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The Endogenous Relative Price of Investment

by Joel Wagner

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Abstract

This paper takes a full-information model-based approach to evaluate the link between investment-specific technology and the inverse of the relative price of investment. The two-sector model presented includes monopolistic competition where firms can vary the markup charged on their product depending on the number of firms competing. With these changes to the standard two-sector model, both total factor productivity as well as a series of non-technological shocks can impact the high-frequency volatility of the relative price of investment. Utilizing a Bayesian estimation approach to match the model to the data, we find that investment-specific technology can explain at most half of the growth rate of the relative price of investment. Last of all, we compare the benchmark model results with endogenous movement in the relative price of investment to a model where all movement in the relative price of investment is derived exogenously. This is done by allowing technologies across sectors to move together over time. Comparison of these two methods finds that the exogenous approach is incapable of capturing changes in the relative price of investment as found in the data. This paper adds to the growing list of research, like that of Fisher (2009) and Basu et al. (2013), that suggests that the quality-adjusted relative price of investment may be a poor indicator of investment-specific technology.

JEL classification: E32, L11, L16
Bank classification: Business fluctuations and cycles

Résumé

L'auteur utilise une méthode d'analyse basée sur un modèle structurel et des données complètes pour évaluer le lien entre les chocs technologiques spécifiques à l'investissement et l'inverse du prix relatif de l'investissement. Il propose un modèle bisectoriel intégrant une structure de concurrence monopolistique où les entreprises peuvent faire varier le taux de marge appliqué à leur produit en fonction du nombre de firmes concurrentes. Compte tenu de cette adaptation du modèle bisectoriel standard, aussi bien la productivité totale des facteurs qu'une série de chocs non technologiques peuvent avoir une incidence sur la volatilité à haute fréquence du prix relatif de l'investissement. L'auteur emploie une méthode d'estimation bayésienne pour faire concorder le modèle et les données, et il constate que les chocs technologiques spécifiques à l'investissement peuvent expliquer tout au plus la moitié du taux de croissance du prix relatif de l'investissement. Enfin, il compare les résultats obtenus à l'aide du modèle de référence dans lequel les variations du prix relatif de l'investissement sont endogènes aux résultats d'un modèle où tout mouvement du prix relatif de l'investissement est déterminé de façon exogène. À cette fin, les technologies suivent une évolution parallèle au fil du temps tous secteurs confondus. La comparaison de ces deux méthodes permet de constater que l'approche exogène ne peut pas rendre compte des changements du prix relatif de l'investissement qui ressortent des données.

Cette étude enrichit le corpus de recherches qui, comme celles de Fisher (2009) et de Basu et coll. (2013), laissent entrevoir que le prix relatif de l'investissement corrigé des variations de qualité est peut-être un mauvais indicateur des chocs technologiques spécifiques à l'investissement.

Classification JEL : E32, L11, L16
Classification de la Banque : Cycles et fluctuations économiques

Non-Technical Summary

Motivation and Question

What drives the business cycle? In the literature, two types of technology shocks have been identified as potential sources of business cycle volatility. These include neutral technology shocks (which improve productivity economy-wide) and investment-specific technology shocks (such as innovations in communication technology). Traditionally, investment-specific technology (IST) has been identified by the inverse of the relative price of investment (RPI). This identification scheme assumes, however, that the RPI is not influenced by shifts in investment demand. This paper explores the validity of this assumption. That is, does the RPI vary with investment demand? Given their relevance in explaining business cycle volatility, this subject warrants investigation.

Methodology

This paper proposes a two-sector model adapted to allow for monopolistic competition in the production of both consumption and investment sector intermediate goods. It is assumed that the number of firms operating within each sector is finite, thus allowing firms to vary their markup depending on the number of existing competitors. The model is then perturbed by both stationary and non-stationary neutral and investment-specific technology shocks as well as three non-technological shocks. This model is compared to an alternative model where business cycle movement in the RPI is modeled exogenously by allowing both neutral and investment-specific technology to follow a common stochastic trend. Bayesian estimation is used for the model's parameterization, followed by a decomposition to assess each shock's contribution to the volatility of the RPI. These two models are then assessed by the ability of neutral technology as well as shifts in investment demand (relative to consumption demand) to generate volatility in the growth rate of the RPI.

Key Contributions

This paper finds that over half of the volatility of the RPI can be attributed to shifts in investment demand, with the remainder due to shifts in IST. This result is in sharp contrast to the longstanding assumption in the macroeconomic literature that movement in the RPI is purely a technological phenomenon. This paper adds to the current business cycle literature by demonstrating that IST shocks are not only incapable of generating realistic business cycles (as is the current assumption in the literature), but are incapable of generating volatility in the RPI itself. These results suggest that calibrating IST shocks to the inverse of the RPI overestimates the relative importance of IST. This paper also validates the current assumption in the literature that changes in the marginal efficiency of investment (such as changes in the firm's ability to access credit) do not impact the RPI.

Future Research

This paper addresses the co-movement of total factor productivity and IST endogenously over the short run. It is, however, incapable of reproducing the common trend component between these two technologies observed in the data in the long run. Given the potential relevance of both neutral and investment-specific technology in generating growth in the Canadian economy, future research should be done to uncover the source of this relationship.

1 Introduction

Since Greenwood, Hercowitz, and Huffman (GHH) (1988) first identified investment-specific technology (IST) as a potential source of business cycle volatility, this type of shock has become a common feature in the business cycle literature. Likewise, identification of IST has remained roughly in line with the method used by Greenwood, Hercowitz, and Krusell (GHK) (1997), who built upon the work done by GHH (1988). Since their seminal work, the business cycle literature has shifted over time in its assessment of the relative importance of IST. At first, research such as that by Fisher (2006) as well as Justiniano, Primiceri, and Tambalotti (2010) found IST to be an important source of both low-frequency and high-frequency volatility. Each time, the relative importance of IST is assessed by either analyzing the variance decomposition or by growth accounting (as done by Fisher (2006)). Recent research, such as that of Justiniano et al. (2011) and Schmitt-Grohé and Uribe (2011), has, however, found that IST, when correctly adapted to reflect movement in the relative price of investment (RPI), lacks the ability to generate any business cycle volatility.

Beaudry and Lucke (2009) take an alternative approach. In their research, rather than analyzing a dynamic stochastic general-equilibrium model's variance decomposition, they quantitatively assess the relative importance of IST against a menu of alternative shocks using an approach based on a cointegrated structural vector autoregression (SVAR). They conclude that expected changes in neutral technology, as well as preference and monetary shocks, play a far more significant role in explaining high-frequency movements in the data in their forecast-error variance decomposition than IST. All of the aforementioned research relies heavily on the assumption that IST can be uniquely identified by the inverse of the RPI. Using micro-level data, Basu et al. (2013) show that the RPI responds slowly to changes in IST, often taking up to three quarters for the effect of an IST shock to impact the RPI. This could be due to either sticky investment prices, or, alternatively, investment prices that are driven by forces other than IST. Through an SVAR-based approach, Kim (2009) finds that IST shocks could at most explain 27 percent of the RPI from 1955Q1 to 2000Q4. The assumption that IST is an independent stochastic process implies that the RPI is orthogonal to any other type of economic disturbance, such as neutral technology shocks, wage shocks or preference shocks, which are commonly included in the literature. Therefore, the adequacy of the RPI to correctly indicate movements in IST could, for example, be assessed by the independence of the RPI from any one of these disturbances. If the inverse of the RPI, as GHK (1997) suggested, is a good indication of IST, then these technology shocks should in theory be unrelated to neutral technology as measured by total factor productivity (TFP).

As can be seen in Figure 1, upturns in log TFP (*tfp*) are typically followed by a decrease in the log RPI (*rpi*). The *tfp* plotted in Figure 1 is calculated as in Beaudry and Lucke (2009) (the log of non-farm output less the log of both non-farm hours and capital services, each scaled by its share of output).[1] As for the *rpi*, we use the quality-adjusted *rpi* time

[1] Data on the real non-farm gross value-added output are calculated by the Bureau of Economic Analysis 1947Q1 to 2013Q4. Non-farm hours worked are calculated by the Bureau of Labor Statistics (BLS) 1947Q1 to 2013Q4. Capital services time series are calculated from the (BLS) private sector non-farm business sector

series as calculated by Fisher (2006). This data series adjusts the relative price of equipment estimated by using the Gordon-Cummins-Violante equipment price deflator and divides it by the quarterly price deflator for consumption goods found in the U.S. national income and product accounts (NIPA) tables. With these two time series, Fisher (2006) obtains a quarterly measure of the *rpi* adjusted for changes in quality. Information on the data used is available in the appendix.

With a correlation between detrended *tfp* and *rpi* of approximately -0.216, it would appear that the *rpi* moves countercyclically to *tfp*. This fact has been addressed in countless papers, such as that of Letendre and Luo (2007), who adapt the standard AR(1) set-up to allow for spillovers between *tfp* and *rpi* in order to replicate the countercyclical nature of the *rpi*. Thus, it appears that in the short run, the theory suggested by GHK (1997) that relative prices can be used to determine changes in relative technologies across sectors is less than robust.

Schmitt-Grohé and Uribe (2011) have furthered the disconnect between IST and the *rpi* by also demonstrating that *tfp* and the *rpi* are cointegrated in the long run. With both *tfp* and the *rpi* integrated of order 1 stationary in the United States, they apply a Johansen's test for cointegration in which they show that in addition to *tfp* and *rpi* being non-stationary, they are also cointegrated.[2] With both *tfp* and the *rpi* cointegrated, there exists a cointegration coefficient β such that the difference in levels between each time series remains I(0) stationary in the long run. To highlight this fact, Figure 2 plots *tfp* along with the inverse of the *rpi* adjusted by the cointegration coefficient $\beta = 0.623$. Figure 2 demonstrates that these two time series follow a common stochastic trend. Given the assumption made by GHK (1997) that relative technologies across sectors are reflected in the relative price, it would be expected that these two time series would not follow a common stochastic trend, as both cointegration tests, as well as Figure 2, appear to suggest. As is shown in section 4, when the standard business cycle model is adapted to replicate the co-movement of *tfp* and the *rpi*, 31 percent of *rpi* growth from one period to the next can be explained by shifts in neutral technology.

Given the aforementioned relationships between *tfp* and the *rpi*, both in the long run and over the business cycle, the assumption that the *rpi* is orthogonal to any form of economic disturbance can be safely rejected. Further tests could be done to assess the orthogonality of the *rpi* with any other form of economic disturbances, such as wage markup, preference shocks or shifts in the marginal efficiency of investment.

Is the cyclical movement in the *rpi* a technological phenomenon, or is movement in the *rpi* due to changes in the relative demand for investment goods over consumption goods? In response to this question, this paper proposes a two-sector model adapted to incorporate endogenous markup variation within each sector. Endogenous price markups are incorpo-

(NAICS 113-81) 2009 index 1987-2012.

[2]Schmitt-Grohé and Uribe (2011) apply both an Augmented Dickey Fuller Test (ADF) as well as a Kwiatkowski, Phillips, Schmidt, and Shin (KPSS) test to determine the stationarity of both *tfp* and the *rpi*. As can be seen in Tables 1 and 2 of Schmitt-Grohé and Uribe (2011), both of these tests conclude that these two series are non-stationary.

rated by assuming that each sector (consumption and investment) is populated by a finite number of firms, each selling a differentiable good. Each of these firms is capable of not only influencing its own price, but also the price charged across all firms. An alternative model is presented in section 4, where movement in the *rpi* is generated entirely by technological spillovers. As section 4 demonstrates, when the assumption of orthogonality between technologies is relaxed, approximately 31 percent of the *rpi* can be attributed to shifts in *tfp*. In contrast, when the *rpi* moves in response to changes in demand, as is the case in the benchmark model, the explanatory power of IST drops further to 45 percent. Stationary and non-stationary *tfp* shocks explain approximately 29 percent of the volatility of the *rpi*. Non-technological shocks contribute 26 percent. With the vast majority of business cycle research assuming that the *rpi* is determined exogenously, the results of this paper are particularly poignant.

These two approaches are compared to Kim's (2009) research to assess whether one approach outperforms the other. As outlined in section 4, our results indicate that an endogenous approach to modeling movement in the *rpi* outperforms the exogenous-based approach, due to the impact non-technological shocks have on the relative demand for investment goods over consumption goods. Through the endogenous-based approach, 45 percent of the volatility of the *rpi* is explained by unanticipated changes in IST. These results bring into question the current convention in linking IST to the inverse of the *rpi*.

