probability $1 - \chi$ every period. Note that if $\chi = 0$, then prices are flexible. When a monopolistic producer adjusts the price, she maximizes the present discounted value of profits while the price remains fixed:

$$\max_{P_{j,t}} E_t' \left[\sum_{s=t}^{\infty} \chi^{s-t} \Lambda_{t,s} \left(P_{j,t} - P_s f_s \right) y_{j,s} \right], \tag{5}$$

subject to the production function (2) and the demand function (3), taking the probability distribution of $\Lambda_{t,s}$, f_s, y_s and P_s as given for all $s \geq t$. The operator E_t' in this equation is the subjective conditional expectation operator, and the variable $\Lambda_{t,s}$ is the discount factor between periods t and s for intermediate-input producers. The variable f_s is the real marginal cost of production for intermediate-input producers. Cost minimization by intermediate-input producers in competitive factor markets implies that:

$$f_t = \left(\frac{r_{K,t}}{\alpha} \right)^{\alpha} \left[\frac{w_t}{(1-\alpha) A_t} \right]^{1-\alpha}, \tag{6}$$

where $r_{K,t}$ and w_t are the rental price of capital and the effective real wage rate, respectively. Factor demand, $\{k_{j,t}, l_{j,t}\}_{j \in [0,1]}$, is determined by cost minimization among intermediate-input producers:

$$k_{j,t} = \frac{\alpha f_t \, y_{j,t}}{r_{K,t}}, \tag{7}$$

$$l_{j,t} = \frac{(1-\alpha) f_t \, y_{j,t}}{w_t}, \tag{8}$$

for $j \in [0, 1]$.

3.2 Households

There are two types of households. One type has a higher time-discount rate than the other. Label the former type "savers" and the latter type "home-buyers". As savers value current consumption less than home-buyers, savers lend to home-buyers in the neighbourhood of the deterministic steady state. The saver fraction of the population is μ ($\in (0, 1]$), and the home-buyer fraction is $1 - \mu$.

Consider a 'cash-less' economy where the money balance is negligible as part of financial assets. Each saver maximizes the utility function:

$$E'_t \left\{ \sum_{s=t}^{\infty} (\beta')^{s-t} \left[\ln(c'_s) + \gamma \ln(h'_s) - \frac{(l'_s)^{\xi'}}{\xi'} \right] \right\}, \tag{9}$$

where E'_t is the subjective expectation operator conditional on the information set at t for savers, β' ($\in (0, 1)$) is the time-discount rate, c'_t is consumption, h'_t is housing stock, l'_t is hours-worked, and $\gamma > 0$ and $\xi' > 1$. The prime symbol ($'$) denotes variables and parameters for savers. Savers are subject to the following flow-of-funds constraint and the law of motion for capital stock:

$$c'_t + \frac{\zeta_K}{2} \left(\frac{i'_t}{s'_{t-1}} \right)^2 s'_{t-1} + q_t (h'_t - h'_{t-1}) + b'_t = w_t l'_t + r_{K,t} s'_{t-1} + \frac{R_{t-1}}{\pi_t} b'_{t-1} + \Gamma_t, \tag{10}$$

$$s'_t = i'_t + (1 - \delta) s'_{t-1}, \tag{11}$$

where i'_t is the inclement of capital stock, s'_t is the amount of capital stock at the end of period t, q_t is the real price of housing stock, b'_t is the real balance of bonds, R_t is the gross nominal interest rate, and π_t is the gross rate of inflation, i.e., P_t/P_{t-1}. The second term on the left-hand side of Equation (10) is the investment cost function, where $\zeta_K > 0$. In the equilibrium analysis below, the convex investment cost will lead to co-movement between consumption and investment.

As shareholders, savers receive the transfer of profits from intermediate-input producers, which is denoted by Γ_t on the right-hand side of Equation (10). The real value of the transferred profits per saver, Γ_t, is given by:

$$\Gamma_t = \frac{1}{\mu} \int_0^1 \left(\frac{P_{j,t}}{P_t} - f_t \right) y_{j,t} \, dj. \tag{12}$$

Assume that intermediate-input producers and savers share the same subjective expectation operator, E'_t. Also, they share a common stochastic discount factor, i.e., $\Lambda_{t,s} = (\beta')^{s-t} c'_t / c'_s$. These assumptions ensure that intermediate-input producers behave as if they maximize the utility function of savers.

Each home-buyer maximizes the utility function:

$$E''_t \left\{ \sum_{s=t}^{\infty} (\beta'')^{s-t} \left[\ln(c''_s) + \gamma \ln(h''_s) - \frac{(l''_s)^{\xi''}}{\xi''} \right] \right\}, \tag{13}$$

where β'' $(\in (0, \beta'))$, subject to the following flow-of-funds and borrowing constraints:

$$c''_t + q_t(h''_t - h''_{t-1}) + b''_t = w_t l''_t + \frac{R_{t-1}}{\pi_t} b''_{t-1}, \tag{14}$$

$$b''_t \geq -m E'_t \left[\frac{\pi_{t+1} q_{t+1} h''_t}{R_t} \right]. \tag{15}$$

The double-prime symbol $('')$ denotes variables for home-buyers. The assumption that β'' is smaller than β' implies that home-buyers value current consumption more than savers. This difference induces home-buyers to be borrowers and savers to be lenders in the neighbourhood of the deterministic steady state.[11]

The flow-of-funds constraint (14) implies that home-buyers do not invest in capital. This assumption lets this paper abstract from the optimal investment allocation among households under the convex investment cost.

The borrowing constraint (15) implies that home-buyers can only borrow up to the collateral value of their housing, which is determined by the expected liquidation value of housing for lenders.[12] Thus the collateral value is evaluated by the saver expectations represented by E_t'. The parameter m controls the loan-to-value ratio for residential mortgages. Later, this paper will analyze the sensitivity of model dynamics to the value of m to discuss the effect of the availability of mortgage debt on house price dynamics.

3.3 Monetary policy

If goods prices are flexible – that is, if the probability of price adjustment for each firm, $1-\chi$, is 1 – then simply assume that the aggregate nominal price level, P_t, remains constant for all t without loss of generality. Thus, the gross rate of inflation, π_t, satisfies:

$$\pi_t = 1, \qquad \text{if } \chi = 0. \tag{16}$$

If $\chi > 0$, so that goods prices are sticky, then assume that the central bank sets the nominal interest rate, R_t, following a standard Taylor rule:

$$\widehat{R}_t = \phi_\pi \widehat{\pi}_t + \phi_Y \widehat{y}_t + \phi_R \widehat{R}_{t-1}, \qquad \text{if } \chi \in (0,1), \tag{17}$$

[11] See Iacoviello (2005) for more details.

[12] See Kiyotaki and Moore (1997) for the bargaining environment behind the borrowing constraints. In short, lenders can only foreclose on collateral if borrowers walk away from debt contracts. Borrowers renegotiate debt contracts if the value of future debt service exceeds the value of collateral. Lenders expect this and lend only up to the value of collateral. This paper assumes that borrowers can renegotiate the debts only before the realization of aggregate shocks in the next period, and hence lenders can seize the labour income of borrowers in period $t+1$ if debts exceed the realized value of collateral.

where the hat symbol denotes log deviations of the variables from the steady state values.[13]

3.4 Shock process, public signals, and heterogeneous beliefs

Assume that labour augmenting technology, A_t, is driven by an AR(1) process:

$$\ln(A_t) = \rho_A \ln(A_{t-1}) + \epsilon_{A,t}, \tag{18}$$

where $\epsilon_{A,t}$ is an i.i.d. shock distributed by $N(0, \sigma_A^2)$. Households receive public signals of $\epsilon_{A,t+\tau}$ in period t, where τ is a positive integer. Signals are generated by the following process:

$$z_{A,t} = \epsilon_{A,t+\tau} + \omega_{A,t}, \tag{19}$$

where $\omega_{A,t}$ is an i.i.d. noise distributed by $N(0, \nu_A^2)$.

Assume that households disagree on the accuracy of public signals, which is represented by ν_A. Denote the beliefs of savers and home-buyers by $\nu_A^{'}$ and $\nu_A^{''}$, respectively. Their beliefs are fixed regardless of ex-post realizations of $\epsilon_{A,t}$. Then Equations (18) and (19) imply:

$$E'[\epsilon_{A,t+\tau}|z_{A,t}] = \frac{\sigma_A^2 z_{A,t}}{\sigma_A^2 + (\nu_A^{'})^2}, \tag{20}$$

$$E''[\epsilon_{A,t+\tau}|z_{A,t}] = \frac{\sigma_A^2 z_{A,t}}{\sigma_A^2 + (\nu_A^{''})^2}. \tag{21}$$

Thus, heterogeneous household beliefs generate heterogeneous expectations in response to public signals.[14]

Even though rational households would update their beliefs on the accuracy of public

[13] Here the monetary policy rule does not include the expected value of a future inflation rate or future output. This assumption is convenient, since it requires no specification of the central bank's subjective expectation formation process. The main results of this paper will sustain even if an expected future inflation rate is included in the monetary policy rule instead of the current inflation rate, provided that the central bank does not share the over-optimism of home-buyers.

[14] Bolton, Scheinkman and Xiong (2006) generate heterogeneous expectations by heterogeneous beliefs of the same form as in Equations (20) and (21).

signals, note that public signals in the model are a proxy for news about future technological progress in reality. Since every discovery of new technology is different, it is difficult to guess the accuracy of news about an expected discovery from past experience. The time-invariant household beliefs are a short-cut to reflect this difficulty. While it remains a question how the distribution of heterogeneous household beliefs is determined in reality, this paper utilizes the type-specific household beliefs to generate heterogeneous household expectations between home-buyers and savers, as suggested by the evidence described in Section 2.2.[15]

3.5 Equilibrium conditions

Market prices are determined to satisfy market clearing conditions for hours-worked, capital stock, housing stock, and bonds:

$$\int_0^1 l_{j,t} dj = \mu l'_t + (1-\mu) l''_t, \tag{22}$$

$$\mu s'_{t-1} = \int_0^1 k_{j,t} dj, \tag{23}$$

$$\mu h'_t + (1-\mu) h''_t = 1, \tag{24}$$

$$\mu b'_t + (1-\mu) b''_t = 0, \tag{25}$$

where the supply of housing stock is fixed to 1 in Equation (24). Thus, housing stock is land. Equation (23) implies that investments in capital by savers materialize one period later.

Equilibrium conditions are defined as follows: Every period, $\{c'_t, h'_t, l'_t, b'_t, i'_t, s'_t\}$ and $\{c''_t, h''_t, l''_t, b''_t\}$ solve the maximization problems for savers and home-buyers, respectively; $P_{j,t}$ for $j \in [0,1]$ solves the maximization problem for intermediate-input producers if the producer of the variety j can adjust the price in period t, and otherwise $P_{j,t}$ equals $P_{j,t-1}$; agents hold rational expectations of the determination of $\{w_s, r_{K,s}, q_s, f_s, y_s, \pi_s, R_s, \Gamma_s\}_{s=t}^{\infty}$ conditional on each realization of technological shocks and public signals, but the subjective likelihood

[15] Regarding the endogenous dynamics of the belief distribution, Yu (2009) presents a housing-market model where heterogeneous beliefs on fundamentals among market participants remain significant for a long period of time despite Baysian learning by each market participant.

of the realization of future shocks for each agent is determined by the agent-specific time-invariant belief of the value of ν_A; and $\{w_t,\ r_{K,t},\ q_t,\ f_t,\ y_t,\ \pi_t,\ R_t,\ \Gamma_t\}$ and $\{y_{j,t},\ k_{j,t},\ l_{j,t}\}$ for $j \in [0,1]$ are determined to satisfy Equations (1), (2), (6)-(8), (12), (16), (17), and (22)-(25).

The equilibrium in the model is similar to standard competitive equilibrium, as agents hold rational expectations of equilibrium dynamics for each possible realization of shocks and public signals. The only difference from standard competitive equilibrium is that households disagree on the likelihood of future shocks, given public signals.

