Working Paper 97-5 / Document de travail 97-5

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Bank of Canada Banque du Canada

ISSN 1192-5434
ISBN 0-662-25462-7

Printed in Canada on recycled paper

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February 1997

ACKNOWLEDGMENTS

The authors would like to thank Alain DeSerres, Paul Fenton, Robert Lafrance, René Lalonde, John Murray and Alain Paquet for useful comments and discussions. They also want to thank Jennifer Page and Robert Vigfusson for their excellent research assistance. Of course, since the authors are solely responsible for the paper's content, none of the above mentioned is responsible for any remaining errors.

ABSTRACT

In this paper, the authors survey some of the recent techniques proposed in the literature to measure the trend component of output or potential output. Given the reported shortcomings of mechanical filters and univariate approaches to estimate potential output, the paper focusses on three simple multivariate methodologies: the multivariate Beveridge-Nelson methodology (MBN), Cochrane's methodology (CO), and the structural VAR methodology with long-run restrictions applied to output (LRRO). The foundation of these methodologies is first discussed and then applications to U.S. output and consumption data are considered. The LRRO estimates provide significant evidence of a diffusion process for shocks to potential output. This suggests that permanent shocks have more complex dynamics than a random walk, which is the basic assumption of the CO and MBN approaches. However, it is also found that the estimation of the output gap on the basis of an estimated VAR is imprecise, which is consistent with results obtained by Staiger, Stock and Watson (1996) with a different methodology. The spectra of the transitory components (output gaps) resulting from the empirical applications of the CO, MBN and LRRO methodologies differ from one another. Indeed, only the LRRO transitory component has a peak at business-cycle frequencies, i.e., cycles lasting between 6 and 32 quarters.

RÉSUMÉ

Les auteurs passent en revue quelques-unes des techniques proposées dans les études récentes pour mesurer la composante tendancielle de la production, soit la production potentielle. En raison des lacunes connues des filtres appliqués de façon mécanique aux séries temporelles et des méthodes de décomposition univariées, les auteurs font plutôt appel à trois méthodes multivariées relativement simples pour estimer la production potentielle : la méthode de Beveridge-Nelson (MBN), la méthode de Cochrane (CO) et la méthode vectorielle autorégressive structurelle qui repose sur l'imposition de restrictions à la production en longue période (LRRO) . Ils exposent d'abord l'assise théorique de ces trois méthodes, pour ensuite les appliquer à tour de rôle aux chiffres de la production et de la consommation américaines. Les estimations obtenues à l'aide de la méthode LRRO font clairement ressortir la présence d'un processus de diffusion dans le cas des chocs que subit la production potentielle. S'il faut en croire ces résultats, la dynamique des chocs permanents serait plus complexe qu'une marche aléatoire, laquelle est au coeur des méthodes CO et MBN. Les auteurs constatent cependant que l'écart de production calculé à l'aide d'un vecteur autorégressif estimé manque de précision, ce qui est conforme aux résultats obtenus par Staiger, Stock et Watson (1996) au moyen d'une autre méthode. Les spectres des composantes transitoires (écarts de production) qui résultent de l'application empirique des méthodes CO, MBN et LRRO diffèrent les uns des autres. Seul le spectre de la composante transitoire dégagée au moyen de la méthode LRRO atteint un sommet au voisinage des fréquences correspondant à un cycle économique (c'est-à-dire celles comprises entre 6 et 32 trimestres).

CONTENTS

1 Introduction

Most structural macroeconomic models that are used for forecasting and policy analysis require an estimate of potential output. In these models, the gap between actual and potential output is a key variable determining the evolution of prices and wages. A level of real GDP above potential will often be seen as a source of inflationary pressures and a signal that the monetary authorities should tighten their policy. A level of real GDP below potential will have the opposite implications.[1] Potential output is often associated with the permanent component (or trend) of output. The output gap then corresponds to the transitory component of output.

Since the publication of Nelson and Plosser's (1982) influential paper suggesting that output series are best characterized as integrated series, there has been increasing recognition that measuring the permanent component of output, or potential output, with any degree of accuracy is a difficult task. The presence of a stochastic permanent component implies that potential output cannot be treated as a deterministic trend. As a result, various methods have been proposed to uncover the permanent and transitory components of output.