The remainder of the paper is organized as follows. Section 2 outlines the benchmark model, consisting of the varying elements that allow the *rpi* to move endogenously over time. Section 3 outlines the Bayesian estimation process, which is used to estimate the parameter values. Section 4 outlines the results of the benchmark model with variations of this model, to assess the relative importance of each aspect in generating these results. Section 5 concludes.

2 Benchmark Model

The benchmark model for this paper involves a two-sector real business cycle model with monopolistic competition in both consumption- and investment-goods-producing sectors. This model is set up in such a way that firms are able to vary the markup charged above production costs depending on the number of competing firms within that industry. We begin with an outline of the various stages of production in the consumption sector.

Production of each good can be divided into three stages of production. These stages include a finite number of monopolistically competitive firms that produce their product using both capital and labour inputs. These goods are then aggregated at an industry level by firms that assemble them into a composite good to be sold at the sector level. Lastly, there is a perfectly competitive firm that purchases these industry-level goods and assembles them into a composite good ready to sell to consumers. For ease of illustration, we begin our dissection of the various stages of consumption production at the sector level.

2.1 Consumption Sector

2.1.1 Sector Level

At the aggregate level, the consumption good produced in this economy C_t is a composite good consisting of a continuum of unit measure one industry-level goods produced using the following constant-returns-to-scale production function:

$$C_t = \left[\int_0^1 Q_t^c(j)^{\omega} \mathrm{d}j \right]^{\frac{1}{\omega}}, \tag{1}$$

where $Q_t^c(j)$ refers to the quantity of output produced in industry j, with the elasticity of substitution between industry-level goods equal to $\frac{1}{1-\omega}$. The total profit earned by assembling these industry-level goods at the sector-level Π_t^c is equal to

$$\Pi_t^c = \left\{ P_t^c C_t - \int_0^1 P_t^c(j) Q_t^c(j) \mathrm{d}j \right\}, \tag{2}$$

where P_t^C is the price of the sector-level consumption good and $P_t^c(j)$ is the price paid for industry j's composite good. Solving the production problem for the sector-level consumption-goods producer implies a conditional input demand of

$$Q_t^c(j) = \left(\frac{P_t^c(j)}{P_t^c} \right)^{\frac{1}{\omega-1}} C_t \tag{3}$$

of industry j's good by the sector-level producer, where the price index P_t^c is equal to

$$P_t^c = \left[\int_0^1 P_t^C(j)^{\frac{\omega}{\omega-1}} \mathrm{d}j \right]^{\frac{\omega-1}{\omega}}. \tag{4}$$

2.1.2 Industry Level

The industry-level consumption good is produced using a constant-returns-to-scale production function which aggregates output produced by a finite number of firms within industry j. Firm i within industry j produces a differentiable good $x_t^c(j,i)$. This good is used as an input at the industry level through the following production function:

$$Q_t^c(j) = \big[N_t^c(j)\big]^{1-\frac{1}{\tau}} \Bigg[\sum_{i=1}^{N_t^c} x_t^c(j,i)^{\tau}\Bigg]^{\frac{1}{\tau}}. \tag{5}$$

$N_t^c(j)$ denotes the number of firms competing in industry j and $1/(1-\tau)$ is the elasticity of substitution between industry-level goods. Given this production function, the profit function for the firm producing the industry j good $\Pi_t^c(j)$ is determined as

$$\Pi_t^c(j) = \left\{ P_t^c(j) Q_t^c(j) - \sum_{i=1}^{N_t^c(j)} x_t^c(j,i) p_t^c(j,i) \right\}, \tag{6}$$

where $P_t^C(j,i)$ denotes the price of firm i's output in industry j. This profit function implies a conditional demand

$$x_t^c(j,i) = \Big(\frac{P_t^c(j,i)}{P_t^c(j)}\Big)^{\frac{1}{\tau-1}} \frac{Q_t^c(j)}{N_t^c} \tag{7}$$

by industry j for firm i's product. Analogous to the sector level of production, the industry j consumption-good price index is equal to

$$P_t^c(j) = N_t^c(j)^{\frac{1}{\tau}-1} \Bigg[\sum_{i=1}^{N_t^c(j)} P_t^c(j,i)^{\frac{\tau}{\tau-1}}\Bigg]^{\frac{\tau-1}{\tau}}. \tag{8}$$

2.1.3 Firm Level

The last stage of production consists of a finite number of monopolistically competitive firms within each industry. These firms produce a good using both capital and labour as inputs. We assume that these firms can costlessly differentiate their product, and thus, given a finite number of firms competing, have the ability to not only influence its own price $P_t^c(j,i)$, but also the industry-level price $P_t^c(j)$. While firm i in industry j has the ability to influence $P_t^c(j,i)$ as well as $P_t^c(j)$, it does not have the ability to influence the sector-level price P_t^c. In industry j, firm i's good is produced using the following constant-returns-to-scale production function:

$$x_t^c(j,i) = Z_t\big(k_t^c(j,i)\big)^{\alpha}\big(X_t^z h_t^c(j,i)\big)^{1-\alpha} - \phi^c \quad where \quad \phi^c > 0, 0 < \alpha < 1, \tag{9}$$

where $k_t^c(j,i)$ and $h_t^c(j,i)$ denote the capital and labour used by firm i in industry j, respectively, α is the capital share of output, and ϕ^c denotes the fixed cost of production. We assume that there are two types of technology shocks affecting production of consumption goods. These include a stationary technology shock, Z_t, and a non-stationary labour-augmenting technology, X_t^z, where the stochastic growth rate of X_t^Z is given by

$$\mu_t^z \equiv \frac{X_t^z}{X_{t-1}^z}. \tag{10}$$

TFP in the consumption sector is

$$TFP_t = Z_t \big(X_t^z\big)^{1-\alpha}. \tag{11}$$

Given the conditional input demand for industry-level consumption goods by the sector-level firm (equation (3)) and industry j's conditional input demand by industry j for firm i's consumption good (equation (7)), we can write the conditional demand for firm i's good as

$$x_t^c(j,i) = \left[\frac{P_t^c(j,i)}{P_t^c(j)}\right]^{\frac{1}{\tau-1}} \left[\frac{P_t^c(j)}{P_t^c}\right]^{\frac{1}{\omega-1}} \frac{C_t}{N_t^c(j)}. \tag{12}$$

Thus, firm i maximizes profits by choosing its capital and labour demand as well as a price $P_t^c(j,i)$ that maximizes profit:

$$\Pi_t^c(j,i) = \left\{P_t^c(j,i)x_t^c(j,i) - w_t^c h_t^c(j,i) - r_t^c k_t^c(j,i)\right\}, \tag{13}$$

subject to its production function (9).

Solving the firm-level problem, we get

$$P_t^c(j,i) = \mu_t^c(N_t^c(j))MC_t^c(j,i) = \frac{(1-\omega)N_t^c(j) - (\tau-\omega)}{\tau(1-\omega)N_t^c(j) - (\tau-\omega)} MC_t^c(j,i), \tag{14}$$

where $MC_t^c(j,i)$ is the marginal cost of production by firm i in sector j and $\mu_t^c(N_t^c(j))$ is the markup charged by this firm. The firm's optimal labour demand implies a wage rate in the consumption sector,

$$w_t^c = \frac{P_t^c(j,i)}{\mu_t^c(j,i)} \alpha Z_t^c \left(\frac{k_t^c(j,i)}{h_t^c(j,i)}\right)^{\alpha} X_t^{z\,1-\alpha}, \tag{15}$$

and a rental rate

$$r_t^c = \frac{P_t^c(j,i)}{\mu_t^c(j,i)}(1-\alpha)Z_t^c\left(\frac{k_t^c(j,i)}{h_t^c(j,i)}\right)^{\alpha-1}X_t^{z1-\alpha}. \tag{16}$$

The markup charged over production costs by this firm is determined by both the number of firms competing in their industry and the substitutability of its goods within and across industries.

It is assumed that firm-level technology is identical both within and across industries in the consumption sector. This assumption implies that for every firm $i \in [0, N_t^c(j)]$ and for every industry $j \in [0,1]$, firms make identical decisions when choosing both labour and capital services $(h_t^c(j,i) = h_t^c, k_t^c(j,i) = k_t^c)$. This implies that the quantity of goods produced by each firm will also be the same across all firms $(x_t^c(j,i) = x_t^c)$. Furthermore, with this assumption we can generalize the price charged by firms along with the price index at both an industrial level (equation (8)) and at a sector level (equation (4)), implying that $P_t^c(j,i) = P_t^c(j) = P_t^c$.

As mentioned earlier, the firm incurs a fixed cost of production ϕ_t^c, which we set according to the following zero-profit condition:

$$\phi_t^c = x_t^c(\mu_t^c - 1), \tag{17}$$

along a balanced growth path (BGP). Given N_t^c firms in each industry, we can calculate the quantity of consumption goods produced C_t as

$$C_t = Q_t^c = N_t^c x_t^c = \frac{Z_t}{\mu_t^c}(k_t^c)^\alpha (X_t^z h_t^c)^{1-\alpha}. \tag{18}$$

With this equation along with the zero-profit condition outlined in equation (17), we can calculate the total number of firms operating within each industry as

$$N_t^c = \frac{\mu_t^c - 1}{\mu_t^c \phi^c} Z_t^c (k_t^c)^\alpha (X_t^Z h_t^c)^{1-\alpha}. \tag{19}$$

2.2 Investment Sector

Thus far we have outlined the various stages of production in the consumption-goods sector. The investment sector shares a similar structure to the consumption-goods sector, having a finite number of monopolistically competitive firms selling their differentiable products to a continuum of unit measure one industry-level firms, who in turn sell these goods to the sector-level producer. Similar to the consumption sector, we begin our description of the investment-goods sector at the sectoral level.

2.2.1 Sector Level

Sector-level investment goods are produced by amalgamating a continuum of industry-level investment goods according to the following constant-returns-to-scale production function:

$$I_t = \left[\int_0^1 Q_t^I(j)^{\omega} \mathrm{d}j \right]^{\frac{1}{\omega}}. \tag{20}$$

As was the case in the consumption sector, the final good produced in the investment sector I_t is a composite good consisting of a continuum of industry-level investment goods $Q_t^I(j)$ of unit measure one. The profit function for the investment-good producer at the sector level is

$$\Pi_t^I = \left\{ P_t^I I_t - \int_0^1 P_t^I(j) Q_t^I(j) \mathrm{d}j \right\}, \tag{21}$$

where $P_t^I(j)$ is the price of industry j's investment good and $Q_t^I(j)$ denotes the quantity of investment goods produced in industry j. As was the case in the consumption sector, industry-level investment goods are not perfect substitutes but rather have an elasticity of substitution determined by $1/(1-\omega)$.