3.6 The numerical solution method

This paper solves equilibrium dynamics numerically by log-linearizing the equilibrium conditions. The log-linearized equilibrium conditions are different from the standard rational equilibrium conditions in terms of the agent-specific expectation operators. Use the undetermined coefficient method to find the solution to the log-linearized equilibrium conditions. See Appendix B for more details.

4 Basic results in the model with flexible prices

This section analyzes the model with flexible goods prices (i.e., $\chi = 0$), showing that an ex-post wrong public signal of future technological progress causes housing-market boom-bust cycles when home-buyers believe that the signal is accurate (i.e., $\nu''_A = 0$) and savers consider the signal uninformative (i.e., $\nu'_A = \infty$). The analysis is numerical, using standard parameter values in the literature. The unit of time in the model is a quarter. The capital share of aggregate factor income, α, is 0.33. The quarterly depreciation rate of capital, δ, is 0.025. The loan-to-value ratio for residential mortgages, m, is set to 0.88, which is implied by the average ratio of downpayment to housing value among first-time home-buyers in the U.S. reported by Fisher and Gervais (2007).[16] The lead of public signals, τ, is assumed to be 4 periods, following Beaudry and Portier (2004). The fraction of credit-constrained

[16] They report that the average down-payment ratio was 0.13 in 1986 and 0.12 in 1996.

home-buyers, $1-\mu$, is 0.25, which is the credit-constrained fraction of households estimated by Hajivassiliou and Ioannides (2007) using the PSID data. The following parameter values are set as in Iacoviello (2005): the time discount rates of savers, β', and home-buyers, β'', are 0.99 and 0.95, respectively; the coefficient of the investment function, ζ_K, is $2/\delta$; the weight on housing preference, γ, is 0.1; and the elasticity of substitution between varieties of inputs, θ, is 6, which implies a 5% mark-up. The persistence of realized productivity shocks, ρ_A, is 0.9. To analyze the sensitivity of equilibrium dynamics to the elasticity of labour supply, this section shows equilibrium dynamics under $\xi' = \xi'' = 1.5$ and $\xi' = \xi'' = 3$. Section 5 will calibrate the values of ξ', ξ'' and ρ_A to the standard deviations of detrended age-specific hours-worked and the lag-1 autocorrelation of detrended GDP in U.S. data, using the model with sticky prices.

4.1 Expectation-driven boom-bust cycles in the housing market with heterogeneous household expectations

Figure 6 shows the impulse response of the model when: households receive a public signal of future technological progress in period 0; home-buyers believe that the signal is accurate, but savers consider the signal uninformative; and no technological progress is realized in period 4 (i.e. $z_{A,0} = 1$, $\epsilon_{A,4} = 0$, $\nu'_A = \infty$, and $\nu''_A = 0$). Thus the optimism of home-buyers and subsequent corrections of their expectations are the sources of the dynamics in the figure.

In the figure, over-optimism of home-buyers causes simultaneous boom-bust cycles in house prices, output, investment, and hours-worked. A housing boom occurs in response to a public signal of future technological progress, since home-buyers increase housing investments on expectations that future house prices will rise. At the same time, aggregate output rises during the housing boom, as credit-constrained home-buyers work more to raise internal funds for financing their housing investments. On the other hand, as savers do not share the optimism of home-buyers, they instead expect the boom to be temporary and increase savings for a future recession. As a consequence, investments into capital expand. A housing bust occurs subsequently when the optimistic expectations of home-buyers are not realized in period 4. In response, savings and labour supply decline, and aggregate investment and output fall.

Savers reduce labour supply during the housing boom in the figure, since the presence of the convex investment cost induces them to consume a large part of the revenue from the sales of their housing, which results in an increase in disutility of labour.[17] The pro-cyclical fluctuations in home-buyers' labour supply and the counter-cyclical fluctuations in savers' labour supply in the model are largely consistent with the stylized age-specific employment dynamics during housing-market boom-bust cycles over 1985-2000 shown in Figure 4, given that young and old households are taken as proxies for home-buyers and savers, respectively.

Figure 6 indicates that aggregate consumption co-moves with house prices if the elasticity of labour supply is high (i.e., the values of ξ' and ξ'' are low). In this case, overly optimistic home-buyers increase labour supply so much that increased wage income prevents a large decline in their consumption. As a result, aggregate consumption increases through a rise in savers' consumption during the housing boom. On the other hand, if the elasticity of labour supply is low, then overly optimistic home-buyers do not increase labour supply very much, but cut their consumption substantially to increase their housing investments. A large decline in home-buyers' consumption leads to a drop in aggregate consumption during the housing boom.

Figure 7 shows the case in which a positive technological shock is realized in period 4 as signaled (i.e., $z_{A,0} = \epsilon_{A,4} = 1$). In this case, house prices remain high after period 4, confirming the optimistic expectations of home-buyers.

[17] Without the convex investment cost, savers would smooth their consumption intertemporally by adjusting investments into capital. This behaviour reduces an increase in savers' consumption during housing booms, leading to a decline in aggregate consumption through a drop in home-buyers' consumption.

4.2 The effects of over-optimism without heterogeneous household expectations or borrowing constraints

Sensitivity analysis indicates that both heterogeneous household expectations and borrowing constraints on home-buyers are necessary to generate expectation-driven boom-bust cycles in the housing market in the model.

First, Figure 8 shows that the impulse response of the model to an ex-post wrong public signal of future technological progress ($z_{A,0} = 1$ and $\epsilon_{A,4} = 0$) when all the households are savers ($\mu = 1$) and consider the signal accurate ($\nu'_A = 0$). This is a standard set-up in news-shock models with representative agents. The figure shows that over-optimism of households leads to a rise in the real interest rate through higher expected future consumption, causing a decline in house prices.[18] Figure 9 shows that this result remains robust even when only half of the households (i.e., savers) consider the signal accurate and the other half believe that the signal is uninformative, so that households have heterogeneous expectations without borrowing constraints.[19]

Second, Figure 10 shows that, when both home-buyers and savers consider the signal accurate ($\mu = 0.75$ and $\upsilon'_A = \upsilon''_A = 0$), an ex-post wrong public signal of future technological progress does not cause a housing-market boom-bust cycle, since the real interest rate rises due to a decline in savings by savers who expect that an increase in future income will

[18] In the figure, over-optimism leads to an increase in aggregate investment in capital, since expected future technological progress will raise the rate of return on capital. This effect is so strong that aggregate consumption drops, which leads to an increase in labour supply and aggregate output. It can be numerically shown that, if the elasticity of substitution between varieties of inputs, θ, is high, then over-optimism increases aggregate consumption and house prices through higher expected consumption, leading to a decline in aggregate investment and output. In general, over-optimism leads to a negative correlation between house prices and aggregate output. This result is similar to the findings of Beaudry and Portier (2004, 2007) in the neo-classical growth model.

[19] Only for this case, where all the households are savers and only half of the households believe the signal, a small adjustment cost for bonds, $\frac{\zeta_B}{2}(b'_t)^2$, with $\zeta_B = 1e-6$ is added to the left-hand side of Equation (10). This assumption ensures that the log-linearized equilibrium dynamics around the deterministic steady state is stationary when savers have credit transactions among them due to heterogeneous expectations. For the sake of simplicity, it is assumed that savers do not trade shares of intermediate producers. Also, note that, with flexible prices, the intermediate-input producers set $P_{j,t}/P_t = \theta f_t/(\theta-1)$ every period, regardless of their subjective expectation operators and time-discount rates. Thus, it is not necessary to specify how heterogeneous saver expectations change the subjective expectation operator for intermediate-input producers.

finance their future consumption.[20] Similarly, it is possible to show that an ex-post wrong public signal of future technological progress leads to a decline in aggregate output if savers consider the signal accurate and home-buyers do not ($v'_A = 0$ and $v''_A = \infty$). Overall, these results indicate that both borrowing constraints on home-buyers and the existence of non-optimistic savers are necessary for over-optimism of home-buyers to cause expectation-driven boom-bust cycles.

5 Analysis of the model with sticky prices

This section analyzes the model with sticky goods prices (i.e., $\chi > 0$). For all the dynamics shown in this section, home-buyers believe that public signals of future technological progress are accurate ($v''_A = 0$), but savers consider them uninformative ($v'_A = \infty$). Also, positive signals in period 0 turn out to be wrong ex-post ($z_{A,0} = 1$ and $\epsilon_{A,4} = 0$) for all the dynamics.

In addition to the parameter values specified in Section 4, the benchmark monetary policy rule coefficients, ϕ_π, ϕ_Y and ϕ_R, are set to 0.245, 0.097, and 0.86, respectively, which are the least square estimates by Gertler (1999) for the U.S.[21] The probability of price-adjustment by intermediate-input producers, $1 - \chi$, is set to 0.5. While the implied value of χ is slightly smaller than the standard values in the literature, the number is not unrealistic. For example, Amirault, Kwan, and Wilkinson (2005) report that half of Canadian firms changed prices at least once every three months from July 2002 to March 2003 in a survey conducted by the Bank of Canada. Also, Bunn and Ellis (2009) report that the average duration of price changes in UK monthly CPI microdata is 5.3 months for all items and 6.7 months for all

[20] In contrast with Figure 8, output and house prices increase very strongly after the expected technological progress is not realized. This is due to the existence of credit-constrained home-buyers. As in Figure 8, expectations of future technological progress lead to a rise in the real interest rate. In response to a resulting rise in the cost of housing investments, home-buyers reduce housing investments. When the public signal of future technological progress turns out to be wrong, home-buyers start working more to raise funds for replenishing their housing stock. This increase in labour supply gives rise to a strong increase in aggregate output. Also, home-buyers buy housing from savers and savers add the revenues to their savings, which lowers the real interest rate. This development causes a strong increase in house prices after the signal turns out to be wrong.

[21] The estimates are for the Taylor rule that responds to the contemporaneous inflation rate as in this paper.

items excluding temporary discounts. These numbers imply that χ equals 0.45 and 0.57, respectively.[22]

In the model with sticky prices, it turns out that over-optimism of home-buyers does not cause simultaneous boom-bust cycles in house prices and aggregate output when all the households have an identical elasticity of labour supply, i.e., $\xi' = \xi''$.[23] Instead, this section sets realistic parameter values by jointly calibrating the values of ξ' and ξ'' and the persistence of realized productivity shocks, ρ_A, to the standard-deviations of detrended hours-worked of young and old workers divided by the standard deviation of detrended GDP and to the lag-1 autocorrelation of detrended GDP in U.S. data, given the other parameter values. The data on hours-worked of young and old workers are used as proxies for hours-worked of home-buyers and savers, respectively. Since the specification of heterogeneous household beliefs in the model is too stylized for quantitative exercise, the calibration uses the model without public signals, $\omega_{A,t}$, which is a standard business cycle model very similar to Iacoviello (2005).[24] The calibration yields $\xi' = 2.7$, $\xi'' = 1.01$, and $\rho_A = 0.6$.[25] The results for the flexible-price case shown in Section 4 sustain with these values of ξ', ξ'' and ρ_A.

5.1 Underlying mechanism for the stylized features of housing-market boom-bust cycles

Figure 11 shows the impulse response of the model with sticky prices. It is largely consistent with the stylized features of housing-market boom-bust cycles in developed countries

[22]If χ is high, i.e., the probability of price adjustment is low, then the inflation rate does not fluctuate much. In this case, the policy interest rate becomes more responsive to output, given the values of ϕ_π and ϕ_Y, which prevents simultaneous boom-bust cycles in house prices and output.

[23]In this case, aggregate labour supply is strongly affected by savers' labour supply. The negative correlation between savers' labour supply and house prices leads to a negative correlation between output and house prices during housing-market boom-bust cycles.