One of these methods consists of using mechanical filters such as the Hodrick-Prescott (HP) filter or the band-pass filter (BK) proposed by Baxter and King (1995). However, mechanical filters have been criticized. For example, Harvey and Jaeger (1993) and Cogley and Nason (1995) show that spurious cyclicality can be induced by the HP filter when it is used with integrated or nearly integrated data. Guay and St-Amant (1996) reach the more general conclusion that the HP and BK filters perform poorly in identifying the cyclical component of time series that have a spectrum or pseudo-spectrum with Granger's typical shape, i.e., the shape characteristic of most macroeconomic time series.

Baxter and King (1995) and others note that two-sided filters such as the HP and BK filters become ill-defined at the beginning and the end of samples. For this reason, they recommend discarding three years of quarterly data at both ends of the sample when using the HP filter. Van Norden (1995) stresses the fact that this is a very significant limitation for policymakers interested in estimating

1. For a discussion of how the estimation of potential output can affect the formulation of monetary policy, see Boschen and Mills (1990) or Laxton and Tetlow (1992).

the current level of the output gap.

Another strategy for identifying the permanent and transitory components of output involves the use of univariate techniques such as the unobserved components approach suggested by Watson (1986) and the Beveridge-Nelson (1981) method. However, Quah (1992) has shown that “without additional ad hoc restrictions those [univariate] characterizations are completely uninformative for the relative importance of the underlying permanent and transitory components.”

Partly in response to this criticism, a variety of multivariate methods have been proposed to generate permanent-transitory decompositions that appear less arbitrary, or that at least can be given a structural interpretation. One example is a decomposition method proposed by Cochrane (CO) which is based on the permanent-income theory and uses consumption to define the permanent component of output. Multivariate extensions of the Beveridge-Nelson decomposition method (MBN) have also been applied to identify the trend component of output (Evans and Reichlin 1994). However, a major restriction in the univariate context, which is maintained in the multivariate extensions, is that the permanent component of output behaves like a random walk. This assumption is difficult to reconcile with the widely held view that the permanent component of output is, at least in part, driven by technological innovations. As underlined by King et al. (1991), “productivity shocks set off transitional dynamics, as capital is accumulated and the economy moves towards a new steady-state.” Lippi and Reichlin (1994) go even further, arguing that modelling the trend in output as a random walk is inconsistent with standard views concerning the dynamics of productivity shocks. Adjustment costs on capital and labour, learning and diffusion processes, habit formation and time to build all imply richer dynamics than a random-walk process for technology shocks.

In this paper, we compare some of the techniques briefly introduced above with the structural vector autoregression methodology based on long-run restrictions imposed on output (LRRO) proposed by Blanchard and Quah (1989), Shapiro and Watson (1988), and King et al. (1991) in theory (Section 2) and in applications (Section 3). In Section 2, we note that one characteristic of the LRRO method is that it does not impose restrictions on the short-run dynamics of the permanent component of output. Instead, it allows for a permanent component incorporating an estimated diffusion process for permanent shocks that can differ from the random walk. We argue that it will often be interesting for researchers and policymakers to include the dynamics of permanent shocks in potential output (as opposed to the output gap) since they are more likely to reflect the production capacity of the economy.

In Section 3, we compare simple applications of the CO, MBN, LRRO approaches using U.S. data on output and consumption and ask whether the choice of a particular methodology really makes a difference statistically. We find that the answer is “yes” when the entire output gap series is considered, but “not really” when one is interested in estimating the output gap at a specific point in time. In the latter case, the estimation of potential output and the output gap is indeed imprecise. This is consistent with recent results reported by Staiger, Stock and Watson (1996) for the estimation of the NAIRU. Another interesting result is that, of the methods we consider, only the LRRO-based one generates an output gap with a peak at business cycle frequencies as defined by Burns and Mitchell (1946), i.e., cycles lasting between 6 and 32 quarters.