2.2.2 Industry Level

At the industry level, the investment-goods sector is symmetric in construction to the consumption-goods sector at the same level of production. The industry-level composite good is produced using the following constant-returns-to-scale production function:

$$Q_t^I(j) = \left(N_t^I(j)\right)^{1-\frac{1}{\tau}} \left[\sum_{i=1}^{N_t^I(j)} x_t^I(j,i)^{\tau} \right]^{\frac{1}{\tau}}. \tag{22}$$

The conditional input demand for the firm-level good $x_t^I(j,i)$ by industry j is then calculated as

$$x_t^I(j,i) = \left(\frac{P_t^I(j,i)}{P_t^I(j)} \right)^{\frac{1}{\tau-1}} \frac{Q_t^I(j)}{N_t^I}, \tag{23}$$

with the price index $P_t^I(j)$ in industry j equal to

$$P_t^I(j) = N_t^I(j)^{\frac{1}{\tau}-1}\left[\sum_{i=1}^{N_t^I(j)} P_t^I(j,i)^{\frac{\tau}{\tau-1}}\right]^{\frac{\tau-1}{\tau}}. \tag{24}$$

2.3 Firm Level

The firm-level investment good, $x_t^I(j,i)$, is produced using the following production function:

$$x_t^I(j,i) = Z_t A_t k_t^I(j,i)^{\alpha}\big(X_t^Z X_t^A h_t^I(j,i)\big)^{1-\alpha} - \phi^I, \tag{25}$$

where $k_t^I(j,i)$ and $h_t^I(j,i)$ denote the capital and labour services used by firm i in industry j, ϕ^I is the fixed cost of production, and α denotes the capital share of output. As was the case in the consumption-goods sector, technology in the investment sector can be broken down into two separate components. There is a stationary IST shock A_t as well as the stationary *tfp* shock Z_t. There is also a non-stationary labour-augmenting technology $X_t^A(j)$ specific to the investment sector, along with the neutral technology $X_t^Z(j)$. The non-stationary IST is assumed to follow a stochastic growth rate, defined as

$$\mu_t^A \equiv \frac{X_t^A}{X_{t-1}^A}. \tag{26}$$

IST is measured as

$$IST_t = A_t\big(X_t^A\big)^{(1-\alpha)}. \tag{27}$$

With each firm i selling a differentiable good in industry j, firms compete on price, thus allowing investment firms to sell their product at a markup μ_t^I above their respective marginal cost $MC_t^I(j,i)$:

$$P_t^I(j,i) = \mu_t^I(N_t^I(j))MC_t^I(j,i) = \frac{(1-\omega)N_t^I - (\tau-\omega)}{\tau(1-\omega)N_t^I - (\tau-\omega)}MC_t^I(j,i). \tag{28}$$

As was the case in the consumption sector, with symmetric technologies across industries, we can drop all indexes. The fixed cost of production is set equal to

$$\phi_t^I = x_t^I(\mu^I - 1). \tag{29}$$

This is used to remove firm profits along a BGP. With this information, we can calculate the number of firms in the investment sector as

$$N_t^I = \frac{(\mu_t^I - 1)}{\mu_t^I \phi^I} Z_t A_t k_t^{I\alpha} \left(X_t^Z X_t^A h_t^I\right)^{1-\alpha}. \tag{30}$$

This implies a total output in the investment sector of

$$I_t = \frac{Z_t A_t}{\mu_t^I} k_t^{I\alpha} \left(X_t^Z X_t^A h_t^I\right)^{1-\alpha}. \tag{31}$$

The real wage and rental rates in the investment sector are

$$w_t^I = \frac{P^I}{\mu_t^I} \alpha Z_t A_t k_t^{I\alpha} h_t^{I-\alpha} X_t^Z X_t^{A(1-\alpha)} \tag{32}$$

$$r_t^I = \frac{P^I}{\mu_t^I} (\alpha - 1) Z_t A_t k_t^{I\alpha - 1} (X_t^Z X_t^A h_t^I)^{(1-\alpha)}. \tag{33}$$

With both labour and capital perfectly mobile between sectors, we have

$$w_t^I = w_t^C \quad and \quad r_t^I = r_t^C. \tag{34}$$

Dividing the wage rate in the investment sector by the wage rate in the consumption sector, we can estimate the *rpi* as

$$\frac{P_t^I}{P_t^c} = \frac{\mu_t^I}{\mu_t^C} \frac{1}{A_t} \left(\frac{k_t^C}{h_t^C}\right)^{\alpha} \left(\frac{k_t^I}{h_t^I}\right)^{-\alpha} X_t^{A(\alpha-1)}. \tag{35}$$

2.4 Households

The economy consists of a large number of identical and infinitely lived households who, by choosing consumption C_t and hours worked H_t, maximize their expected lifetime utility subject to their budget constraint, with a lifetime utility of

$$E_0 \sum_{t=0}^{\infty} \beta^t U(C_t, H_t), \tag{36}$$

where $0 < \beta < 1$ is the subjective discount factor. The households' periodic utility function is represented using Jaimovich and Rebelo preferences:

$$U(C_t, H_t) = \frac{b_t(C_t - \chi C_{t-1} - \Gamma H_t^{\Theta} X_t)^{1-\sigma} - 1}{1-\sigma} \tag{37}$$

$$X_t = (C_t - \chi C_{t-1})^{\eta} X_{t-1}^{1-\eta}, \tag{38}$$

where $\Gamma > 0$, $\Theta > 1$, $\sigma > 0, \chi > 0$, and $0 < \eta < 1$. Here Θ determines the level of labour supply elasticity and σ determines the curvature of household utility, χ is the habit persistence parameter, and η determines the effect wealth has on household labour supply decisions. The elements included in the periodic utility function that are distinctive to the style of preferences used by Jaimovich and Rebelo (2009) are the preference parameter η and the latent variable X_t. These preferences have become popular due to their ability to dial up or dial down the wealth effect on labour supply. When η is close to 1, we have King, Plosser, and Rebelo (1988) preferences (strong wealth effect). When η is closer to 0, we have GHH (1988) preferences, with a limited wealth effect on labour supply. Last of all, preference shocks b_t, which alter the households' intertemporal consumption and labour supply decisions are included in the benchmark model.

Households can accumulate capital according to the following capital accumulation equation:

$$K_{t+1} = (1-\delta)K_t + v_t I_t, \tag{39}$$

where K_t is the households' capital stock and I_t is the real quantity of investment goods purchased in period t. We also include a marginal efficiency of investment (MEI) v_t. These shocks have become popular in the literature since Justiniano et al. (2011) demonstrated that they are an important determinant of volatility in investment growth over the business cycle. The households' labour and capital services are used by both capital- and consumption-goods-producing firms. The household budget constraint is given by the following formula:

$$P_t^C C_t + P_t^I I_t = \frac{w_t H_t}{\mu^w} + r_t K_t^H + \Pi_t^C + \Pi_t^I + \Psi_t. \tag{40}$$

Given that wages earned in each sector are equal (labour supply is perfectly mobile), the household earns a labour income of $w_t H_t/\mu^w$ for hours worked in each sector, where H_t denotes the number of hours supplied by households in period t, w_t denotes the wages paid, and w_t/μ^w denotes the wages earned by the household adjusted by a wage markup shock. Here I assume that the portion of wages taken from the household through the wage markup shock are rebated back to the household via a lump sum transfer Ψ_t. Households also earn a rental income from capital services provided to both sectors $r_t K_t$. Since households own both consumption- and investment-goods-producing firms, any profits Π_t^C and Π_t^I are accrued to the household. Given the prices P_t^C and P_t^I for consumption and investment goods, respectively, households purchase C_t consumption goods and I_t investment goods, all measured in real terms.

With households as the only source of labour in this model, the market-clearing conditions in the labour markets imply that labour supply H_t equals the sum of labour demand in both sectors. With N_t^C firms operating within the consumption sector and N_t^I firms within the investment sector, this equilibrium condition implies that

$$H_t = N_t^C h_t^C + N^I h_t^I. \tag{41}$$

Normalizing the population of entrepreneurs to 1, the capital market clears when

$$K_t = N_t^C k_t^C + N_t^I k_t^I. \tag{42}$$

Last of all, with all firms within each sector identical, the total amount of consumption and investment goods produced is calculated as follows:

$$C_t = N_t^C x_t^C, \tag{43}$$

and

$$I_t = N_t^I x_t^I. \tag{44}$$

2.5 Exogenous Shocks

Altogether, we have seven types of shocks. There are technology shocks, which include both stationary and non-stationary *tfp* as well as stationary and non-stationary IST shocks. The non-technology shocks include wage markup, preference and MEI, each of which is assumed to be stationary. For stationary shocks Z_t and A_t, we assume the following AR(1) processes:

$$ln(Z_t) = \rho_Z ln(Z_{t-1}) + \epsilon_t^Z \tag{45}$$

$$ln(A_t) = \rho_A ln(A_{t-1}) + \epsilon_t^A, \tag{46}$$

where $0 < \rho_Z < 1$, $0 < \rho_A < 1$ refers to the level of persistence for each shock, while e_t^Z and ϵ_t^A are unanticipated shocks to $ln(Z_t)$, and $ln(A_t)$, respectively. The steady-state values of Z_t and A_t are normalized to one. Innovations ϵ_t^Z and ϵ_t^A, have an expected value of zero, with variance σ_t^Z and σ_t^A respectively. As for the non-stationary components for neutral and investment-specific technology, we assume that each technology experiences stochastic growth rates according to the following laws of motion:

$$ln(\mu_t^Z/\bar{\mu}^Z) = \rho_{\mu^Z} ln(\mu_{t-1}^Z/\bar{\mu}^Z) + \sigma^{\mu^Z}\epsilon_t^{\mu^Z} \tag{47}$$

$$ln(\mu_t^A/\bar{\mu}^Z) = \rho_{\mu^A} ln(\mu_{t-1}^A/\bar{\mu}^A) + \sigma^{\mu^A}\epsilon_t^{\mu^A}, \tag{48}$$

where the growth rates in TFP and IST are calculated as in equations (10) and (26), respectively. The persistence of each disturbance ρ_μ^z and ρ_μ^A is assumed to be between 0 and 1. The innovations in *tfp* growth $\epsilon_t^{\mu^z}$ and IST growth $\epsilon_t^{\mu^A}$ are unanticipated, with a standard deviation $\sigma_t^{\mu^Z}$ and $\sigma_t^{\mu^A}$, respectively. Lastly, $\bar{\mu^Z}$ and $\bar{\mu^A}$ denote the steady-state values of μ_t^Z and μ_t^A, which are discussed in the next section.

There are three stationary non-technological shocks – wage markup, preference and MEI shocks – which move according to the following laws of motion, respectively:

$$ln(\frac{\mu^w}{\bar{\mu}^w}) = \rho_{\mu^w} ln(\frac{\mu_{t-1}^w}{\bar{\mu}^w}) + \sigma^{\mu^w}\epsilon_t^{\mu^w} \tag{49}$$

$$ln(b_t) = \rho_b ln(b_{t-1}) + \sigma_b \epsilon_t^b \tag{50}$$

$$ln(v_t) = \rho_v ln(v_{t-1}) + \sigma_v \epsilon_t^v. \tag{51}$$

Each of the persistence parameters ρ_{μ^w}, ρ_b and ρ_v is between 0 and 1. Each innovation listed above is assumed to be i.i.d. with mean 0 and variance of 1 and with a standard deviation of σ_{μ^w}, σ_b and σ_v, respectively.

With both non-stationary neutral and investment-specific technology, each variable discussed thus far must be detrended wherever a trend is present. With the trend in neutral technology denoted by X_t^Z, the trend in output X_t^Y has the following form:

$$X^Y = X_t^Z \left(X_t^A\right)^\alpha, \tag{52}$$

where consumption, nominal investment, output and the fixed cost of production in the consumption sector all share this same trend. As for the trend of the capital stock X_t^k, the trend in the fixed cost of investment production and the trend of real investment X_t^I, we have

$$X^I = X_t^Z X_t^A. \tag{53}$$

We normalize the price of consumption goods P_t^C to 1. The trend in the *rpi* is equal to

$$X_t^{PI} = \left(X_t^A\right)^{\alpha-1)}. \tag{54}$$

There is no growth in hours, price markups or the number of firms within an industry. Letting $\mu^Y \equiv X_t^Y / X_{t-1}^Y$, and $\mu^K \equiv X_t^K / X_{t-1}^K$, the system of equations for the detrended model includes

$$\tilde{Y}_t = \tilde{C}_t + \tilde{P}_t^I \tilde{I}_t \tag{55}$$

$$\tilde{C}_t = \frac{Z_t}{\mu^c}\left(\frac{\tilde{K}_t^c}{\mu^K}\right)^{\alpha} H_t^{C\,1-\alpha} \tag{56}$$

$$\tilde{I}_t = \frac{Z_t A_t}{\mu^I}\left(\frac{\tilde{K}_t^I}{N_t^I \mu^K}\right)^{\alpha}\left(\frac{H^I}{N_t^I}\right)^{1-\alpha} \tag{57}$$

$$\tilde{K}_{t+1} = (1-\delta)\frac{\tilde{K}_t}{\mu^K} + v_t \tilde{I}_t \tag{58}$$

$$N_t^C = \left(\frac{\mu_t^C - 1}{\mu_t^C \tilde{\phi}^C}\right) Z_t \left(\frac{\tilde{K}_t^C}{\mu_t^V}\right)^{\alpha} H_t^{C\,1-\alpha} \tag{59}$$

$$N_t^I = \frac{\mu_t^I - 1}{\mu_t^I \tilde{\phi}^I} Z_t A_t \left(\frac{\tilde{K^I}_t}{\mu_t^K}\right)^{\alpha} H_t^{I\,1-\alpha} \tag{60}$$

$$\tilde{\lambda}_t \tilde{P}_t^{\,I} = E_t\left\{\frac{\tilde{\lambda}_{t+1}\beta \mu_{t+1}^{Y\;\;1-\sigma}}{\mu_{t+1}^K}\left(\tilde{r}_{t+1} + \tilde{P}_{t+1}^I(1-\delta)\right)\right\} \tag{61}$$