[24]The calibration chooses the values of ξ', ξ'', and ρ_A that minimize the sum of squares of percent gaps between the three moments in the model without public signals and the data. In the data, both hours-worked and GDP are log-linearly detrended. See Appendix A for data details and comparison of the targeted moments in the model and the data. The minimization problem is numerically solved by grid search. The intervals between grid points are 0.1 for ξ', 1 for $\log_{10}\xi''$, and 0.01 for ρ_A. Note that, without public signals, the standard deviations of variables in the log-linearized model are proportional to the standard deviation of technological shocks, σ_A. Since all the standard deviations of hours-worked are divided by the standard deviation of output, it is not necessary to specify the value of σ_A.

[25]Similarly, Campbell and Hercowitz (2004) take into account the difference in labour supply between home-buyers and savers by assuming that savers do not supply labour in their model.

described in Section 2.1: aggregate output, investment, consumption and hours-worked co-move with the house price; the inflation rate and the nominal interest rate are low during a housing boom and rising at the end of the boom; and the labour supply of home-buyers is pro-cyclical while that of savers is counter-cyclical.

The underlying mechanism for the dynamics of real variables is the same as in the previous section. The model explains the dynamics of the inflation rate and the nominal interest rate as follows. As in the flexible-price case, aggregate labour supply and savings rise during expectation-driven housing booms, lowering effective real wages and the real interest rate. Given sticky prices, a resulting decline in the marginal cost of production leads to a drop in the inflation rate through the pricing behaviour of producers.[26] In response, the central bank cuts the nominal policy interest rate. When the signaled technological progress is not realized ex-post, a housing bust occurs. Resulting declines in aggregate labour supply and savings raise effective real wages and the real interest rate, which leads to rises in the inflation rate and the policy interest rate.

5.2 Sensitivity of housing-market boom-bust cycles to monetary policy

The next two subsections will show that monetary policy and the availability of mortgage debt in the residential mortgage market affect housing-market boom-bust cycles through housing investments by home-buyers, which are determined by the following first-order condition with respect to h''_t:

$$\frac{\gamma c''_t}{h''_t} = q_t - \frac{mE'_t[\pi_{t+1} q_{t+1}]}{R_t} - E''_t \left[\frac{\beta'' c''_t}{c''_{t+1}} \left(q_{t+1} - mE'_t q_{t+1} \right) \right]. \tag{26}$$

The left-hand side of the equation is the marginal utility derived from housing services in terms of final goods. The right-hand side is the effective user cost of housing for home-buyers. Roughly speaking, the user cost is determined by a weighted average of the present

[26] This relationship between the marginal cost of production and the inflation rate appears in the new-Keynesian phillips curve implied by the maximization problem for intermediate-input producers.

discounted values of housing for savers (the second term on the right-hand side) and home-buyers (the last term on the right-hand side).

Figure 11 illustrates the sensitivity of housing-market boom-bust cycles to the anti-inflationary weight, ϕ_π, in the monetary policy rule. The figure indicates that larger anti-inflationary weight stabilizes boom-bust cycles, given the parameter values specified in Section 4 and this section. In this case, strong policy commitment to inflation stabilization reduces inflation volatility. Especially in period 3 (i.e., one period before the non-realization of signaled future technological progress), savers expect that the inflation rate will not increase much in period 4. This saver expectation of stable inflation prevents a large drop in the savers' subjective real interest rate in period 3, which reduces an expansion of borrowing capacity for home-buyers, given the collateral constraint (15). In Equation (26), this effect appears as a smaller increase in $E'_t \pi_{t+1}(R_t)^{-1}$ in the second term on the right-hand side of the equation in period 3. As a result, an increase in housing investments by overly optimistic home-buyers during housing booms becomes less, which reduces the overall amplitude of boom-bust cycles.

This result, however, is sensitive to the elasticity of savers' labour supply, ξ'. Figure 12 shows that the impulse response of the model with a lower value of ξ', 1.1, given the other parameter values fixed. In contrast to Figure 11, Figure 12 indicates that larger anti-inflationary weight in the monetary policy rule amplifies boom-bust cycles. In this case, high elasticity of savers' labour supply increases the counter-cyclical fluctuations in savers' labour supply over boom-bust cycles, stabilizing the pro-cyclical fluctuations in aggregate labour supply and savings.[27] As a result, effective real wages and the real interest rate are stabilized, and so are the inflation rate. Given a relatively stable inflation rate, larger anti-inflationary weight in the monetary policy rule does not add much to inflation stabilization. On the other hand, it increases the counter-cyclical fluctuations in R_t, fueling housing investments by overly-optimistic home-buyers through expanding their borrowing capacity during housing

[27]Larger counter-cyclical fluctuations in savers' labour supply stabilize fluctuations in savings through less fluctuations in savers' income.

booms, given the collateral constraint (15).[28] This effect appears as a larger increase in $E_t'\pi_{t+1}(R_t)^{-1}$ in the second term on the right-hand side of Equation (26) during housing booms.

Overall, Figures 11 and 12 indicate that the monetary policy effect on expectation-driven boom-bust cycles in the housing market depends on the trade-off between inflation stabilization through policy commitment and destabilization of the nominal policy interest rate due to active policy responses to inflation fluctuations.[29] The balance between these two factors is determined by the elasticity of savers' labour supply in the model.

5.3 Effect of higher availability of mortgage debt on housing-market boom-bust cycles

Figure 13 shows the sensitivity of housing-market boom-bust cycles to the availability of mortgage debt controlled by m. The figure indicates that the amplitude of boom-bust cycles is larger with higher availability of mortgage debt. Equation (26) is helpful to describe the underlying mechanism. As the value of m increases, the user cost of housing becomes more sensitive to savers' evaluation of future housing value, represented by the second term on the right-hand side of the equation. This makes housing investments cheaper for home-buyers in period 3 (i.e., one period before the non-realization of the signaled technological progress), since savers' non-optimistic expectations induce them to increase savings, which results in a lower subjective real interest rate for them. This development strengthens the housing boom in period 3, which feeds back into house prices in periods 0-2. Hence, higher availability of mortgage debt enlarges the amplitude of housing-market boom-bust cycles.

[28] In fact, Figure 12 indicates that greater anti-inflationary weight in the monetary policy rule amplifies fluctuations in the inflation rate. This is because the inflation rate fluctuates more as households adjust their factor-supply behaviour more during housing-market boom-bust cycles. If the anti-inflationary weight in the monetary policy rule is sufficiently enlarged, then fluctuations in the inflation rate are attenuated.

[29] It can be shown that the effect of 'leaning-against-the-wind' policy through a high value of ϕ_Y is similar to the effect of weak policy commitment to inflation stabilization through a low value of ϕ_π.

5.4 Total CPI inflation rate

Figure 1 shows the stylized feature of total CPI inflation rates during past housing-market boom-bust cycles. The total CPI inflation rate is different from the inflation rate for goods prices, $\hat{\pi}_t$, since the total CPI inflation rate includes the inflation rate for shelter (housing) cost, such as rents. Now introduce the total CPI inflation rate in the model by following its definition in practice:

$$\begin{aligned} dCPI_t =& \frac{(1-\lambda)\frac{P_t}{P_{SS}} + \lambda \frac{P_t r_{h,t}}{P_{SS} r_{h,SS}}}{(1-\lambda)\frac{P_{t-1}}{P_{SS}} + \lambda \frac{P_{t-1} r_{h,t-1}}{P_{SS} r_{h,SS}}} \\ =& \pi_t \cdot \frac{(1-\lambda) + \lambda \frac{r_{h,t}}{r_{h,SS}}}{(1-\lambda) + \lambda \frac{r_{h,t-1}}{r_{h,SS}}}, \end{aligned} \tag{27}$$

where $dCPI_t$ denotes the gross total CPI inflation rate, P_t is the nominal price of final goods, λ is the weight on shelter cost in the total CPI, and $r_{h,t}$ is the real value of shelter cost. The subscript SS denotes steady state values.[30] Since shelter cost is correlated with house prices, assume a reduced-form equation for shelter cost:

$$\widehat{r}_{h,t} = \kappa \widehat{u}_t, \text{ where } u_t \equiv q_t - \frac{E_t' \left[\pi_{t+1} q_{t+1}\right]}{R_t}. \tag{28}$$

Note that u_t is the imputed real user cost of housing for savers. The value of λ takes the 2005-6 CPI weight for shelter cost in the U.S. CPI, and κ takes the value of the correlation coefficient between the quarterly growth rate for real shelter cost in the CPI and that for the imputed ex-post real user cost of housing in U.S. data.[31] It is clear in Figures 12 (with

[30] The total CPI weights the nominal price indices for shelter cost and the rest of goods and services by expenditure shares in the base period of the indices. The steady-state value is used for the base-year value of each price index.

[31] $(\lambda, \kappa) = (0.325, 0.119)$. To calculate imputed ex-post user cost of housing, u_t, use the nationwide house price index from the Federal Housing Finance Agency for the house price in the model and the real 90-day treasury bill rate plus 0.5 % risk premium for the quarterly real interest rate in the model. This paper follows Kiyotaki and West (2004) for the degree of risk premium for discounting future house prices. The GDP deflator is used to convert nominal variables in real terms. If the imputed ex-post user cost of housing is negative in a quarter, then that quarter is excluded from the sample. The sample period of the data used for calculating the correlation coefficient is for 1981:1-1999:4, since the house price index from the Federal Housing Finance Agency is only available from 1975 and the ex-post user cost becomes negative for most of

$\xi' = 1.1$) and 13 (with $\xi' = 2.7$) that the total CPI inflation rate starts rising before the end of the housing boom in the model, since an inflation in shelter cost during housing booms contributes to a rise in the total CPI inflation rate. This result is consistent with the stylized feature of total CPI inflation rates during past housing-market boom-bust cycles shown in Figure 1.

6 Conclusions

This paper has presented a business cycle model that captures the stylized features of housing-market boom-bust cycles observed in developed countries. The model indicates that over-optimism of home-buyers can cause boom-bust cycles accompanied by these features, if home-buyers are credit-constrained and savers who supply mortgage loans to home-buyers do not share the over-optimism. Policy experiments suggest that the sensitivity of boom-bust cycles to monetary policy depends on the trade-off between inflation stabilization through policy commitment and destabilization of the nominal policy interest rate due to active policy responses to inflation fluctuations. This paper also finds that higher availability of mortgage debt in the residential mortgage market amplifies boom-bust cycles.

While this paper analyzes the effects of type-specific beliefs of home-buyers and savers, the cause of heterogeneous beliefs remains a question. Also, while this paper obtains a rich set of results using a linearized business cycle model, this approach makes it difficult to consider occasionally binding borrowing constraints on households. It is possible that optimistic agents become credit-constrained endogenously if the paper incorporates occasionally binding borrowing constraints. It is left for future research to address these remaining questions.

the quarters during the second half of 1970s and after 2000.

Table 1: The peak quarter of each housing boom in developed countries

Australia	1974:1, 1981:2, 1989:2, 1994:3
Belgium	1979:3
Canada	1976:4, 1981:1, 1989:1
Denmark	1973:3, 1979:2, 1986:1
Finland	1974:1, 1989:2, 2000::2
France	1981:1, 1991:1
Germany	1974:1, 1982:1, 1994:2
Ireland	1979:2, 1990:3
Italy	1974:4, 1981:2, 1992:2
Japan	1973:4, 1990:4
Netherlands	1978:2
New Zealand	1974:3, 1983:1, 1996:2
Norway	1976:4, 1987:2
Spain	1978:2, 1991:4
Sweden	1979:3, 1990:1
Switzerland	1973:1, 1989:4
United Kingdom	1973:3, 1980:3, 1989:3
United States	1973:4, 1979:2, 1989:4

Source: Ahearne, et al. (2005).

Table 2: Regression of the U.S. real house price growth rate on heterogeneous household expectations.