2 Methodologies used for estimating the trend in output

2.1 The approach based on the LRRO methodology

In this section, we briefly present the LRRO decomposition methodology involving long-run identifying restrictions (LRRO) and explain how it can be used to estimate potential output.[2]

Let Z_t be a n x 1 stationary vector including a n_1-vector of I(1) variables and a n_2-vector of I(0) variables such that $Z_t = (\Delta X_{1t}', X_{2t}')'$.[3] By the Wold decomposition theorem, Z_t can be expressed as the following reduced form:

$$Z_t = \delta(t) + C(L)\varepsilon_t \tag{1}$$

where $\delta(t)$ is deterministic, $C(L) = \Sigma_{i=0}^{\infty} C_i L^i$ is a matrix of polynomial lags, $C_0 = I_n$ is the identity matrix, the vector ε_t is the one-step-ahead forecast errors

2. See Watson (1993) for a more detailed presentation of the LRRO approach.
3. I(d) denotes a variable that is integrated of order d.

in Z_t given information on lagged values of Z_t, $E(\varepsilon_t) = 0$, and $E(\varepsilon_t\varepsilon_t') = \Omega$ with Ω positive definite. We suppose that the determinantal polynomial $|C(L)|$ has all its roots on or outside the unit circle, which rules out the non-fundamental representations emphasized by Lippi and Reichlin (1993).

Equation (1) can be decomposed into a long-run component and a transitory component:

$$Z_t = \delta(t) + C(1)\varepsilon_t + C^*(L)\varepsilon_t \quad (2)$$

where $C(1) = \Sigma_{i=0}^{\infty} C_i$ and $C^*(L) = C(L) - C(1)$. This decomposition corresponds to the multivariate Beveridge-Nelson decomposition — see Evans and Reichlin (1994) and King et al. (1991). We define $C_1(1)$ as the long-run multiplier of the vector X_{1t}. If the rank of $C_1(1)$ is less than n_1, there exists at least one linear combination of the elements in X_{1t} that is I(0). In other words, there exists at least one cointegration relationship between these variables — see Engle and Granger (1987).

The LRRO approach assumes that Z_t has the following structural representation:

$$Z_t = \delta(t) + \Gamma(L)\eta_t \quad (3)$$

where η_t is a n-vector of structural shocks, $E(\eta_t) = 0$, and $E(\eta_t\eta_t') = I_n$ (a simple normalization). From the estimated reduced form, we can retrieve the structural form (1) using the following relationships: $\Gamma_0\Gamma_0' = \Omega$, $\varepsilon_t = \Gamma_0\eta_t$, and $C(L) = \Gamma(L)\Gamma_0^{-1}$.

The long-run covariance matrix of the reduced form is equal to $C(1)\Omega C(1)'$. From (2) and (3) we have:

$$C(1)\Omega C(1)' = \Gamma(1)\Gamma(1)' \quad (4)$$

This relation suggests that we can identify matrix Γ_0 with an appropriate number of restrictions on the long-run covariance matrix of the structural form. Blanchard and Quah (1989) and Shapiro and Watson (1988) use long-run restrictions to identify shocks with $C(1)$ having full rank. King et al. (1991) work in a context where the rank of $C(1)$ is less than n_1 and they use cointegration restrictions.

Let us assume that the log of output is the first variable in the vector Z_{1t}. It is then equal to:

$$\Delta y_t = \mu_y + \Gamma_1^p(L)\eta_t^p + \Gamma_1^c(L)\eta_t^c \tag{5}$$

where η_t^p is the vector of permanent shocks affecting output and η_t^c is the vector containing shocks having only a transitory effect on output. Potential output based on the LRRO method is then:

$$\Delta y_t^p = \mu_y + \Gamma_1^p(L)\eta_t^p \tag{6}$$

Thus, “potential output” corresponds to the permanent component of output. The part of output due to transitory shocks is defined as the “output gap.” It is important to note that we do not talk in terms of “demand” or “supply” shocks as in Blanchard and Quah (1989), but simply in terms of permanent and transitory shocks.