$$\tilde{w}_t = \frac{\tilde{P}_t^I}{\mu_t^I}(1-\alpha) Z_t A_t \left(\frac{\tilde{K}_t^I}{N_t^I \mu_t^K}\right)^{\alpha}\left(\frac{H_t^I}{N_t^I}\right)^{-\alpha} \tag{62}$$

$$\tilde{w}_t = \frac{1}{\mu_t^C}(1-\alpha) Z_t \left(\frac{\tilde{K}_t^C}{\mu_t^K H_t^C}\right)^{\alpha} \tag{63}$$

$$\tilde{r}_t = \frac{\tilde{P}_t^I}{\mu_t^I}\alpha Z_t A_t \left(\frac{\tilde{K^I}_t}{N_t^I \mu_t^K}\right)^{\alpha-1}\left(\frac{H_t^I}{N_t^I}\right)^{(1-\alpha)} \tag{64}$$

$$\tilde{r}_t = \frac{1}{\mu_t^C} \alpha Z_t \left(\frac{\widetilde{K_t^C}}{\mu_t^K H_t^C} \right)^{\alpha - 1} \tag{65}$$

$$b_t \Theta \Gamma H_t^{\Theta - 1} \tilde{X}_t \left(C_t - \chi \frac{\widetilde{C_{t-1}}}{\mu^Y} - \Gamma H_t^{\Theta} \tilde{X}_t \right)^{-\sigma} = \frac{\tilde{\lambda}_t \tilde{w}_t}{\mu^w} \tag{66}$$

$$\begin{aligned} \tilde{\lambda_t} \quad = \quad & b_t(\tilde{C}_t - \frac{\chi}{\mu_t^y}\widetilde{C_{t-1}} - \Gamma H_t^{\theta} \tilde{X}_t)^{-\sigma} - E_0 b_{t+1} {\mu_{t+1}^y}^{-\sigma} \beta \chi (\widetilde{C_{t+1}} - \frac{\chi}{\mu_{t+1}^y} \tilde{C}_t - \Gamma H_{t+1}^{\theta} \widetilde{X_{t+1}})^{-\sigma} \dots \\ & - \widetilde{\lambda_{2t}} \eta {\mu_t^y}^{\eta - 1} (\tilde{C}_t - \frac{\chi}{\mu_t^y} \widetilde{C_{t-1}})^{\eta - 1} {\widetilde{X_{t-1}}}^{1-\eta} \dots \\ & + E_0 {\mu_{t+1}^y}^{1-\sigma} \beta \widetilde{\lambda_{2t+1}} \eta {\mu_{t+1}^y}^{\eta - 1} \frac{\chi}{\mu_{t+1}^y} (\widetilde{C_{t+1}} - \frac{\chi}{\mu_{t+1}^y} \tilde{C}_t)^{\eta - 1} {\tilde{X}_t}^{1-\eta} \end{aligned} \tag{67}$$

$$b_t \Gamma H_t^{\theta} (\tilde{C}_t - \frac{\chi}{\mu_t^y} \widetilde{C_{t-1}} - \Gamma H_t^{\theta} \tilde{X}_t)^{-\sigma} = \widetilde{\lambda_{2t}} - \beta E_0 {\mu_{t+1}^y}^{1-\sigma} \widetilde{\lambda_{2t+1}} (1 - \eta) {\mu_{t+1}^y}^{\eta - 1} (\widetilde{C_{t+1}} - \frac{\chi}{\mu_{t+1}^y} \tilde{C}_t)^{\eta} {\tilde{X}_t}^{-\eta} \tag{68}$$

$$\tilde{X}_t = (\tilde{C}_t - \frac{\chi}{\mu_t^y} \widetilde{C_{t-1}})^{\eta} ({\widetilde{X_{t-1}}}^{1-\eta})(\mu_t^y)^{\eta - 1} \tag{69}$$

$$\mu_t^I = \frac{(1 - \omega^I) N_t^I - (\tau^I - \omega^I)}{\tau^I (1 - \omega^I) N_t^I - (\tau^I - \omega^I)} \tag{70}$$

$$\mu_t^C = \frac{(1 - \omega^C) N_t^C - (\tau^C - \omega^C)}{\tau^c (1 - \omega^C) N_t^C - (\tau^C - \omega^C)} \tag{71}$$

$$H = H_t^C + H_t^I \tag{72}$$

$$ln(Z_t) = \rho_Z ln(Z_{t-1}) + \sigma^Z \epsilon_t^Z \tag{73}$$

$$ln(A_t) = \rho_A ln(A_{t-1}) + \sigma^A \epsilon_t^A \tag{74}$$

$$ln(v_t) = \rho_v ln(v_{t-1}) + \sigma_v \epsilon_t^v \tag{75}$$

$$ln(b_t) = \rho_b ln(b_{t-1}) + \sigma_b \epsilon_t^b \tag{76}$$

$$ln(\frac{\mu^w}{\bar{\mu}^w}) = \rho_{\mu^w} ln(\frac{\mu_{t-1}^w}{\bar{\mu}^w}) + \sigma^{\mu^w} \epsilon_t^{\mu^w} \tag{77}$$

$$ln(\mu_t^Z / \bar{\mu}^Z) = \rho_{\mu^Z} ln(\mu_{t-1}^Z / \bar{\mu}^Z) + \sigma^{\mu^Z} \epsilon_t^{\mu^Z} \tag{78}$$

$$ln(\mu_t^A / \bar{\mu}^A) = \rho_{\mu^A} ln(\mu_{t-1}^A / \bar{\mu}^A) + \sigma^{\mu^A} \epsilon_t^{\mu^A}, \tag{79}$$

where λ and λ_2 are Lagrangian multipliers.

3 Model Estimation

We use a Bayesian estimation process to determine the value of the majority of the parameters included in the benchmark model, while calibrating some of the more well-known parameters directly. This method of parameterization has become one of the more common methods for estimating parameters in the business cycle literature, due to its ability to take the best aspects of two formerly used methods of parameterization: maximum likelihood and direct calibration. The Bayesian estimation process involves three components: a list of observables, the model and a set of priors. The priors are chosen based on either micro-level data or economic theory, which assigns a higher weight to a given area of the parameter subspace. It is with these priors that Bayesian estimation can be understood as bridging both maximum likelihood and direct calibration. As the proportion of the parameter subspace included within the prior distribution decreases, Bayesian estimation becomes akin to direct calibration. Conversely, as the given area of the parameter subspace increases to infinity, the Bayesian estimation will be where the log-likelihood function peaks, thus maximum likelihood. For the more frequently estimated parameters, we choose priors that match those used in the literature. To facilitate our Bayesian estimation, we will be using Dynare. For readers who are interested in a more in-depth discussion of the mechanisms involved in the Bayesian estimation process, we recommend An and Schorfheide (2007).

The list of observables included in our Bayesian estimation process includes log differences in output, investment, consumption, hours worked and the *rpi*, all measured in percentage terms. Letting Υ_t denote the vector of observables, we have

$$\Upsilon_t = \begin{bmatrix} \Delta ln(Y_t) \\ \Delta ln(C_t) \\ \Delta ln(I_t) \\ \Delta ln(H_t) \\ \Delta ln(RPI_t) \end{bmatrix} \times 100 + \begin{bmatrix} \epsilon^{ME}_{Y,t} \\ \epsilon^{ME}_{C,t} \\ \epsilon^{ME}_{I,t} \\ \epsilon^{ME}_{H,t} \\ \epsilon^{ME}_{RPI,t} \end{bmatrix}, \tag{80}$$

where measurement errors are included for all observables, following Ireland (2004).

Thus far, for notational simplicity we have assumed that the elasticities of substitution between firm-level and industry-level goods were identical across sectors. However, this assumption could be potentially restrictive, hence from this point on we assume that each sector differs in its elasticity of substitution, between both industry- and firm-level goods. Thus, τ_c and τ_i govern the elasticity of substitution between firm-level goods in the consumption and investment sectors, respectively. Likewise, ω_c and ω_i govern the elasticity of substitution between industry-level consumption goods and industry-level investment goods. As with Floetotto et al. (2009), we assume that the elasticity of substitution in both sectors must be greater at the firm level than at the industry level ($\frac{1}{1-\omega^C} < \frac{1}{1-\tau^C}$ and $\frac{1}{1-\omega^I} < \frac{1}{1-\tau^I}$). As pointed out by Floetotto et al. (2009), there is no clear estimate for these elasticities in the literature. The value assigned to these elasticities depends heavily on the markup charged above marginal costs within each industry, along with the number of firms which either enter or exit each industry. Combining equations (43) and (44), respectively, the zero-profit conditions for each sector described in equations (17) and (29) and equations (14) and (28), we can calculate the percentage change in markup charged in both sectors, denoted by

$\hat{\mu_t^C}$ and $\hat{\mu_t^I}$ respectively, as follows:

$$\hat{\mu_t^C} = \frac{(1 - \tau^C \bar{\mu_t^C})}{\tau^C \bar{\mu_t^C}} \hat{C}_t \tag{81}$$

$$\hat{\mu_t^I} = \frac{(1 - \tau^I \bar{\mu_t^I})}{\tau^I \bar{\mu_t^I}} \hat{I}_t. \tag{82}$$

Log linearizing equations (14) and (28), we can calculate the percentage change in markup charged by firms as a function of the number of firms competing within each industry:

$$\hat{\mu_t^C} = \frac{\tau^C(\bar{\mu^C} - 1)(\bar{\mu^C}\tau^C - 1)}{\bar{\mu^C}\tau^C(\tau^C - 1)} \hat{N_t^C} \tag{83}$$

$$\hat{\mu_t^I} = \frac{\tau^I(\bar{\mu^I} - 1)(\bar{\mu^I}\tau^{-}1)}{\bar{\mu^I}\tau^C(\tau^I - 1)} \hat{N_t^I}. \tag{84}$$

Combining equations (81) with (83) and (82) with (84), we can then estimate the values τ^c and τ^i with data on the number of firms within each sector N_t^I and N_t^C as well as data on both consumption and investment. To calculate the number of firms operating within each sector, we (1) estimate the number of firms operating within each of the non-agriculture Standard Industrial Classification (SIC) supersectors, (2) scale each sector by its average contribution to total payroll, and then (3) subdivide each sector by its contributions to either consumption or investment production by using data from the input-output use tables available from the Bureau of Economic Analysis (BEA).[3] A detailed list of the data used and the steps involved in estimating the number of firms competing within each sector appears in the appendix. With data on the number of firms competing within each sector $\hat{N}^I$ and $\hat{N}^C$ from 1997 to 2012 in the United States accompanied with data on aggregate consumption and investment, we can estimate the value of τ^C and τ^I.

Jaimovich and Floetotto (2008) estimate the firm-level markup μ in their one-sector model as low as 1.05 using value-added data, and as high as 1.4 using data they collected on gross output. Given this range, we set the markups in steady state $\bar{\mu^C}$ and $\bar{\mu^I}$ equal to 1.3, as done by Floetotto et al. (2009). With these values for $\bar{\mu^C}$ and $\bar{\mu^I}$, we regress $\hat{N_t^C}$ with $\hat{C}_t$ and $\hat{N_t^I}$ with $\hat{I}_t$, as suggested above, and use the coefficient estimates to calculate the values for τ^c and τ^i as listed in Table 1. Given this information, a normal prior distribution is chosen for τ^C and τ^I with mean and standard deviations around their estimated values reported in Table 1. Governed by the assumption that the elasticity of substitution between firm-level goods is greater than the elasticity of substitution across industries, ω^I and ω^C are

[3] The method we use to estimate the number of firms operating within each sector is the same approach used by Floetotto et al. (2009).

set equal to 0.6. The values of these parameters do not impact our results.

For the preference parameters, we assume a gamma distribution with a mean of 3 and variance of 0.75 for θ, which determines the elasticity of labour supply. The habit persistence parameter χ is assigned a beta distribution with a mean of 0.5 and variance of 0.1. As for η, which determines the wealth effect on labour supply, we assign a uniform distribution between 0 and 1. Lastly, since steady-state hours are left to be calculated in our Bayesian estimation, they are assigned a normal distribution around a mean of 0.3 with a standard deviation of 0.03.

For observable $i \in \{Y, I, C, H, RPI\}$, the measurement error ϵ_{it}^{ME} has a mean of zero and standard deviation σ_i^{ME} governed by a uniform prior distribution bound between 0 and one-quarter of the standard deviation of the observable. The remaining parameters to be estimated include the persistence and variance for the seven shocks discussed in the previous section. The priors chosen for these parameters, along with all other priors used in the Bayesian estimation, are available in Table 2.