Regressors	Coefficient estimates			
	0-Lag	1-Lag	2-Lag	3-Lag
ICE for young - ICE for old ($\times 10^{-4}$)	4.99** (1.88)	-3.61 (2.03)	-1.18 (1.96)	-1.05 (1.86)
ICE for all age cohorts ($\times 10^{-4}$)	2.14 (1.10)	-0.58 (0.14)	2.40 (1.49)	1.70 (1.10)
Real GDP growth rate	-0.21 (0.10)	-0.13 (0.11)	0.28** (0.09)	-0.026 (0.097)
Real 3-month T-bill rate	-0.55** (0.20)	0.68** (0.22)	-0.43 (0.24)	0.098 (0.22)
Lagged real house price growth rate		0.49** (0.09)	-0.25* (0.09)	0.44** (0.08)
Constant	-0.014** (0.004)			
Observations:	123			
R^2:	0.66			
t-statistics for "$H_0 : \sum_{i=0}^{3} \beta_i = 0$": 0.38 (significance level: 0.70)				

Notes: The dynamic equation for the real house price growth rate (y) is:

$$y(t) = \alpha + \sum_{i=0}^{3} \beta_i x(t-i) + \sum_{i=1}^{3} \gamma_i y(t-i) + \sum_{i=0}^{3} \theta_i' Z(t-i) + \epsilon_t,$$

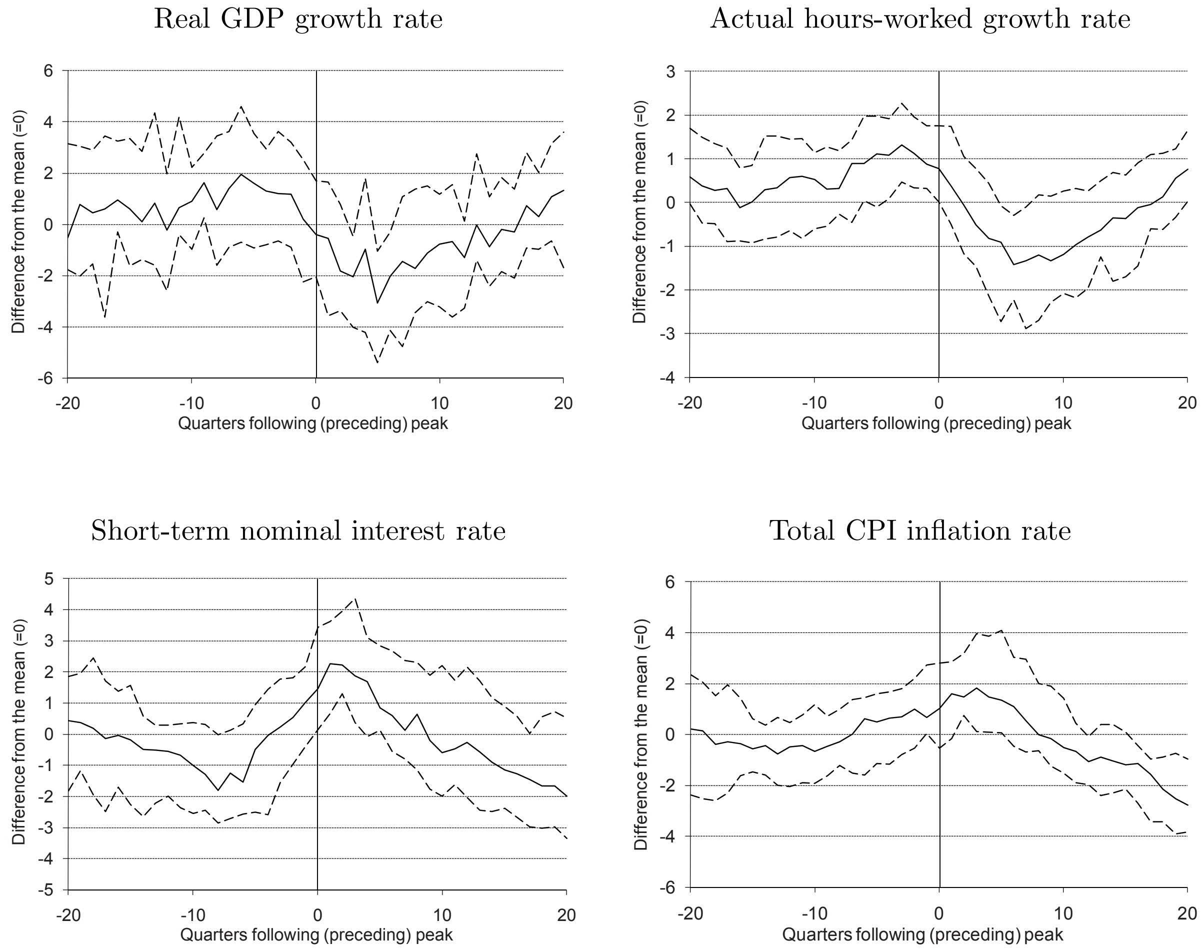
where x is the Index of Consumer Expectations (ICE) for young households (under 44 years old) minus the ICE for old households (over 45 years old), $Z = \{$the average level of the ICE across all age cohorts, the real GDP growth rate, the ex-post real 3-month T-bill rate$\}$, and ϵ_t is an error term. The regression uses OLS estimators. The sample period is for 1978:1-2009:3. Standard errors are in the parentheses. ** and * mark 1% and 5% levels of significance, respectively. See Appendix A for data details.

Table 3: Moments of aggregate variables in the calibrated model and the data. (For Appendix A.)

	Variables	Sticky-price model without public signals	Log-linearly detrended U.S. data
Standard deviations	Hours-worked of savers (old workers)†	0.792	0.794
	Hours-worked of home-buyers (young workers)†	1.767	1.767
	Private investment	1.896	5.206
	Private consumption	0.848	0.764
	Aggregate hours-worked	0.687	1.711
Lag-1 auto-correlation	GDP†	0.851	0.952

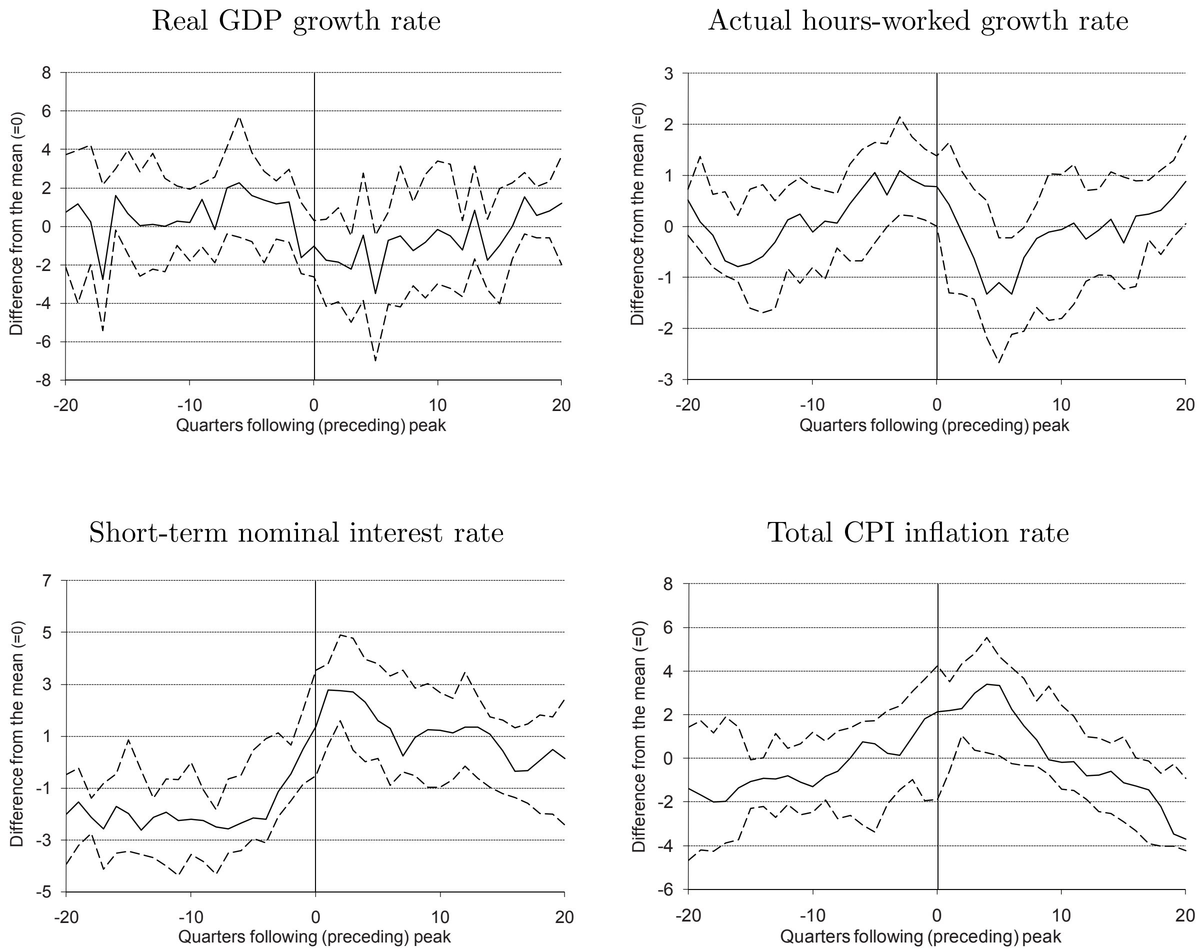
Notes: Standard deviations of the variables are divided by the standard deviation of final goods or GDP. † marks the moments targeted by the calibration of ξ', ξ'', and ρ_A using the sticky-price version of the model without public signals. In the data, private investment is gross private domestic investment and private consumption is personal non-durable consumption expenditure. GDP, consumption, and investment are in real terms. Hours-worked are actual hours-worked. Hours-worked data for young and old workers are used as proxies for hours-worked of home-buyers and savers, respectively. See Appendix A for data details.

Figure 1: Stylized features of housing-market boom-bust cycles in developed countries over 1970-2000