2.2 Comparison with other multivariate methods

In this section, we examine the features of two alternatives to the LRRO approach: the multivariate Beveridge-Nelson decomposition (MBN) and Cochrane’s output-consumption decomposition (CO).[4]

The MBN decomposition defines potential output as the level of output that is reached after all transitory dynamics have worked themselves out. With reference to equation (2), where output is the first element of Z_t, we write the following decomposition:

$$\Delta y_t = \mu_y + C_1(1)\varepsilon_t + C_1{}^*(L)\varepsilon_t \tag{7}$$

Potential output is defined by the first two terms on the right-hand side of (7):

$$\Delta y_t^p = \mu_y + C_1(1)\varepsilon_t \tag{8}$$

4. See Cogley (1995) for another comparison of the MBN and CO methodologies.

Potential output is thus simply a random walk with drift.

Cochrane (1994) uses a two-variable VAR including GNP and consumption to identify the permanent and transitory components of GNP. The bivariate representation is augmented with lags of the ratio consumption to GNP. The permanent-income theory implies that consumption is a random walk (for a constant real interest rate). In addition, if we assume that GNP and consumption are cointegrated, then fluctuations in GNP with consumption unchanged must be perceived to be transitory. It is on this basis that Cochrane decomposes GNP into permanent and transitory components. To extract potential output, the errors of the VAR are orthogonalized so that consumption does not respond contemporaneously to GNP shocks.

Cochrane shows that if GNP and consumption are cointegrated and consumption is a random walk, identification based on the LRRO method and conventional orthogonalization (i.e., a Choleski decomposition) amounts to the same thing. Moreover, if consumption is a pure random walk, Cochrane's decomposition corresponds exactly to the Beveridge-Nelson decomposition. The moving-average representation of this process when the log of consumption is a random walk can be written as:

$$\Delta y_t = \mu_y + C(1)e_t + C^*(L)e_t \tag{9}$$

$$\Delta c_t = \mu_c + C_2(1)e_t \tag{10}$$

where $\mu_y = \mu_c$ and $C_1(1) = C_2(1)$. Potential output is defined as the first two terms on the right-hand side of the output equation, these terms being equal to the change in consumption. A multivariate Beveridge-Nelson decomposition including GNP and consumption would have the same form.

In order to compare the LRRO approach based on long-run restrictions with the CO and MBN approaches, let us first write structural form (2) in terms of the log of GNP (y_t) and the log of consumption (c_t) decomposed between permanent and transitory shocks (we assume that y_t and c_t are cointegrated):

$$\Delta y_t = \mu_y + \Gamma_y^p(1)\eta_t^p + \Gamma_y^{p*}(L)\eta_t^p + \Gamma_y^c(L)\eta_t^c \tag{11}$$

$$\Delta c_t = \mu_c + \Gamma_c^p(1)\eta_t^p + \Gamma_c^{p*}(L)\eta_t^p + \Gamma_c^c(L)\eta_t^c \qquad (12)$$

where $\Gamma^p(1)$ is the long-run multiplier of permanent shocks and $\Gamma_y^{p*}(L) = \Gamma^p(L) - \Gamma^p(1)$ is their transitory component. The MBN method considers only the first component of the permanent shocks plus the drift term, i.e., $\mu + \Gamma_y^p(1)\eta_t^p$. In this case, potential output is restricted to be a random walk. The LRRO approach is different in that it also includes the dynamics of permanent shocks ($\Gamma^{p*}(L)$).

With the CO approach, potential output is constrained to be a random walk to the extent that consumption is a random walk. Indeed, the validity of the permanent-income hypothesis would imply that the last two terms of the consumption equation are equal to zero and that $\Gamma_y^p(1) = \Gamma_c^p(1)$. It is not clear to what the CO decomposition corresponds if consumption is not a random walk.[5] Cochrane (1994) notes that the measure of potential output obtained on the basis of the CO method would be equivalent to the one obtained from the LRRO approach if the transitory effect of permanent shocks to GNP and consumption were exactly the same, i.e., if $\Gamma_y^{p*}(L) = \Gamma_c^{p*}(L)$ and $\Gamma_y^p(1) = \Gamma_c^p(1)$. However, these restrictive conditions are unlikely to occur in practice.