As outlined in section 2, the growth rate of the *rpi* is equal to

$$\mu_t^{RPI} = \left(\mu_t^A\right)^{-(1-\alpha)}, \tag{85}$$

while the growth rate of output is equal to

$$\mu^Y = \mu_t^Z \left(\mu_t^A\right)^{\alpha}. \tag{86}$$

The parameters that have yet to be discussed are those directly calibrated. The steady-state growth rate of the *rpi* is calculated using the time series for the quarterly *rpi* adjusted for changes in quality from 1949Q1 to 2006Q3 mentioned in the introduction of this paper. Using this time series, the estimated growth rate of the *rpi* equals 0.9957. As for the gross growth rate of output, we calculate the steady-state quarterly growth rate of output μ^Y using seasonally adjusted non-farm output from 1949Q1 to 2006Q3 available through the Bureau of Labor Statistics. With this data, we estimate an average quarterly growth rate of output equal to 1.0049. With these two growth rates at hand, we choose a growth rate for non-stationary neutral and investment-specific technology that matches the growth rates of both output and the *rpi*. The preference parameter σ governing the households' risk aversion is set equal to 2. The households' quarterly discount parameter β is set equal to 0.985. The Cobb-Douglas parameter α is set equal to 0.33. The quarterly depreciation rate δ is set equal to 0.025. All calibrated parameters are shown in Table 3.

Given the benchmark model M outlined in section 2, the set of observables Υ_t and a vector of parameters, Θ_M we can begin our Bayesian estimation process. Using these components, along with the likelihood function $L(\Theta_M, \Upsilon_t, M)$ calculated as

$$L(\Theta_M|\Upsilon_T, M) = p(v_0|\Theta_M, M) \prod_{t=1}^{T} p(v_t|\Upsilon_{T-1}, \Theta_M, M), \tag{87}$$

and with a Kalman filter to calculate the unknown likelihood function along with a Metropolis-Hastings algorithm, which generates a random sample of these estimates through a Monte Carlo Markov chain, we calculate the posterior density $P(\Theta_M|\Upsilon_t, M)$. The results of our Bayesian estimation are available in Table 4.

4 Model Results

As the benchmark model of this paper establishes, the cyclical nature of the *rpi* can be reproduced by allowing it to respond to changes in the relative demand of investment goods to consumption goods, in addition to changes in technology. This method is referred to as the *endogenous approach*, since the *rpi* is treated as an endogenous variable. A second approach could alternatively have movements in the *rpi* modeled exogenously by assuming that technologies across sectors move together over time, rather than endogenously. This method is referred to as the *exogenous approach*, since the *rpi* is determined completely by changes in either neutral or investment-specific technology. Both approaches are valid choices and are evaluated in this paper to assess whether one approach outperforms the other. Contrasting these two methods will determine whether future research should model the countercyclical pattern observed in the *rpi* endogenously or exogenously. The following subsection presents a model where cyclical movements in the *rpi* are entirely exogenous.

4.1 Two-Sector Model with Cointegrated TFP and IST

As outlined in section 2, the benchmark model assumes that the *rpi* moves in response to changes in *tfp* and the non-technological disturbances (wage markup, preference and MEI) through the inclusion of endogenous price markups. Rather than have the *rpi* move endogenously, one might be interested in modeling the relationship between *tfp* and the *rpi* exogenously. As demonstrated by Schmitt-Grohé and Uribe (2011), *tfp* and *rpi* in the post-war United States are best characterized by a cointegrating relationship during that era. With both *tfp* and *rpi* cointegrated, any deviation from the equilibrium long-run relationship between *tfp* and the *rpi* by either of the technologies will generate a counteracting response in the other technology so as to maintain the long-run relationship between these two time series. Furthermore, Schmitt-Grohé and Uribe (2011) and Wagner (2013) have shown that cointegration impacts the relative importance of technology shocks when analyzing the variance decomposition. Given our attempt to replicate the true data-generating process governing the co-movement of *tfp* and the *rpi*, along with the research listed above, it seems natural to allow *tfp* and the *rpi* to follow a common stochastic trend in our assessment. To clarify, this model does not allow for any movements of firms in and out of each sector, thus cutting off any endogenous movement in the price markups in each sector. These sectors can be cointegrated by updating equations (47) and (48) to the following:

$$\begin{bmatrix} ln(\mu_t^Z/\bar{\mu}^Z) \\ ln(\mu_t^A/\bar{\mu}^A) \end{bmatrix} = \begin{bmatrix} \rho_{11} & \rho_{12} \\ \rho_{21} & \rho_{22} \end{bmatrix} \begin{bmatrix} ln(\mu_{t-1}^Z/\bar{\mu}^Z) \\ ln(\mu_{t-1}^A/\bar{\mu}^A) \end{bmatrix} + \begin{bmatrix} \kappa_1 \\ \kappa_2 \end{bmatrix} x_{t-1}^{co} + \begin{bmatrix} \epsilon_t^{\mu^Z} \\ \epsilon_t^{\mu^A} \end{bmatrix}, \tag{88}$$

where, as before, μ_t^Z and μ_t^A are the growth rates of the non-stationary neutral technology in the consumption and investment sectors, respectively, and x_t^{co} is the cointegrating term, which equals

$$x_t^{co} = \nu ln(X_t^Z) - ln(X_t^A), \tag{89}$$

where ν is calibrated such that x_t^{co} equals zero in the steady state. As before, ρ_{μ^Z} and ρ_{μ^A} determine the level of persistence, while κ_1 and κ_2 determine the impact that changes in the common trend have on μ^Z and μ^A, respectively; these are the cointegration coefficients. As before, $\epsilon_t^{\mu^Z}$ and $\epsilon_t^{\mu^A}$ are unanticipated shocks to μ_t^Z and μ_t^A, respectively. In addition, equations (45) and (46) are replaced by

$$\begin{bmatrix} ln(Z_t) \\ ln(A_t) \end{bmatrix} = \begin{bmatrix} \rho_Z & \rho_{ZA} \\ \rho_{AZ} & \rho_A \end{bmatrix} \begin{bmatrix} ln(Z_{t-1}) \\ ln(A_{t-1}) \end{bmatrix} + \begin{bmatrix} s_Z & s_{Z,A} \\ s_{A,Z} & s_A \end{bmatrix} \begin{bmatrix} \epsilon_t^Z \\ \epsilon_t^A \end{bmatrix}, \tag{90}$$

where, as before, ρ_Z and ρ_A determine the level of persistence while ρ_{ZA} and ρ_{AZ} determine the degree of spillover between technologies. Lastly, $s_{Z,A}$ and $s_{A,Z}$ allow innovations to be correlated across technologies.

This model develops from the one-sector dynamic stochastic general-equilibrium model studied by Schmitt-Grohé and Uribe (2011). Estimating this model involves a different set of parameters than that included in the benchmark model. For those parameters that are shared between this model and the benchmark, we assume the same prior distribution. For the cointegration coefficients κ_1 and κ_2, we assume a prior with a mean of zero, with lower and upper bounds of -0.4 and 0.4 for both κ_1 and κ_2. The correlation between innovations in neutral and investment-specific technology is given a normal prior distribution with a mean of -0.13 and a variance of 0.1, allowing some flexibility in the estimate. The Bayesian estimation results are reported in Table 5. We next describe our variance decomposition analysis of the two approaches to modeling movement in the *rpi*.

4.2 Variance Decomposition: Endogenous vs Exogenous

The variance decomposition of the endogenous-based approach (benchmark model) highlights the appeal of this approach. As can be seen in Table 6, nearly 55 percent of the *rpi* is determined by non-IST shocks. Approximately 29 percent of that value can be attributed to changes in *tfp*, with the remaining explained by movements in wage markup shocks, preference shocks and shocks to the MEI. These results suggest that movement in the *rpi* is not merely a technological phenomenon, but rather is in part determined by changes in aggregate consumption and investment. With 55 percent of the *rpi* determined by shocks that are not investment-specific, these findings approach those reported by Kim (2009), who finds in his SVAR approach that only 27 percent of the *rpi* can be explained by IST. Lastly, with 29 percent of the *rpi* explained by changes in non-stationary *tfp*, this suggests that changes in the *rpi* from one period to the next reflect low-frequency shifts in the neutral technology.

The exogenous-based modeling approach when estimated captures the cyclical nature of the *rpi* along with the long-run trend in the *rpi* exogenously. It therefore provides some measure of the effectiveness of the endogenous approach to modeling movement in the *rpi*, as is done in the benchmark. The results of the variance decomposition in a two-sector model with co-movement are reported in Table 7. Of notable interest is the high weight assigned to *tfp* in generating volatility in the *rpi*. What can we learn from this experiment? First of all, the assumption that IST can be identified by the inverse of the *rpi* is unreliable at best. With 31 percent of the *rpi* explained by neutral technology shocks, the classical assumption, as assumed by GHK (1997) and later adopted by Fisher (2006), Beaudry and Lucke (2009), and Justiniano et al. (2011), to name a few, that neutral technology shocks do not affect relative prices is invalid.

4.3 The Relative Price of Investment

As was demonstrated in section 2, with perfectly mobile capital and labour, the *rpi* can be calculated as the ratio of the two wage rates (or rental rates), as follows:

$$\frac{P_t^I}{P_t^c} = \frac{\mu_t^I}{\mu_t^C}\frac{1}{A_t}\left(\frac{k_t^C}{h_t^C}\right)^{\alpha}\left(\frac{k_t^I}{h_t^I}\right)^{-\alpha} X_t^{A(\alpha-1)}. \tag{91}$$

Given that capital labour ratios will be the same across sectors, the above formula can be reduced to

$$\frac{P_t^I}{P_t^c} = \frac{\mu_t^I}{\mu_t^C}\frac{1}{A_t} X_t^{A(\alpha-1)}. \tag{92}$$

Linearizing equation (92) we get

$$\hat{P}_t^I = \left(\hat{\mu_t^I} - \hat{\mu_t^C}\right) - \hat{A}_t - (1-\alpha)\hat{X}_t^A. \tag{93}$$

Note that the *rpi* can be broken down into several separate components, including the difference in price markups between sectors, stationary IST and the trend in the IST. In total, approximately 55 percent of the *rpi* variance is explained by changes in price markups. The remaining 45 percent is generated by changes in either stationary or non-stationary IST. This measures against the 100 percent used in most of the literature.

Of particular interest is the ability of endogenous price markups to translate the non-technological shocks into movement in the *rpi* in the benchmark model. In this model, preference shocks, wage markup shocks and MEI shocks combined explain 26 percent of the *rpi* from one period to the next. This suggests that the overall impact of demand shocks occurs through changes in the price markups over the business cycle.

To highlight the proportion of *rpi* variability explained by the inclusion of endogenous price markups in our benchmark model, we simulate a third version of the benchmark model with neither endogenous price markups nor exogenous shock processes designed to generate co-movement between the *tfp* and the *rpi*. This model will be referred to here as the two-sector model. Endogenous movements in price markups are removed from the benchmark model by restricting movements of firms in and out of each sector, thus pinning down the price markup charged in both sectors to $1/\tau^C$ in the consumption sector and $1/\tau^I$ in the investment sector.[4] When the benchmark model is simulated with endogenous price markups removed from the model, we find that the proportion of *rpi* volatility explained by non-IST drops from 55 percent to 0 percent, with stationary and non-stationary IST shocks explaining 70 and 30 percent, respectively.

Figure 3 plots the impulse responses of output, investment, consumption, hours worked and the *rpi* to both a one-standard-error innovation to ϵ_t^z and a one-standard-error innovation to $\epsilon_t^{\mu^Z}$. As Figure 3 shows, a positive shock to stationary *tfp* generates an expansion in output, hours and investment along with a decline in the *rpi*. The immediate increase in production along with the decline in the *rpi* generates a higher-than-normal response in investment, leading to a reduction in household consumption as they accommodate the increase in investment. A non-stationary *tfp* shock generates a similar response in output, hours and investment, with a much more muted response in the *rpi*. The *rpi* declines by less as households respond to a permanent increase in productivity by increasing consumption immediately due to the permanent income hypothesis.

[4] To accomplish this, we remove both the number of firms in both sectors (N_t^C and N_t^I) and the markups μ^C and μ^I as endogenous variables while removing equations governing the number of firms (59) and (60) and the markup equations (70) and (71) from the system of equations listed at the end of section 2. Thus, each markup is set equal to its steady-state value and will not fluctuate over the business cycle.