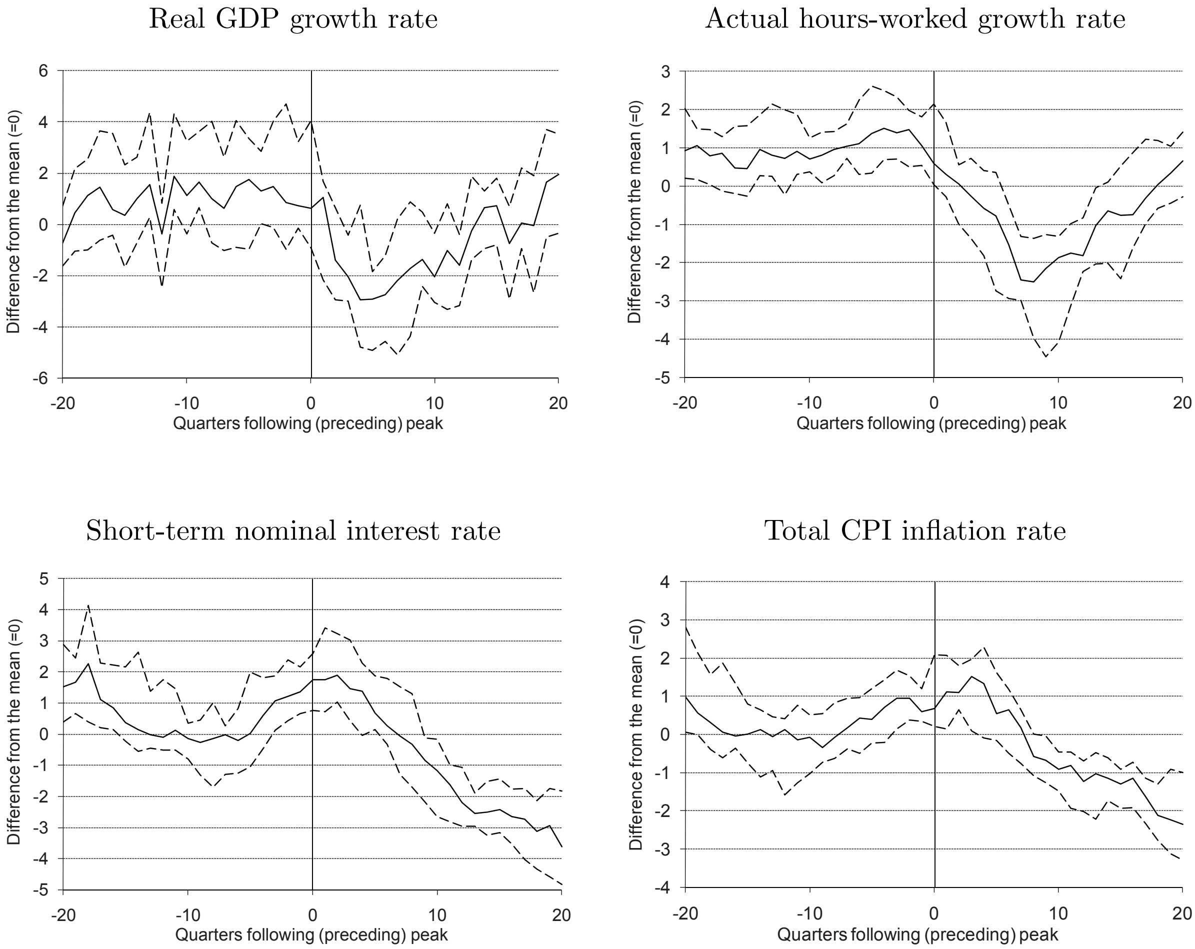
Notes: The solid line is the median of the variable in question in each quarter around the peaks of past housing booms in developed countries for 1970 to 2000. The dashed lines below and above the solid line are the first and the third quartiles in each quarter, respectively. Period 0 corresponds to the peak quarters of housing booms. The unit of each vertical axis is a percentage point on an annualized basis.. See Appendix A for data details.

Figure 2: Stylized features of housing-market boom-bust cycles in developed countries over 1970-1985



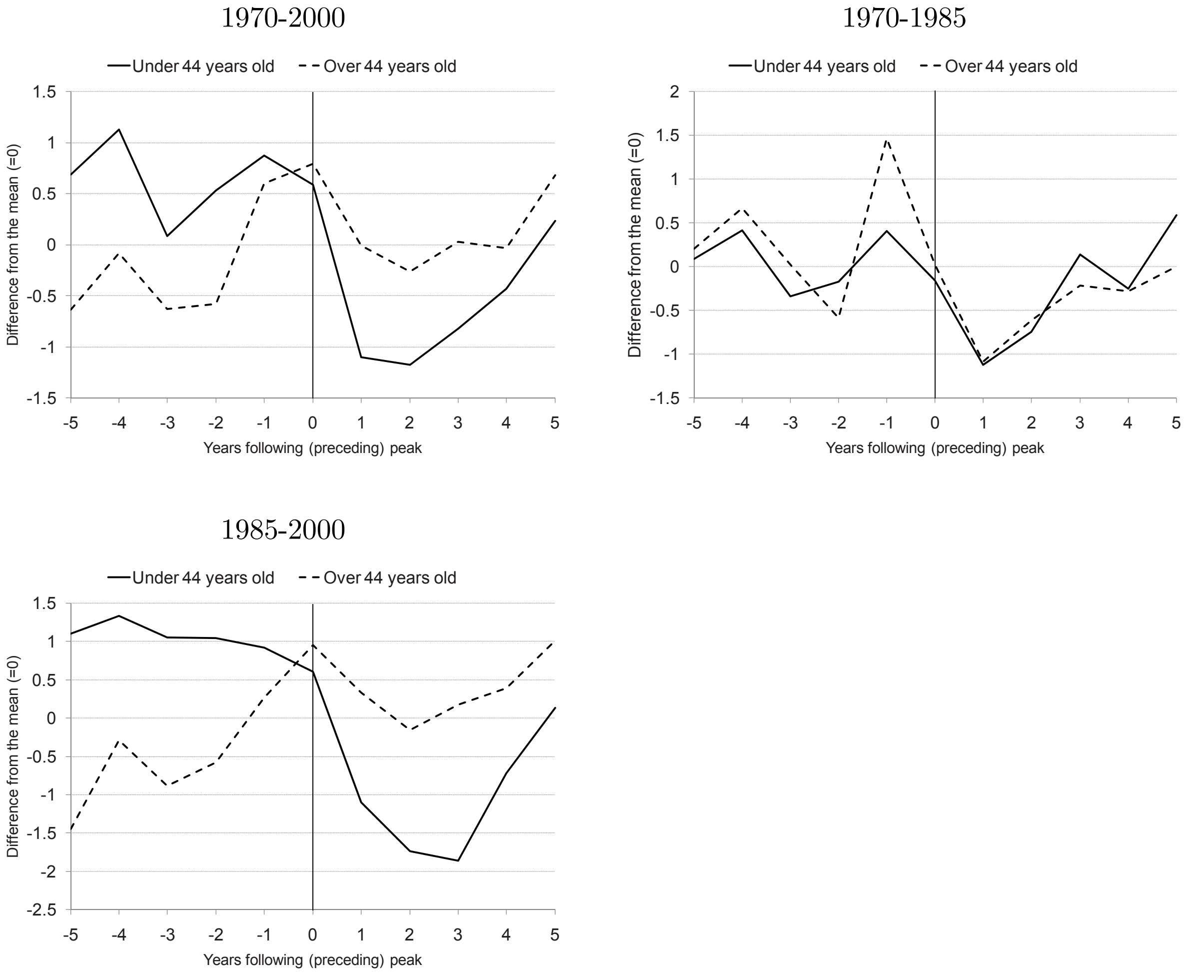
Notes: The solid line is the median of the variable in question in each quarter around the peaks of past housing booms in developed countries for 1970 to 1985. The dashed lines below and above the solid line are the first and the third quartiles in each quarter, respectively. Period 0 corresponds to the peak quarters of housing booms. The unit of each vertical axis is a percentage point on an annualized basis. See Appendix A for data details.

Figure 3: Stylized features of housing-market boom-bust cycles in developed countries over 1985-2000



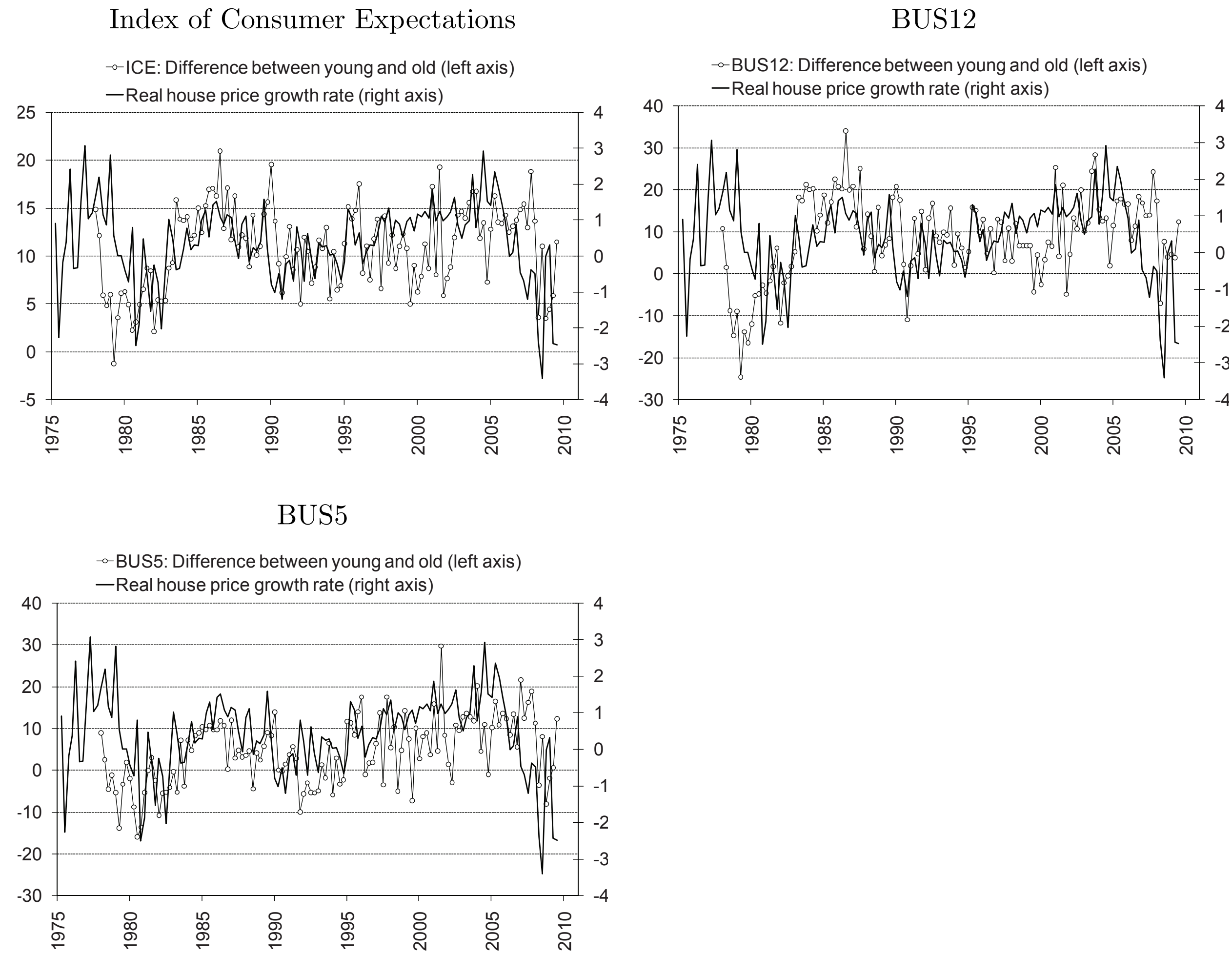
Notes: The solid line is the median of the variable in question in each quarter around the peaks of past housing booms in developed countries for 1985 to 2000. The dashed lines below and above the solid line are the first and the third quartiles in each quarter, respectively. Period 0 corresponds to the peak quarters of housing booms. See Appendix A for data details. The unit of each vertical axis is a percentage point on an annualized basis.