As pointed out by Lippi and Reichlin (1994), modelling the trend in output as a random walk is inconsistent with most economists' interpretation of productivity growth. Indeed, it is generally believed that technology shocks are absorbed gradually by the economy. Adjustment costs for capital and labour, learning and diffusion processes, habit formation, and time to build all imply richer dynamics than a random walk for these shocks. Working in a univariate framework, Lippi and Reichlin must constrain the dynamic of the trend to follow a particular shape (S-shape dynamic) in order to identify the trend and cyclical components. Again, a decisive advantage of the LRRO approach is that it lets the data determine the shape of the diffusion process of permanent shocks.[6]

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5. Stochastic growth models — such as in King et al. (1988) or King et al. (1991) — imply that the ratio of the log of GNP to the log of consumption is stationary but that consumption is not a random walk because the real interest rate is not constant. In these models, the transitory component of permanent shocks to consumption is not equal to zero. The LRRO decomposition is compatible with the prediction of these models.
6. Kuttner (1994) proposes a method based on the univariate unobserved stochastic trend decomposition of Watson (1986) augmented with a Phillips-curve equation. As with the Beveridge-Nelson decomposition, Kuttner's approach constrains potential output to follow a random-walk process.

One implication of defining potential output as a random walk with drift is that when the contemporary effect of a positive permanent shock is smaller (greater) than its long-run effect the output gap, defined as observed output minus potential, is negative (positive). For example, a positive technological shock whose short-term impact is smaller than its long-term impact will cause a transitory negative output gap. Many researchers and/or policymakers will find that this feature of the MBN and CO approaches (in the later case under the assumption that consumption is a random walk) reduces their attractiveness. It will often appear preferable to include the diffusion process associated with permanent shocks in potential output since the economy is likely to remain on its production possibility frontier as adjustments unfold. In many models used for policy analysis there will then be no reason for trend inflation to change.

3 Empirical results

In this section, we first present applications of the LRRO approach to U.S. data. Results are discussed and compared with those obtained from applications of the MBN and the CO approaches. We then compare the spectra of the output gaps estimated on the basis of these methodologies.

3.1 Applications to U.S. data

For our applications of the LRRO methodology we assume that output in first differences (Δy) follows a stationary stochastic process responding to two types of structural shocks: permanent (ε_P) and transitory (ε_T). The difference between the levels of output (y) and real consumption (c) is included in the estimated VARs to help capture the cyclical component of output. The use of this variable, which we suppose to be stationary, makes the comparison between the different methodologies easier. We also consider a case where we add a nominal variable to the information set as recommended by King et al (1991). In a vector form, the structural shocks and the variables used in the VARs can be expressed in the following way:

$$\eta_t = \begin{bmatrix} \varepsilon_P \\ \varepsilon_T \end{bmatrix} \text{ and } Z_t = \begin{bmatrix} \Delta y \\ y - c \end{bmatrix}$$

or

$$\eta_t = \begin{bmatrix} \varepsilon_P \\ \varepsilon_{T1} \\ \varepsilon_{T2} \end{bmatrix} \text{ and } Z_t = \begin{bmatrix} \Delta y \\ y - c \\ \Delta i \end{bmatrix}$$

We use quarterly data on U.S. real GDP and real consumption comprised of non-durables and services (the series used by Cochrane). We also use the federal funds rate when we add a third variable and assume that this series is non-stationary (it is in first differences in the VAR). Our sample extends from the first quarter of 1963 to the fourth quarter of 1995.

The autoregressive reduced-form VAR of the model is first estimated:

$$Z_t = \sum_{i=1}^{q} \Pi_i Z_{t-i} + e_t$$

with q the number of lags and e_t a vector of estimated residuals with $E(e_t e_t) = \Sigma$.

It is crucial that the estimated VARs include a sufficient number of lags. Indeed, Monte Carlo simulations by DeSerres and Guay (1995) show that using a lag structure that is too parsimonious can significantly bias the estimation of the structural components. These authors also find that information-based criteria, such as the Akaike and Schwarz criteria, tend to select an insufficient number of lags, while Wald or likelihood-ratio (LR) tests, using a general-to-specific approach, perform much better. Our LR test statistics selected four lags in the case of the bivariate system and eight lags in the case of the trivariate system.