The decline in the *rpi* occurs for the following reasons. First, as seen in equations (30) and (19), there is an increase in the number of firms operating within each industry as the direct effect of increased productivity causes firms to enter the market. With the number of firms entering into the investment sector N_t^I outpacing the number of firms entering into the consumption sector N_t^C, the markup charged in the investment sector decreases by more than in the consumption sector. Indirectly, with an increase in *tfp*, the demand for investment increases, implying both an increase in the number of firms within the investment sector N_t^I and a decline in the price of investment goods P_t^I due to the drop in the price markup μ_t^I that results from increased competition in this sector. Declining demand for consumption goods leads to a decline in the number of firms competing within this sector, counteracting the initial increase in the number of firms operating due to increased productivity. With the decline in the price of investment goods, the net response to a stationary *tfp* shock is a decline in the *rpi*. Thus far, we have discussed the implications of a temporary shock to *tfp*. In response to a permanent increase in *tfp*, there is an increase in both consumption (permanent income hypothesis) and investment, and consequentially shocks to the growth rate contribute less to the overall variance decomposition of the growth rate of the *rpi*. Since we include multiple stochastic trends, the deviation of the variable from its BGP includes both the variation of the variable from its respective BGP and the variation of the stochastic trend itself.

Figure 4 plots the impulse responses of output, investment, consumption, hours worked and the *rpi* to both a one-standard-error innovation to ϵ_t^A and a one-standard-error innovation to $\epsilon_t^{\mu^A}$. As can be seen in Figure 4, a positive shock to stationary IST generates an expansion in output, hours and investment along with a decline in the *rpi*. The decline in the *rpi* in response to a shock to stationary IST occurs through the following channels. With an increase in IST, the profitability of production in the investment sector causes firms to enter into the investment sector and therefore drives down the markup charged on investment goods. This is the direct effect on the number of firms operating in the investment sector. The second effect comes from households switching from consumption goods to the now relatively cheap investment good, driving up the number of firms entering into the market and driving down the markup charged in this sector. With a decline in consumption, firms exit the consumption sector and we observe an increase in the markup charged by the remaining firms. The overall effect is for the *rpi* to decrease by more than if markups were constant. The same logic holds true for a permanent shift in IST, with an increase rather than a decrease in consumption as households' lifetime permanent disposable income increases. The effect of increased consumption with a permanent shift in IST is a gradual decline in the *rpi* rather than an immediate decline, as is observed in response to a temporary IST shock.

As can be seen in Figure 4, endogenous price markups magnify the response of the relative price of investment to an IST shock. In a two-sector model without endogenous price markups, the response of the RPI matches the inverse of the IST shock exactly. With endogenous price markups, the increased profitability of investment firms drives the number of firms operating within the investment sector, driving down the relative price even further. The exact magnification depends on the steady-state markup. Table 8 outlines the magnification effect endogenous price markups have on RPI over a range of markups. The magnification effect increases with the steady-state markups, ranging from a 102 percent magnification for a steady-state markup of 1.1 to a 153.3 percent magnification for a steady-state markup of 1.4. The magnification effect of endogenous price markups on IST is particularly interesting given the trajectory that IST and their importance have had in the literature. At their inception, GHH (1988), as well as others such as Fisher (2006) and Justiniano, Primiceri and Tambalotti (2010), conclude that IST shocks are capable of reproducing real business cycles. However, since their work, Justiniano, Primiceri and Tambalotti (2011) demonstrated that when the persistence and volatility of IST shocks are set to match movements in the RPI in their Bayesian estimation, this type of shock becomes incapable of generating business cycle volatility: the reason is due to the overestimation of the volatility of the RPI by a factor of three in previous research. Thus, movements in the IST are not relevant in generating movement in output, consumption, investment and hours. This work goes a step further along this line by arguing that matching the IST by the inverse of the relative price of investment overstates the relative importance of the IST. As shown in Figure 4, and as can be observed in Table 8, movement in the *rpi* requires far less volatility in IST.

Lastly, there are three non-technological disturbances included in the benchmark model. These are preference shocks, wage markup shocks and MEI shocks. Figure 5 plots the impulse responses of output, investment, consumption, hours worked and the *rpi* to the standard error innovation to each of the three non-technological disturbances.

The increase in the *rpi* that occurs in response to shocks to either preferences or wage markups occurs through the following channels. With a positive preference shock, households increase their demand for consumption goods over investment goods, thus driving down the markup charged on consumption goods and driving up the markup on investment goods and hence an increase in the *rpi*. With wage markup shocks, both consumption and investment fall in response to a drop in household income. However, due to consumption smoothing, the response in consumption demand and consequently the markup charged are an order of magnitude smaller than the responses in investment. Through the steps listed above, this results in a rise in the *rpi*. In response to an MEI shock, the decline in the *rpi* is due to an increase in investment demand and a decrease in demand for consumption goods. Investment demand increases as households realize that a given amount of savings can be converted into a greater amount of investment goods. This leads households to reduce consumption to free up resources for further investment. Thus, MEI shocks have the exact opposite impact on the *rpi* when compared to preference and wage markup shocks.

The impact that MEI shocks have on the *rpi* via shifts in the relative demand for investment goods over consumption goods is of particular interest. As mentioned in the introduction, the traditional assumption in the business cycle literature has been to assume a one-for-one transformation in the conversion of consumption goods into investment goods. However, as Justiniano et al. (2011) argue, a more realistic version of this transformation would involve two steps, the first being a transformation of consumption goods into investment goods, which is altered by shifts in IST. The second transformation involves taking capital goods fresh off the production line and converting these goods into active capital.

Shocks to this mechanism are referred to as changes in the MEI. Both of these steps are included in the benchmark model. Justiniano et al. (2011) assume that the IST can be identified by the inverse of the *rpi*, while changes in the MEI are driven by changes in the firms' ability to access credit. They make this assertion by linking movements in the MEI in their model to the spread between high-yield and AAA corporate bonds (a measure of risk premium). Incidentally, they assume that changes in the firms' ability to access credit does not impact the *rpi*. As can be seen in Figure 5, shocks to the MEI do in fact have a limited impact on the *rpi*, validating their assumption.

5 Conclusion

Since the seminal work of GHH (1988), IST has become a common feature in most of the business cycle literature. Likewise, the convenient assumption by GHK (1997) that IST can be identified by the inverse of the *rpi* has also remained the same. Assuming that the *rpi* is orthogonal to the business cycle eliminates any possibility that the *rpi* moves in response to changes in the relative demand for investment goods over consumption goods. With 55 percent of the *rpi* determined by non-IST shocks via the endogenous price mechanisms identified above, our results approach those reported by Kim (2009), who finds that IST in the United States explains at most 27 percent of the volatility of the *rpi* in the SVAR estimation, with non-technological disturbances having significant explanatory power. As the benchmark model demonstrates, when the *rpi* moves in part due to changes in aggregate demand via endogenous price markups, IST accounts for less than half of the volatility in the *rpi*. Furthermore, non-technological shocks, such as preference, wage markup and MEI shocks, have an important source of business cycle volatility through their effect on aggregate demand. Lastly, the sizable fraction of the *rpi* explained by non-IST warrants serious skepticism regarding the interpretation of business cycle research where the *rpi* is modeled exogenously, since the *rpi* may not move in tandem with the business cycle. Given these results, future business cycle research regarding the relative importance of IST requires the incorporation of a mechanism to generate endogenous movements in the *rpi* to changes in the relative demand for consumption goods to investment goods.

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Appendix

A.1 Data Used

1. Output Y_t Non-Farm Gross Value-Added NIPA Table 1.3.5 Row 3 1947:Q1-2013:Q4 Chained 2009 Dollars Seasonally Adjusted at Annual Rates

2. Consumption C_t Real Personal Consumption Expenditure BEA 1947:Q1-2013:Q4 Chained 2009 Dollars PCECC96 Seasonally Adjusted at Annual Rates

3. Investment I_t Real Investment Expenditure BEA 1947:Q1-2013:Q4 Chained 2009 Dollars GPDIC96 Seasonally Adjusted at Annual Rates

4. Private Non-Farm Hours Worked Major Sector Multisector Productivity Index Base Year 2009 1947:Q1-2013:Q4 BLS PRS85006033 Seasonally Adjusted at Annual Rates

5. Private Non-Farm Business Sector Capital Services Index 100 2009 BLS MPU4910042 1987-2012 BLS PRS85006033 Annual data on capital services are converted into quarterly data assuming a constant growth rate between quarters.

6. Annual Payroll Information Small Business Administration Data for 20 industry groups. Used to weigh the relevance of each sector.

7. Input-Output Use Table BEA Before Redefinitions 1997-2012. Used to determine the proportion of goods from each industry going to investment projects and consumption-goods production.

8. Number of firms within each SIC Supersector via the BLS data by adding Expansions (firms that hired), Contractions (firms that laid off employees) plus Openings (new start-ups) less Closures (firms that closed).

9. Real Consumption Expenditure Non-Durables and Services NIPA Table 1.1.6 Real Gross Domestic Product Chained 2009 Dollars Seasonally Adjusted at Annual Rates

10. Real Investment Goods Expenditure on Equipment and Consumer Durables NIPA Table 1.1.6 Real Gross Domestic Product Chained 2009 Dollars Seasonally Adjusted at Annual Rates

11. Calculate the number of firms within each industry N_t^I and N_t^C by multiplying the number of firms within each industry (8) by their contribution to total payroll (6), and then subdividing each sector by its contributions to either consumption or investment production.

12. Calculate the elasticities τ^I and τ^C through the following manipulations:

    (a) Calculate $\hat{I}_t$, $\hat{C}_t$, $\hat{N_t^C}$ and $\hat{N_t^I}$ as their log deviation from the HP trend.

    (b) Regress $\hat{N_t^C}$ on $\hat{C}_t$ and $\hat{N_t^I}$ on $\hat{I}_t$

(c) Using the conditions

$$\hat{C}_t = \frac{(\tau^C(\mu^C - 1))}{(1 - \tau^C)} \hat{N}_t^C \tag{94}$$

$$\hat{I}_t = \frac{(\tau^I(\mu^I - 1))}{(1 - \tau^I)} \hat{N}_t^I, \tag{95}$$

and set $\mu^I = \mu^C = 1.3$ and calculate values τ^C and τ^I.

13. Calculate $\hat{\mu_t^C}$ and $\hat{\mu_t^I}$ by equations

$$\hat{\mu_t^C} = \frac{(1 - \tau^C \mu^C)}{(\tau^C \mu^C)} \hat{C}_t \tag{96}$$

$$\hat{\mu_t^I} = \frac{(1 - \tau^I \mu^I)}{(\tau^I \mu^I)} \hat{I}_t. \tag{97}$$

Table 1
Precursor OLS Estimates for τ^C τ^I

Sector	Consumption	Investment
Theory	$\hat{N_t^C} = \left(\frac{1-\tau^C}{\tau^C(\mu^C-1)}\right)\hat{C}_t$	$\hat{N_t^I} = \left(\frac{1-\tau^I}{\tau^I(\mu^I-1)}\right)\hat{I}_t$
Data	$\hat{N_t^C} = \underset{(0.001)}{-6.745e^{-19}} + \underset{(0.11)}{1.123^{***}}\ \hat{C}_t$	$\hat{N_t^I} = \underset{(0.001)}{-3.66e^{-19}} + \underset{(0.027)}{0.394^{***}}\ \hat{I}_t$
R^2	0.5648	0.7252

Note: Data on both the dependent and independent variables are outlined in the appendix. Each variable accompanied by a hat is the log deviation from its Hodrick–Prescott filter trend, where $\lambda = 1600$ with no drift. Data ranges from 1992Q3 to 2013Q4. Significance codes: '***' denotes 0.001, '**' 0.01, and '*' 0.05.

Table 2
Priors

Parameter	Prior Distribution	Lower Bound	Upper Bound	Mean	Variance
τ^I	Normal			0.90	0.05
τ^C	Normal			0.78	0.05
θ	Gamma			3	0.75
χ	Beta			0.5	0.1
η	Uniform	0.01	0.99		
h^{ss}	Normal			0.3	0.03
ρ_Z	Beta			0.9	0.05
ρ_A	Beta			0.9	0.05
ρ_v	Beta			0.9	0.05
ρ_b	Beta			0.9	0.05
ρ_{μ^w}	Beta			0.9	0.05
ρ_{μ^Z}	Beta			0.40	0.20
ρ_{μ^A}	Beta			0.20	0.10
σ^i	Inv gamma			0.5	2
σ^k	Uniform	0	$\frac{1}{4}\sigma^{obs}$		

Note: σ^i refers to the variance of an unanticipated shock to $i = \{Z, A, b, V, \mu^W, \mu^Z, \mu^A\}$, and σ^k the variance of the measurement error for the observable $k = \{Y, I, C, H, RPI\}$.