Figure 4: Employment growth rates for young and old workers during housing-market boom-bust cycles in developed countries



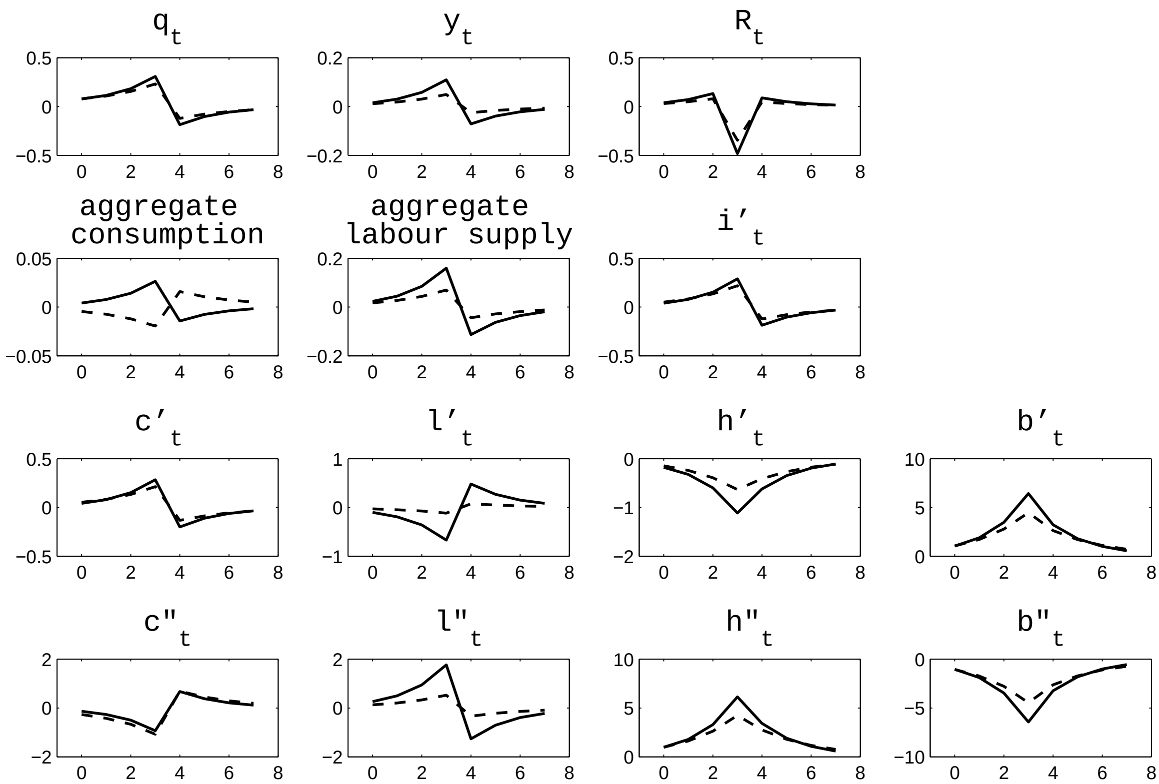
Notes: The solid lines and the dashed lines are the medians of annual employment growth rates for young and old workers, respectively, in each year around the peaks of past housing booms in developed countries over the sample period for each panel. Period 0 corresponds to the peak years of housing booms. See Appendix A for data details. The unit of each vertical axis is a percentage point on an annualized basis.

Figure 5: The real house price growth rate and differences in the Index of Consumer Expectations between young and old households in U.S. data



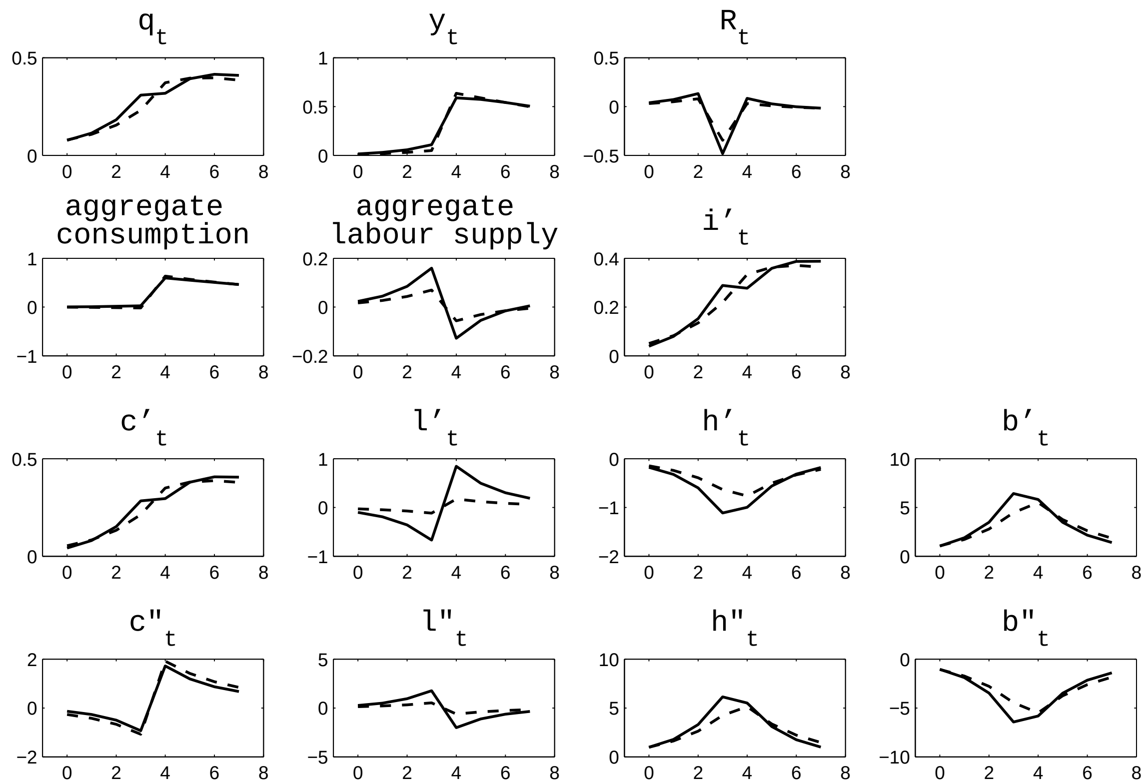
Notes: The unit of real house price growth rates is a percentage point on a quarterly basis. Positive values of "Difference between young and old" in each panel indicate that young households (under 44 years old) have stronger confidence in future economic conditions than old households (over 45 years old). "BUS12" represents household expectations of future financial conditions in the economy during the next 12 months and "BUS5" represents those of future employment conditions in the economy during the next 5 years. See Appendix A for data sources.

Figure 6: Flexible prices: Impulse response to over-optimism of home-buyers when savers do not share the optimism



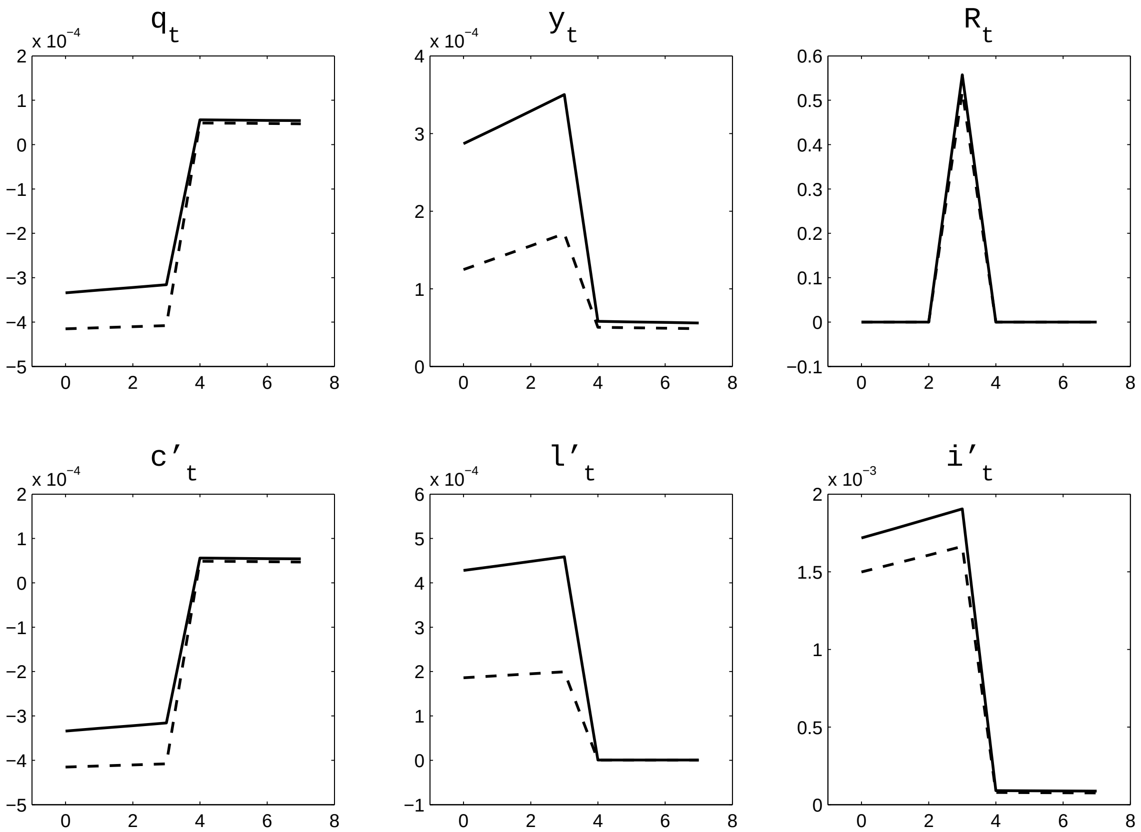
Notes: The solid line: $\xi' = \xi'' = 1.5$. The dashed line: $\xi' = \xi'' = 3$. Figures are % deviations from the deterministic steady-state values. The signal is received in period 0, but is not realized in period 4, i.e., $z_{A,0} = 1$ and $\epsilon_{A,4} = 0$. The economy is at the steady state before period 0. Only home-buyers consider the signal accurate, i.e. $\nu'_A = \infty$ and $\nu''_A = 0$. Note that R_t denotes the real interest rate, as $\pi_t = 1$ in flexible price cases. The third and the forth rows show the actions of savers and home-buyers, respectively.

Figure 7: Flexible prices: Impulse response to correct optimism of home-buyers when savers do not share the optimism



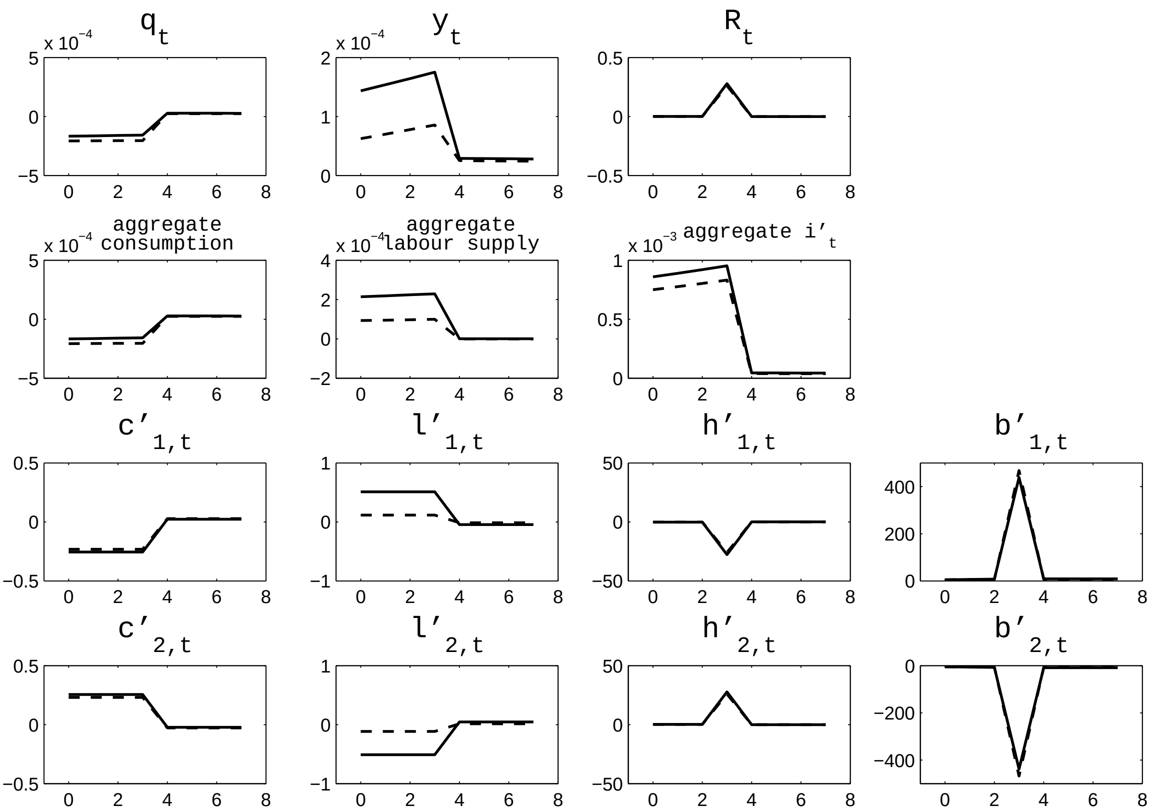
Notes: The solid line: $\xi' = \xi'' = 1.5$. The dashed line: $\xi' = \xi'' = 3$. Figures are % deviations from the deterministic steady-state values. The signal is received in period 0, and realized in period 4, i.e., $z_{A,0} = 1$ and $\epsilon_{A,4} = 1$. The economy is at the steady state before period 0. Only home-buyers consider the signal accurate, i.e. $\nu'_A = \infty$ and $\nu''_A = 0$. Note that R_t denotes the real interest rate, as $\pi_t = 1$ in flexible price cases. The third and the forth rows show the actions of savers and home-buyers, respectively.

Figure 8: Flexible prices: Impulse response to over-optimism of savers in the representative agent case



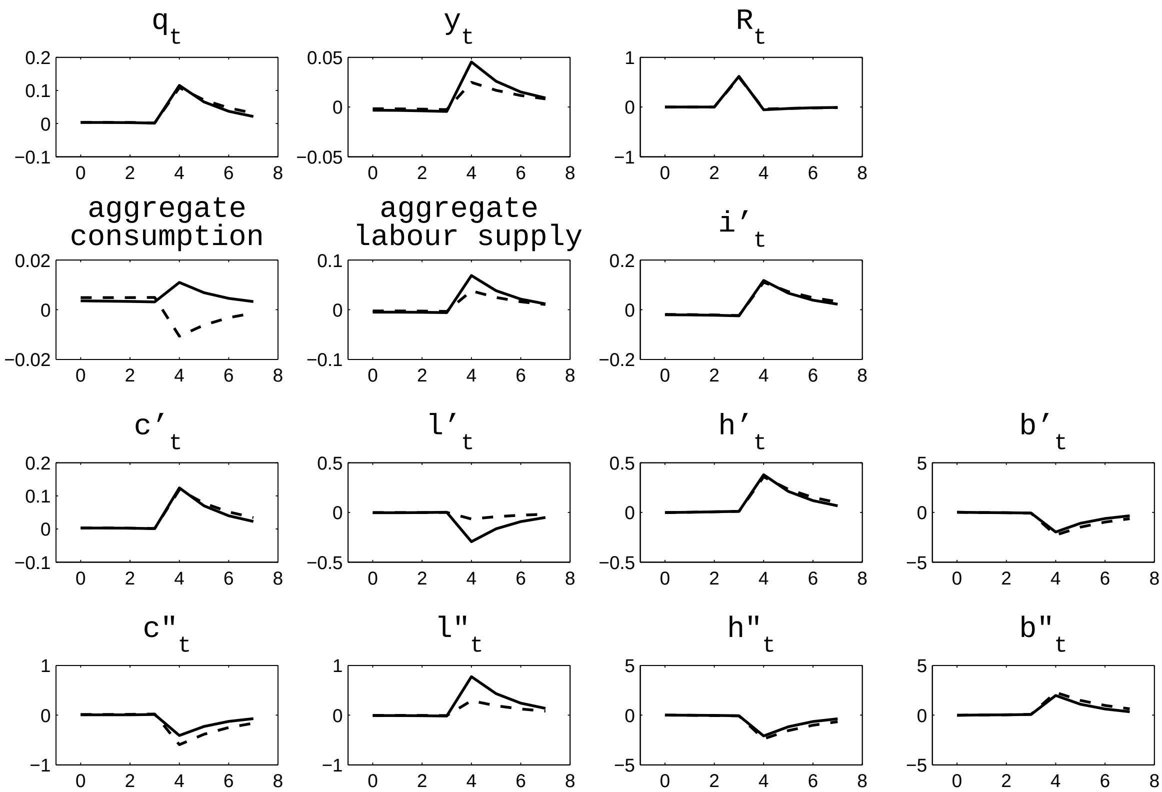
Notes: The solid line: $\xi' = \xi'' = 1.5$. The dashed line: $\xi' = \xi'' = 3$. Figures are % deviations from the deterministic steady-state values. The signal is received in period 0, but is not realized in period 4, i.e., $z_{A,0} = 1$ and $\epsilon_{A,4} = 0$. The economy is at the steady state before period 0. All the households are savers ($\mu = 1$) and consider the signal accurate ($\nu'_A = 0$). Note that R_t denotes the real interest rate, as $\pi_t = 1$ in flexible price cases. As there is only one type in the economy, c'_t, l'_t, and i'_t equal aggregate variables.

Figure 9: Flexible prices: Impulse response to over-optimism of part of savers when the other savers do not share the over-optimism.



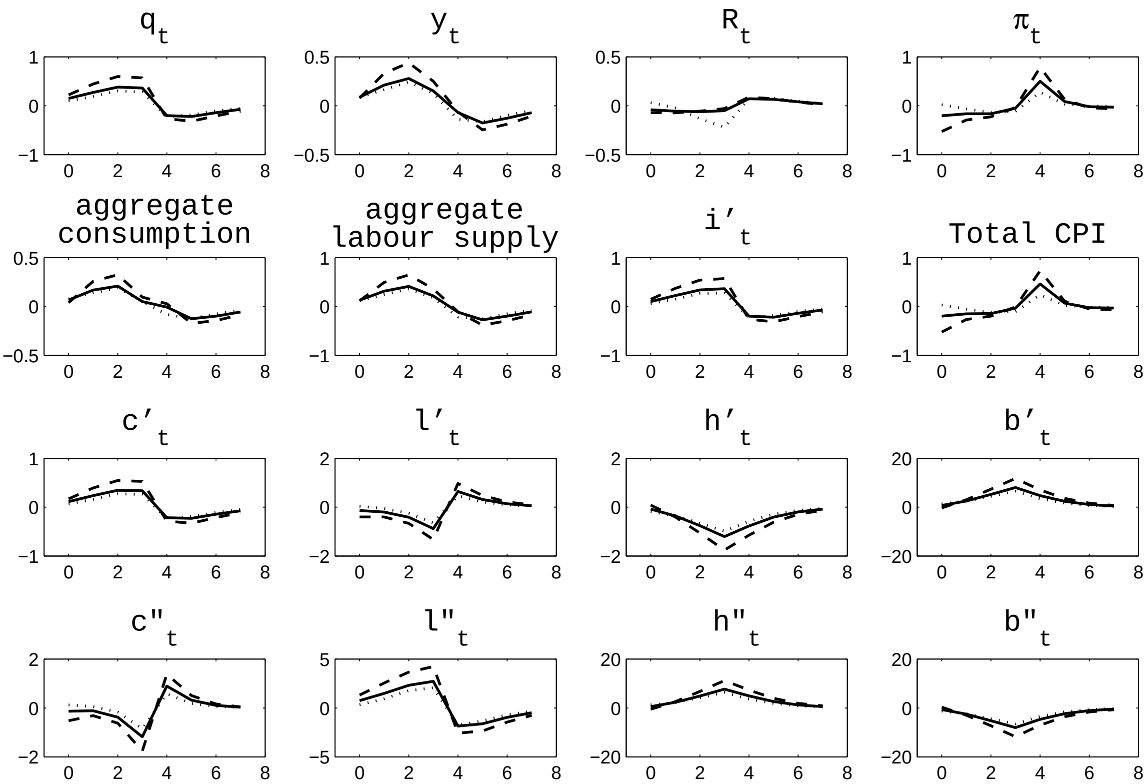
Notes: The solid line: $\xi' = \xi'' = 1.5$. The dashed line: $\xi' = \xi'' = 3$. Figures are % deviations from the deterministic steady-state values. The signal is received in period 0, but is not realized in period 4, i.e., $z_{A,0} = 1$ and $\epsilon_{A,4} = 0$. The economy is at the steady state before period 0. All the households are savers ($\mu = 1$). Half of savers consider the signal accurate ($\nu'_A = 0$), while the other half consider the signal uninformative ($\nu'_A = \infty$). The third and the forth rows show the actions of the two types of savers. Those who consider the signal uninformative are denoted by the subscript "1" and those who consider the signal accurate are denoted by the subscript "2". Note that R_t denotes the real interest rate, as $\pi_t = 1$ in flexible price cases.

Figure 10: Flexible prices: Impulse response to over-optimism of both home-buyers and savers



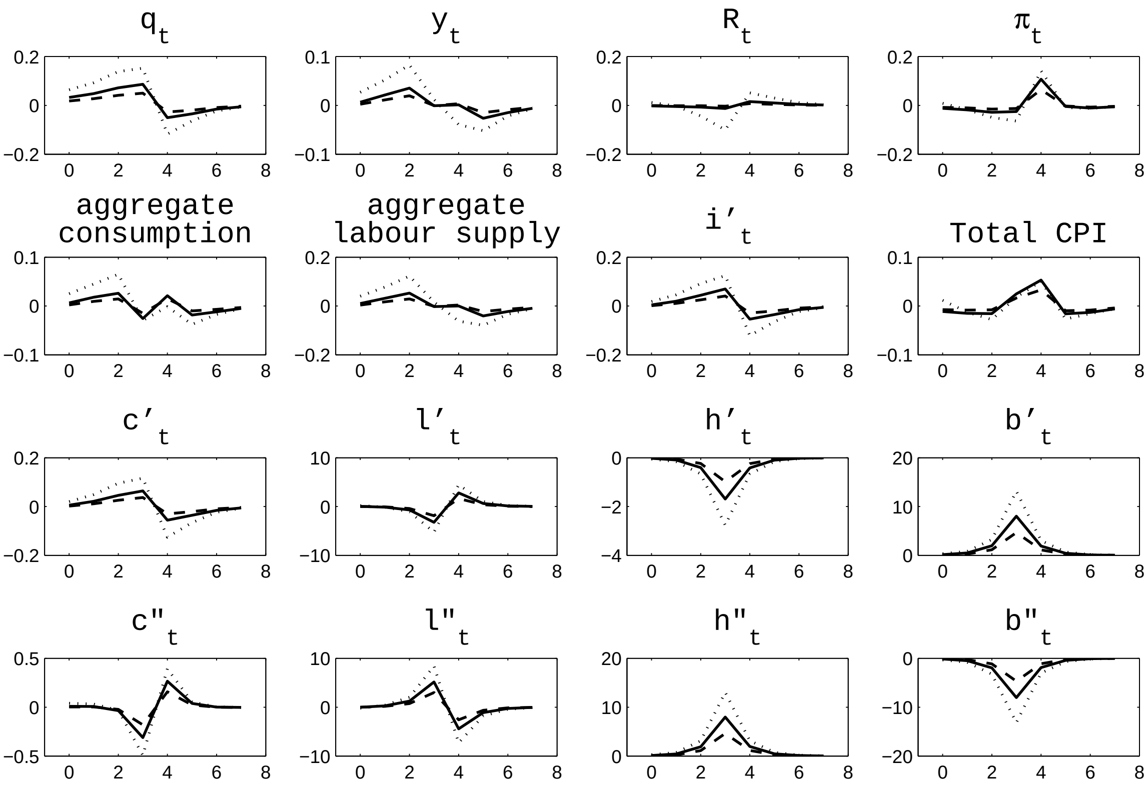
Notes: The solid line: $\xi' = \xi'' = 1.5$. The dashed line: $\xi' = \xi'' = 3$. Figures are % deviations from the deterministic steady-state values. The signal is received in period 0, but is not realized in period 4, i.e., $z_{A,0} = 1$ and $\epsilon_{A,4} = 0$. The economy is at the steady state before period 0. Both types of households consider the signal accurate, i.e. $\nu'_A = \nu''_A = 0$. Note that R_t denotes the real interest rate, as $\pi_t = 1$ in flexible price cases. The third and the forth rows show the actions of savers and home-buyers, respectively.