Table 3
Calibrated Parameters

Parameter	Value	Description
σ	2	Risk aversion
$\bar{\mu}^Y$	1.0049	Per capita output growth along a BGP
$\bar{\mu}^{RPI}$	0.9957	Per capita *rpi* growth along a BGP
δ	0.025	Depreciation rate in steady state
$\bar{\mu}^w$	1.10	Steady-state wage markup
β	0.985	Subjective discount factor
α	0.33	Capital share of output
$\bar{\mu}^I, \bar{\mu}^C$	1.3	Steady-state markup

Note: Parameter values governing the per capita output growth, IST growth, steady-state markup, and households' risk aversion are those used by Schmitt-Grohé and Uribe (2011).

Table 4
Bayesian Estimation

Parameter	Distribution	Prior mean	Standard Deviation	Posterior mean	5%	95%
τ^I	Normal	0.9	0.05	0.9152	0.9151	0.9153
τ^c	Normal	0.78	0.05	0.7848	0.7847	0.7849
θ	Normal	3	0.75	2.9494	2.9481	2.9507
χ	Gamma	0.5	0.1	0.537	0.5368	0.5372
η	Uniform	0.504	–	0.4216	0.4196	0.4236
h^{ss}	Normal	0.3	0.03	0.2791	0.2791	0.2791
ρ_Z	Beta	0.9	0.05	0.8658	0.8656	0.866
ρ_A	Beta	0.9	0.05	0.9266	0.9265	0.9267
ρ_V	Beta	0.9	0.05	0.9927	0.9927	0.9927
ρ_b	Beta	0.9	0.05	0.9436	0.9435	0.9437
ρ_{μ^W}	Beta	0.9	0.05	0.8857	0.8855	0.8859
ρ_{11}	Beta	0.4	0.2	0.3582	0.3579	0.3585
ρ_{22}	Beta	0.2	0.1	0.1632	0.1629	0.1635
σ_z^0	Inverse gamma	0.5	2	0.0608	0.0606	0.061
σ_a^0	Inverse gamma	0.5	2	0.0606	0.0604	0.0608
σ_V^0	Inverse gamma	0.5	2	0.0807	0.0758	0.0856
σ_b^0	Inverse gamma	0.5	2	0.0703	0.0623	0.0783
$\sigma_{\mu^w}^0$	Inverse gamma	0.5	2	0.3654	0.36	0.3708
$\sigma_{\mu^Z}^0$	Inverse gamma	0.5	2	0.0609	0.0597	0.0621
$\sigma_{\mu^A}^0$	Inverse gamma	0.5	2	0.0633	0.0623	0.0643

Note: σ^i refers to the variance of an unanticipated shock to $i = \{Z, A, b, V, \mu^W, \mu^Z, \mu^A\}$.

Table 5
Bayesian Estimation
With Technological Spillovers and Cointegration

Parameter	Distribution	Prior mean	Standard Deviation	Posterior mean	5%	95%
θ	Normal	3	0.75	3.3039	3.2979	3.3099
χ	Gamma	0.5	0.1	0.4934	0.4931	0.4937
η	Uniform	0.504	0.2855	0.6042	0.6018	0.6066
h^{ss}	Normal	0.3	0.03	0.3157	0.3155	0.3159
ρ_Z	Beta	0.9	0.05	0.8984	0.8977	0.8991
ρ_A	Beta	0.9	0.05	0.9116	0.9114	0.9118
ρ_{ZA}	Normal	0	0.3	-0.0383	-0.0416	-0.035
ρ_{AZ}	Normal	0	0.3	-0.109	-0.1106	-0.1074
ρ_V	Beta	0.9	0.05	0.8149	0.8143	0.8155
ρ_b	Beta	0.9	0.05	0.9436	0.913	0.9136
ρ_{μ^W}	Beta	0.9	0.05	0.9428	0.9422	0.9434
ρ_{11}	Beta	0.4	0.2	0.4504	0.4489	0.4519
ρ_{22}	Beta	0.3	0.1	0.2257	0.2252	0.2262
ρ_{12}	Uniform	0	0.17	0.0341	0.0336	0.0346
ρ_{21}	Uniform	0	0.17	-0.2571	-0.2576	-0.2566
κ_1	Uniform	0	0.29	-0.281	-0.2828	-0.2792
κ_2	Uniform	0	0.29	0.4221	0.4204	0.4238
$corr(Z, A)$	Normal	-0.13	0.1	-0.0052	-0.0053	-0.0051
σ_z^0	Inverse gamma	0.5	2	0.061	0.0594	0.0626
σ_a^0	Inverse gamma	0.5	2	0.0609	0.0593	0.0625
σ_V^0	Inverse gamma	0.5	2	0.0656	0.063	0.0682
σ_b^0	Inverse gamma	0.5	2	0.0703	0.0606	0.0652
$\sigma_{\mu^w}^0$	Inverse gamma	0.5	2	0.3654	0.2372	0.2718
$\sigma_{\mu^Z}^0$	Inverse gamma	0.5	2	0.0609	0.0633	0.0645
$\sigma_{\mu^A}^0$	Inverse gamma	0.5	2	0.0628	0.0618	0.0638

Note: All forms of endogenous movement in the *rpi* have been removed. Prior distributions for parameters shared with the benchmark model remain as described in section 3. σ^i refers to the variance of an unanticipated shock to $i = \{Z, A, b, V, \mu^W, \mu^Z, \mu^A\}$.

Table 6
Variance Decomposition: Benchmark Model

	g^y	g^c	g^i	g^h	g^{rpi}
Stationary TFP (Z_t)	37.24	5.47	45.52	3.72	27.22
Stationary IST (A_t)	0.26	8.15	4.62	0.18	37.16
Non-stationary TFP (μ_t^Z)	14.58	42.83	5.47	0.51	1.37
Non-stationary IST (μ_t^A)	0.11	1.21	0.61	0.04	8.25
Preference (b_t)	0.08	4.92	0.65	0.05	0.41
Wage markup (μ_t^W)	47.46	32.84	42.59	95.43	25.25
MEI (V_t)	0.27	4.57	0.55	0.06	0.34

Note: The column headers are defined as follows: g^y growth rate of output, g^c growth rate of consumption, g^i growth rate of investment, g^h growth rate of hours, g^{rpi} growth rate of the *rpi*.

Table 7
Variance Decomposition: Model With Cointegration

	g^y	g^c	g^i	g^h	g^{rpi}
Stationary TFP (Z_t)	42.99	21.87	42.78	2.39	1.26
Stationary IST (A_t)	0.17	7.23	8.1	0.39	46.57
Non-stationary TFP (μ_t^Z)	17.5	14.45	13.64	0.07	29.37
Non-stationary IST (μ_t^A)	1.73	2.56	1.47	0.03	22.79
Preference (b_t)	0.05	4.79	1.05	0.09	0
Wage Markup (μ_t^W)	37.35	42.49	30.96	96.55	0
MEI (V_t)	0.22	6.6	2.01	0.48	0

Note: The column headers are defined as follows: g^y growth rate of output, g^c growth rate of consumption, g^i growth rate of investment, g^h growth rate of hours, g^{rpi} growth rate of the *rpi*.

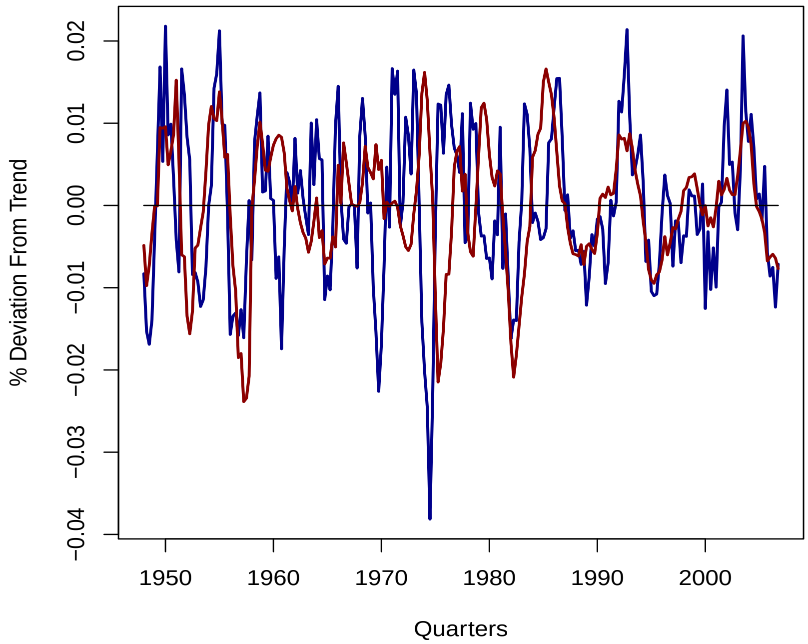
Table 8
Magnification of IST Shocks

		Steady-State Markups			
		1.1	1.2	1.3	1.4
Model	Measure				
Benchmark model	σ^{RPI}/σ^{A}	102.11	118.86	136.22	153.3
	$\sigma^{RPI}/\sigma^{X^A}$	67.16	71.49	75.33	78.6
Exogenous model	σ^{RPI}/σ^{A}	100	100	100	100
	$\sigma^{RPI}/\sigma^{X^A}$	66.57	66.57	66.57	66.57
Two-sector model	σ^{RPI}/σ^{A}, $\sigma^{RPI}/\sigma^{X^A}$	100	100	100	100

Note: Each figure represents the percentage of the measured volatility of the RPI generated by a one-standard-error innovation in either stationary or non-stationary IST. For the benchmark model, τ^I and τ^C are both set equal to 0.92. This is done to keep the number of firms operating within each sector strictly positive across all steady-state values. For the exogenous model, firm entry is fixed, hence the same value appears across all steady-state markups.

Figure 1

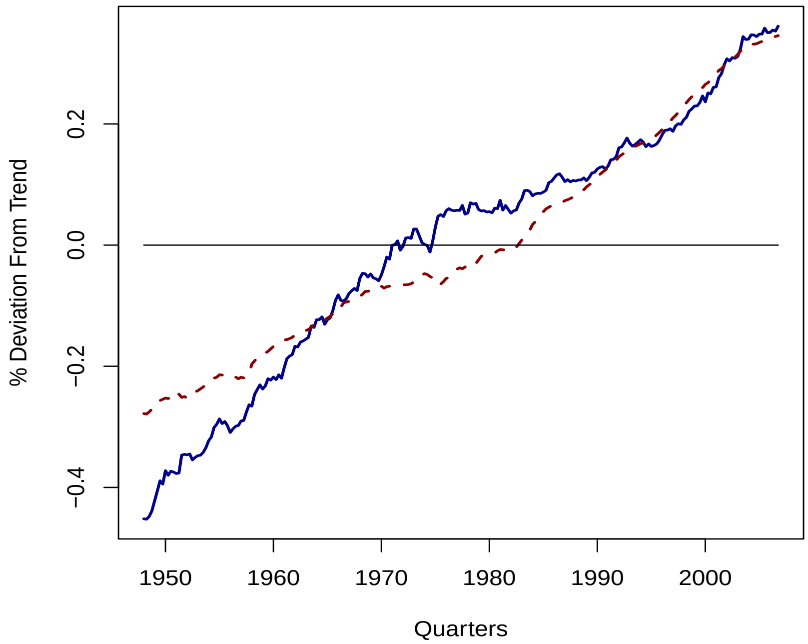
Detrended Log TFP and the Inverse of Detrended Log RPI



Note: Plots of log TFP (tfp) and the log RPI (rpi) for the United States from 1948Q1 to 2006Q3, where each is detrended using a HP filter. The blue line indicates the log detrended tfp, while the red line indicates the inverse of the detrended quality-adjusted rpi over the same period, where the inverse of the log detrended RPI is plotted so as to illustrate the link between these two technologies.