Figure 11: Sticky prices: Impulse response to over-optimism of home-buyers when savers do not share the optimism and their labour supply is inelastic



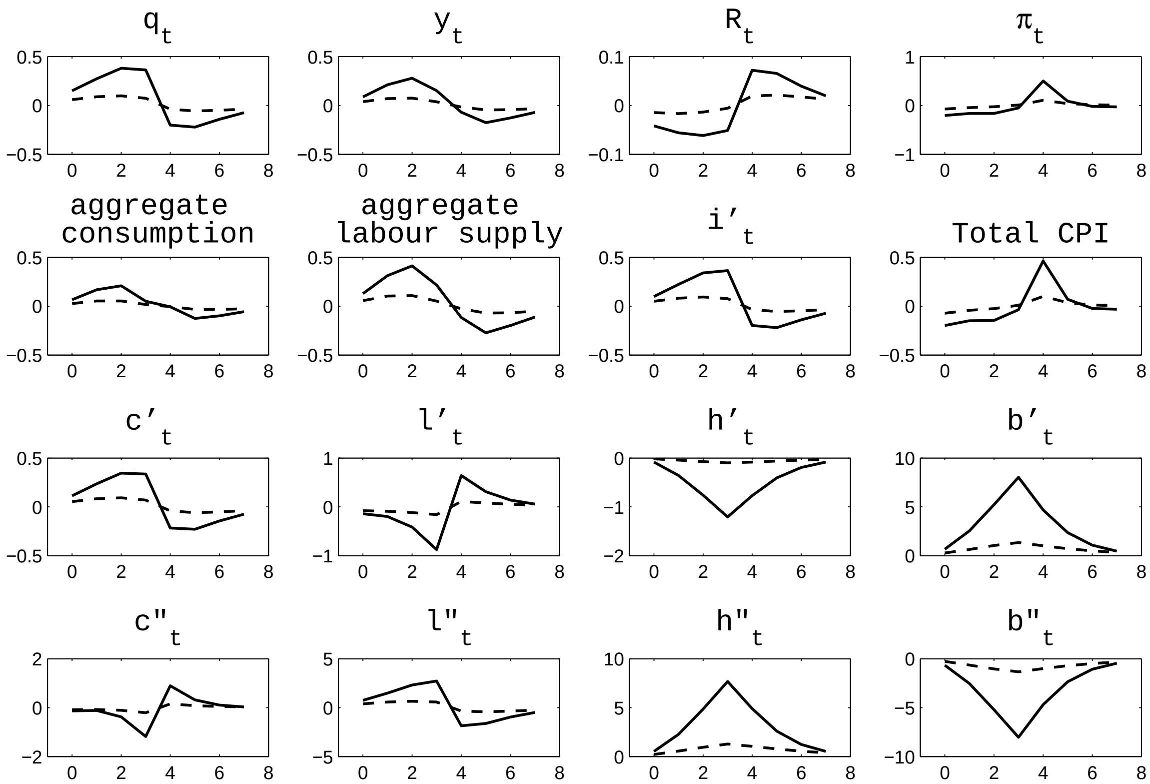
Notes: The solid line: $\phi_\pi = 0.245$ (benchmark value). The dashed line: $\phi_\pi = 0.15$. The dotted line: $\phi_\pi = 1$. For all the lines, $\xi' = 2.7$ (benchmark value). Figures are % deviations from the deterministic steady-state values. The signal is received in period 0, but is not realized in period 4, i.e., $z_{A,0} = 1$ and $\epsilon_{A,4} = 0$. The economy is at the steady state before period 0. Only home-buyers consider the signal accurate, i.e. $\nu'_A = \infty$ and $\nu''_A = 0$.

Figure 12: Sticky prices: Impulse response to over-optimism of home-buyers when savers do not share the optimism and their labour supply is elastic



Notes: The solid line: $\phi_\pi = 0.245$ (benchmark value). The dashed line: $\phi_\pi = 0.15$. The dotted line: $\phi_\pi = 1$. For all the lines, $\xi' = 1.1$. Figures are % deviations from the deterministic steady-state values. The signal is received in period 0, but is not realized in period 4, i.e., $z_{A,0} = 1$ and $\epsilon_{A,4} = 0$. The economy is at the steady state before period 0. Only home-buyers consider the signal accurate, i.e. $\nu'_A = \infty$ and $\nu''_A = 0$.

Figure 13: Sticky prices: Effect of higher availability of mortgage debt on housing-market boom-bust cycles



Notes: The solid line: $m = 0.88$ (benchmark value). The dashed line: $m = 0.5$. For all the lines, $\xi' = 2.7$ (benchmark value). Figures are % deviations from the deterministic steady-state values. The signal is received in period 0, but is not realized in period 4, i.e., $z_{A,0} = 1$ and $\epsilon_{A,4} = 0$. The economy is at the steady state before period 0. Only home-buyers consider the signal accurate, i.e. $\nu'_A = \infty$ and $\nu''_A = 0$.

Appendices

A Data appendix

A.1 Figures 1-4

Ahearne, et al. (2005) use real house prices for determining the peaks of housing booms. See the data appendix of their paper for more details on identification of the peaks of housing booms during past boom-bust cycles.

Real GDP growth rates come from the OECD database. The series are based on seasonally adjusted real GDP. The rates are annualized quarterly growth rates. For Germany, unified German data is used whenever possible. If not available, West German data is used. This is the same for the following data.

Total CPI inflation rates come from the OECD database. These series give year-to-quarter changes in the price index as a percentage.

Short-term nominal interest rate are 3-month nominal money-market rates taken from the OECD database. These rates are normally highly correlated with the target rate adopted by the central bank. If short-term interest rates are not available from the OECD database, this paper uses the money market rates from the IMF database. For a given peak, this paper always uses a single source per country. This paper does this by counting the number of observations available from each source for that country around that peak and choosing the source to maximize this count. All short-term rates are expressed in percentage points on an annualized basis.

Hours-worked are calculated as the product of two series from the OECD database: the total number of workers in the economy and the hours-worked per employee. The growth rate is expressed in percentage points on a year-to-quarter basis. Belgium, Denmark, Spain, and Switzerland do not have data available at quarterly frequency. For these countries, this paper computes year-over-year growth rates, then interpolate quarterly values by estimating a cubic spline, using the year-over-year results as year-end values.

The numbers of employed people for different age cohorts are from the OECD database. The data have a few discontinuous country-year points due to changes in data coverage. These discontinuous points are excluded from the sample.

A.2 Table 2 and Figure 5

The real house price growth rate is the first-order log difference of the nationwide house price index from the Federal Housing Finance Agency divided by the GDP deflator. The Index of Consumer Expectations for young households are the average of the index for 18-34 years old (ice_a1834) and that for 35-44 years old (ice_a3544) in the Reuters/University of Michigan Surveys of Consumers weighted by the numbers of householders in corresponding age groups from Table HH-3, the March CPS, U.S. Census Bureau. The number of householders in each age group in Table HH-3 is annual. This series is converted into a quarterly series by linear interpolation. The Index of Consumer Expectations for old households are similarly constructed from ice_a4554, ice_a4564 and ice_a6597. "Difference between young and old" in the figure is the difference of the index for young households from the index for old households. Real GDP is available from the Bureau of Economic Analysis. The ex-post real interest rate is the 90-day treasury bill rate minus the rate of change in the GDP deflator between the current and the next quarters.

A.3 The standard deviations of hours-worked of young and old workers and the demand components of GDP in the U.S.

Average hours-worked for young and old workers are taken from the March CPS, U.S. Census Bureau. Annual data series are available from `cps.ipums.org/cps` (Miriam King, Steven Ruggles, Trent Alexander, Donna Leicach, and Matthew Sobek. Integrated Public Use Microdata Series, Current Population Survey: Version 2.0. [Machine-readable database]. Minneapolis, MN: Minnesota Population Center [producer and distributor], 2004). Quarterly data series are constructed by linear interpolation. Hours worked are log-linearly detrended when calculating standard deviations. The sample period is for 1980:1-2006:4.

Quarterly GDP and its demand components are available from the Bureau of Economic Analysis. The real values of GDP, private consumption, private investment are log-linearly detrended when calculating standard deviations and lag-1 autocorrelation. The sample periods are for 1980:1-2008:2. See Table 3 for comparison of the moments of aggregate variables between the calibrated model and the data.

B The numerical solution method

Denote the two types of households in question by type 1 and type 2. It is possible to write the log-linearized equilibrium system in the following form:

$$
\begin{aligned}
&A\left[E\zeta_{t+1}',\ E_1\epsilon_{A,t+1},\ E_2\epsilon_{A,t+1},\ k_{t+1}',\ E_1x_{1,t+1}',\ E_2x_{2,t+1}',\ E_1p_{t+1}',\ E_2p_{t+1}'\right]' \\
&\qquad\qquad\qquad\qquad\qquad\qquad = B\left[\zeta_t',\ \epsilon_{A,t},\ k_t',\ x_{1,t}',\ x_{2,t}',\ p_t'\right]',
\end{aligned}
$$

where A and B are constant matrices; E_1 and E_2 are subjective expectation operators for the two types of households; $\zeta_t = [z_t, ..., z_{t-\tau+1}]'$, where z_t is the public signal of $\epsilon_{A,t+4}$; k_{t+1} is the vector of the endogenous state variables determined at period t, including labour augmenting technology A_t; $x_{1,t}$ and $x_{2,t}$ are the vectors of the choice variables for the two types of households, respectively; and p_t is the vector of the aggregate variables and the market prices. The expectation operator to ζ_{t+1} is E, which is the common expectation operators between the two types of households, as there is no disagreement on the likelihood of future signals. The matrices A and B can be written as

$$
A = \begin{bmatrix} A_z & A_{1,\epsilon} & A_{2,\epsilon} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & A_k & A_{1,x} & A_{2,x} & A_{1,p} & A_{2,p} \end{bmatrix}
$$

$$
B = \begin{bmatrix} B_z & 0 & 0 & 0 & 0 & 0 \\ 0 & B_\epsilon & B_k & B_{1,x} & B_{2,x} & B_p \end{bmatrix}.
$$

The first blocks of rows of A and B define the subjective expectations of technological shocks and the law of motion of public signals. The second blocks of rows correspond to the law of motion of labour augmenting technology, the first-order conditions of the maximization problems, market clearing conditions, and the definitions of endogenous variables. Now conjecture that the solution takes the following form:

$$\begin{bmatrix} x_{1,t} \\ x_{2,t} \\ p_t \\ k_{t+1} \end{bmatrix} = \begin{bmatrix} Q_1 & R_1 & S_1 \\ Q_2 & R_2 & S_2 \\ G & H & J \\ U & V & W \end{bmatrix} \begin{bmatrix} k_t \\ \epsilon_{A,t} \\ \zeta_t \end{bmatrix}.$$

Substituting the conjectured solution form into the equilibrium system above, it is possible to show that Q_i and R_i for $i = 1, 2$ and G, H, U and V are identical to those in the case without public signals. Thus, the standard solution method described by Klein (2000) can find these matrices. S_1, S_2, J and W are derived as:

$$[W'_\tau, M'_\tau]' = -\Phi^{-1} \sum_{i=1}^{2} (A_{i,x} R_i + A_{i,p} H) \psi_i$$

$$\left[W'_j, M'_j\right]' = -\Phi^{-1} \Theta \left[M'_{j+1}\right]' \quad \text{for } j = 1, ..., \tau - 1,$$

where

$$W \equiv [W_1, \; ... \;, W_\tau], \quad M \equiv [S'_1, S'_2, J']' \equiv [M_1, \; ..., \; M_\tau]$$

$$\Phi \equiv \left[A_k + \sum_{i=1}^{2} (A_{i,x} Q_i + A_{i,p} G), \; -B_{1,x}, \; -B_{2,x}, \; -B_p \right]$$

$$\Theta \equiv \left[A_{1,x}, \; A_{2,x}, \; \sum_{i=1}^{2} A_{i,p} \right]$$

$$\psi_i \equiv \frac{E_i \left[\epsilon_{A,t+1} | z_{t-\tau+1}\right]}{z_{t-\tau+1}} \quad \text{for } i = 1, 2,$$

given that Φ is invertible.

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