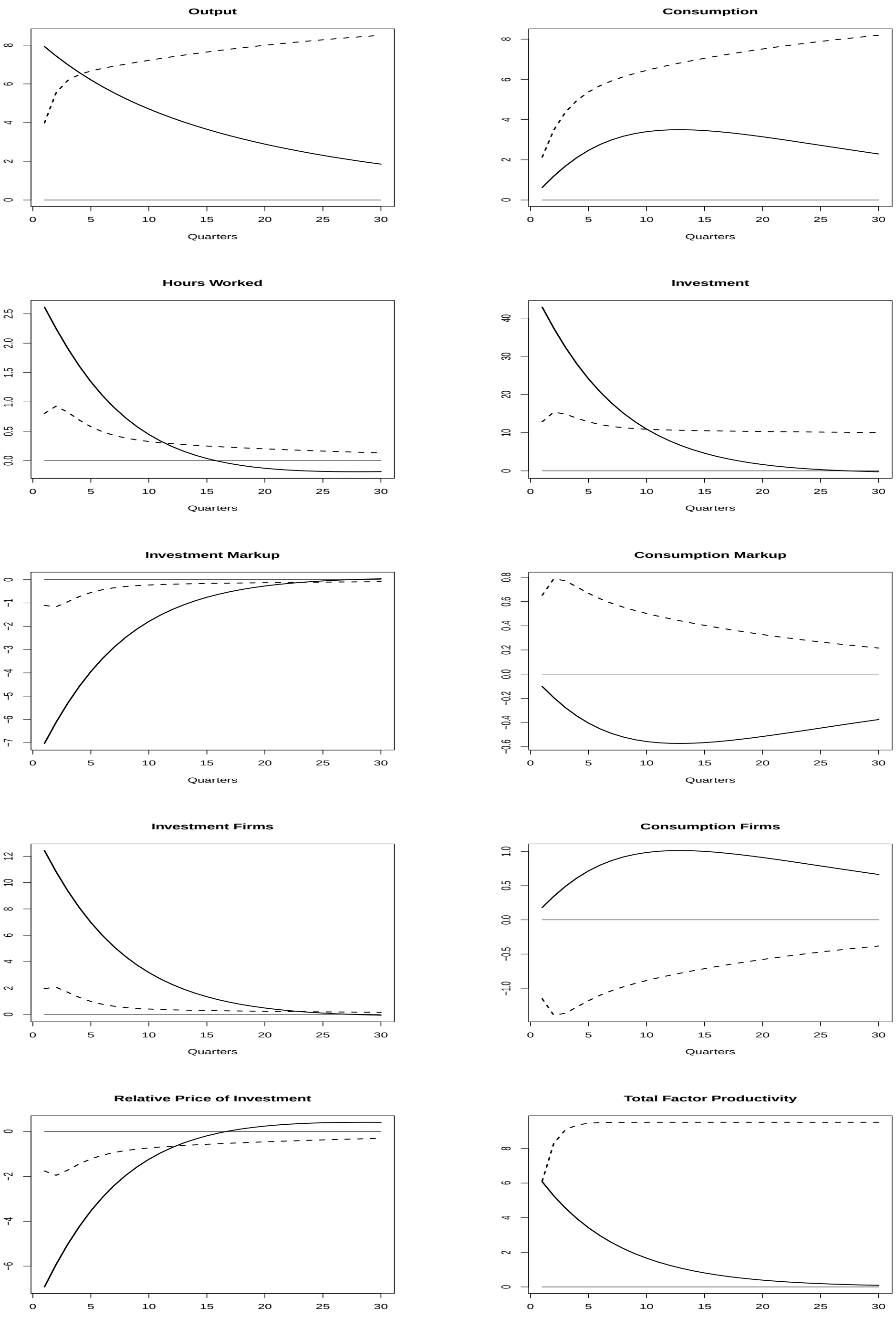
Figure 2

Demeaned Log TFP and the Inverse of Demeaned Log RPI Adjusted by the Cointegration Coefficient



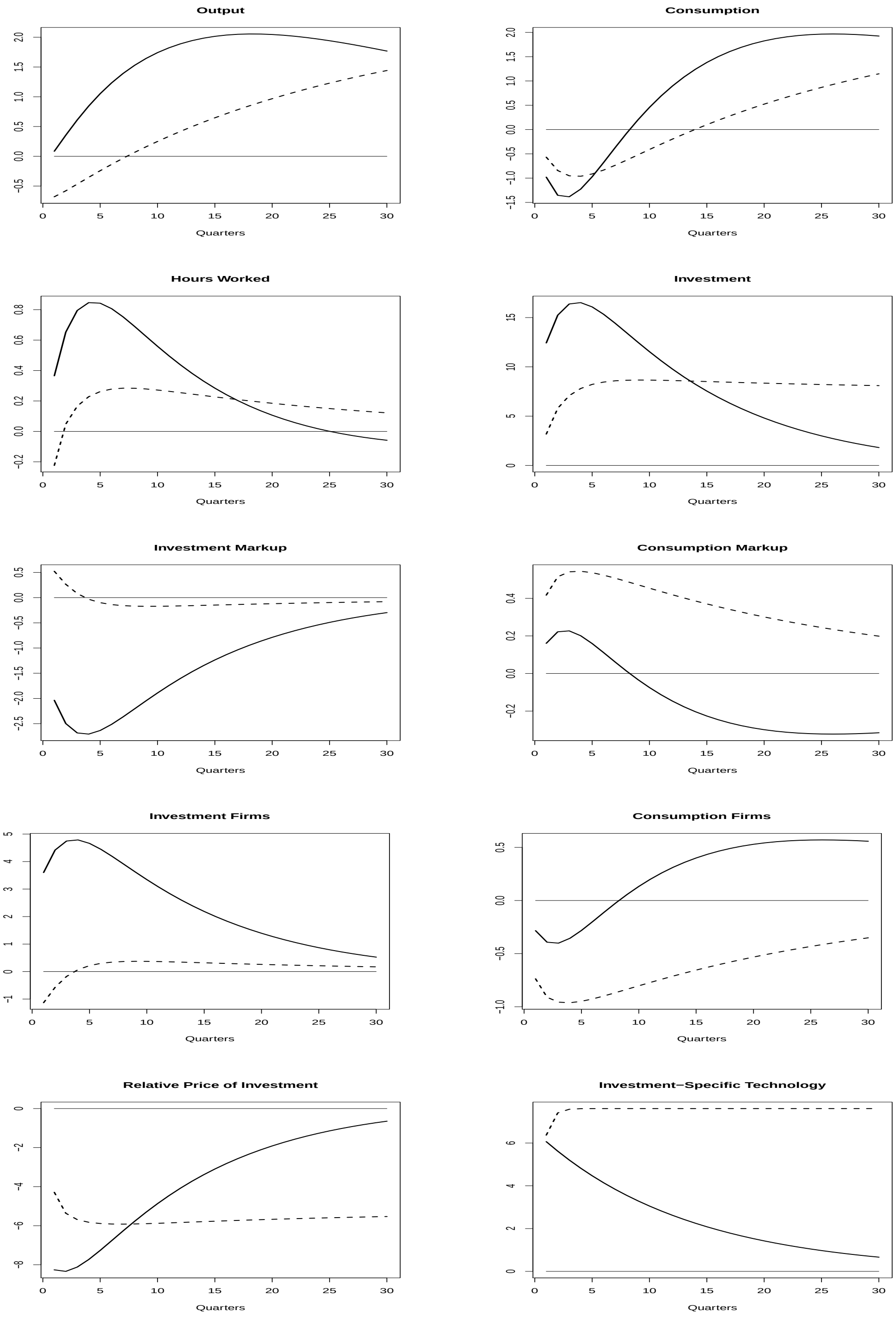
Note: Plots of *tfp* and the *rpi* for the United States from 1948Q1 to 2006Q3. The solid blue line indicates the *tfp* while the dashed red indicates the inverse of the quality-adjusted *rpi* over the same period, where the inverse of the *rpi* is plotted so as to illustrate the link between these two time series. Each time series has been demeaned.

Figure 3
Impulse Responses to Neutral Technology Shock
Benchmark Model



Note: Impulse responses to a one-standard-error innovation to ϵ_t^Z (solid line) and a one-standard-error innovation to $\epsilon_t^{\mu^Z}$ (dashed line), measured as a percent deviation from the respective BGP.

Figure 4
Impulse Responses to IST Shock
Benchmark Model



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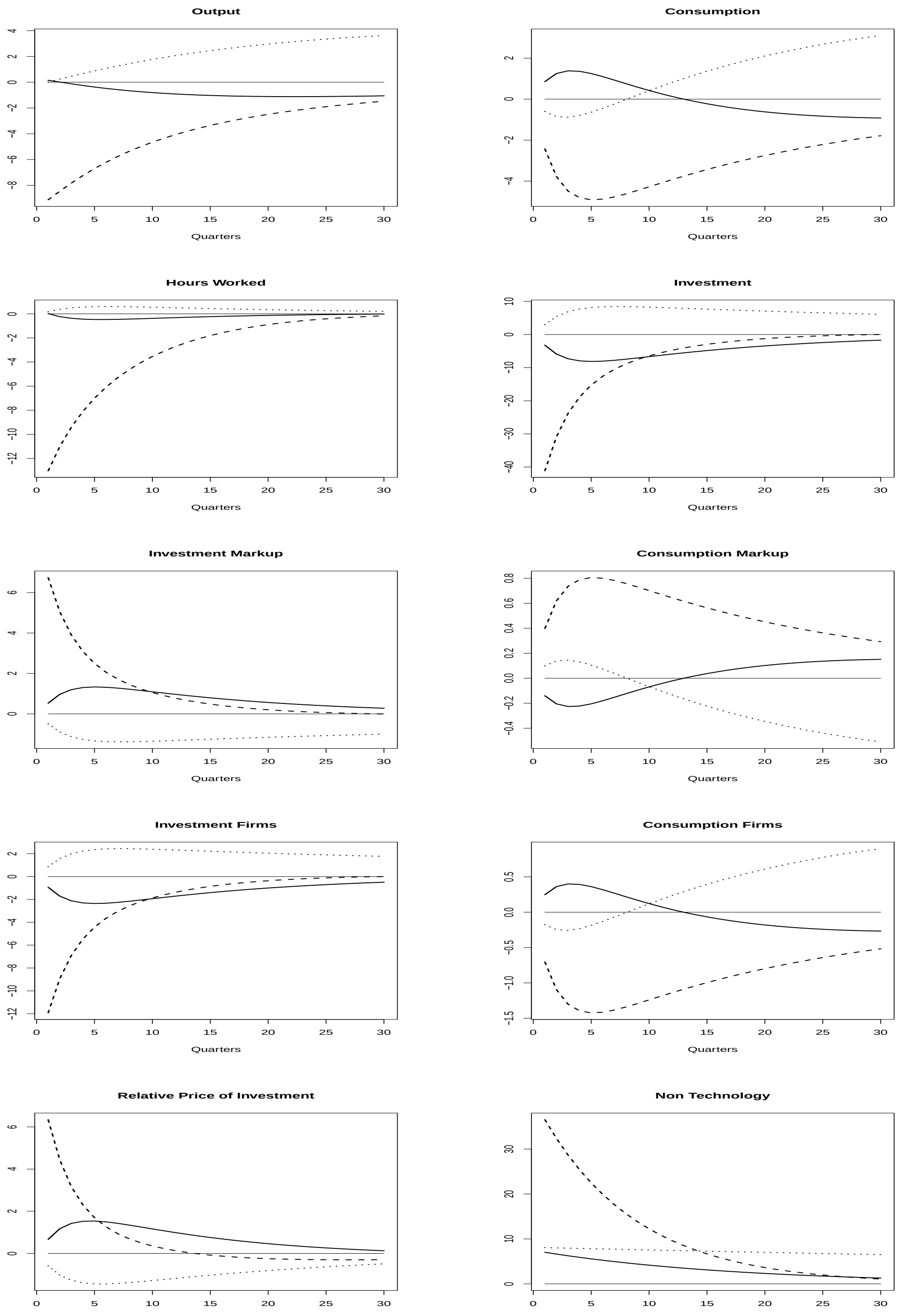
Note: Impulse responses to a one-standard-error innovation to ϵ_t^Z (solid line) and a one-standard-error innovation to $\epsilon_t^{\mu^Z}$ (dashed line), measured as a percent deviation from the respective BGP.

Figure 5
Impulse Responses to Non-Technology Shocks
Wage Markup, Preference and MEI
Benchmark Model



Note: Impulse responses to a one-standard-error innovation to ϵ_t^b (solid line) a one-standard-error innovation to $\epsilon_t^{\mu^W}$ (dashed line), and a one-standard-error innovation to ϵ_t^V (dotted), measured as a percent deviation from its respective steady state.