The LRRO approach involves the identification of structural shocks (ε_t) from reduced-form shocks *(*e_t) and their variance. For this, we need to provide enough identifying restrictions to evaluate the elements in Γ_0. In the two-variable system, Γ_0 has four elements. Given that Σ is symmetric, we need to impose one additional restriction. The matrix of long-run effects of reduced-form shocks, $C(1)$, is related to the equivalent matrix of structural shocks, $\Gamma(1)$, as follows:

$$\Gamma(1) = C(1)\Gamma_0$$

where the matrix $C(1)$ is calculated from the estimated VAR. The additional identifying restriction, which is imposed on $\Gamma(1)$, is simply that shocks to potential output are the ones having a permanent impact on real GDP. In the trivariate case, we impose that $\Gamma(1)$ is triangular to identify the system. The two transitory components are considered as a single element because we are not interested in identifying them.

Charts 1 and 2 present the impulse responses of output to a permanent shock of one standard deviation in size in the bivariate and the trivariate cases. The horizontal axis represents the number of years. Confidence intervals were generated using Monte Carlo simulations in RATS with 1000 replications.

CHART 1: Response of real GDP in the bivariate case

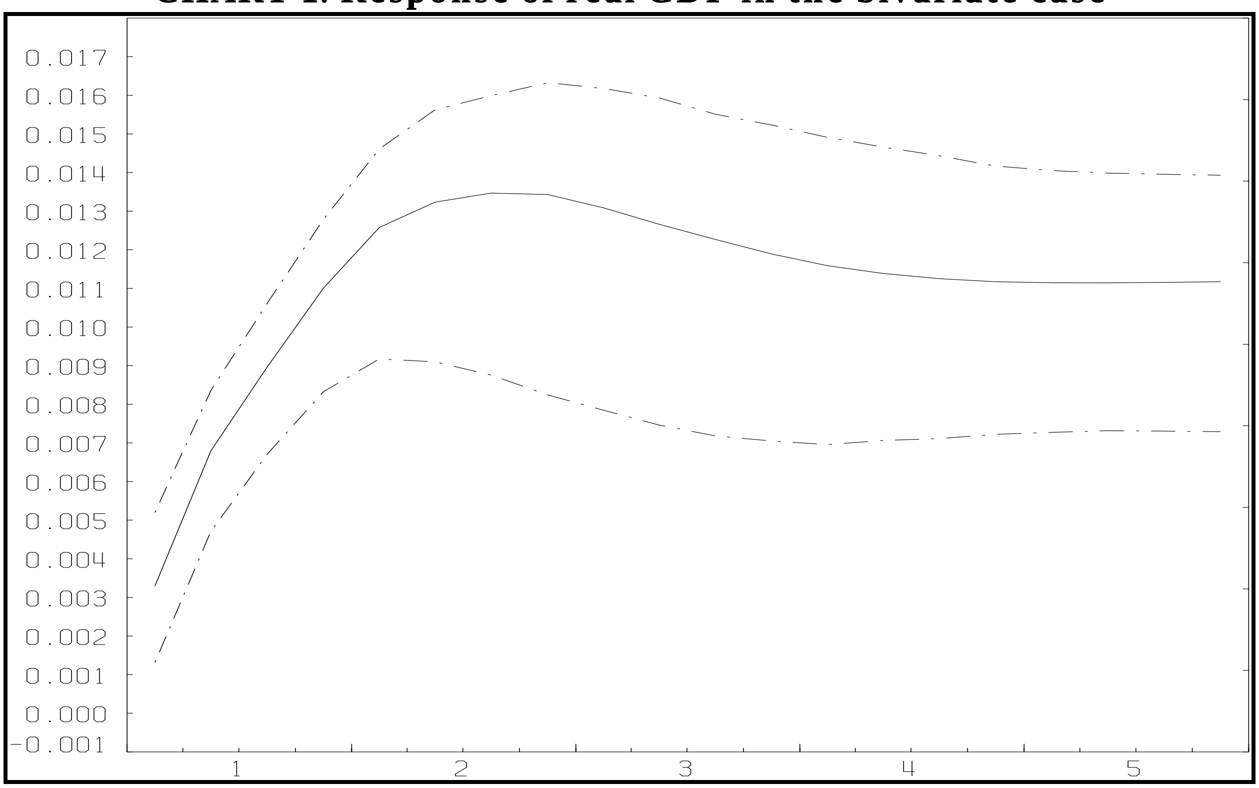
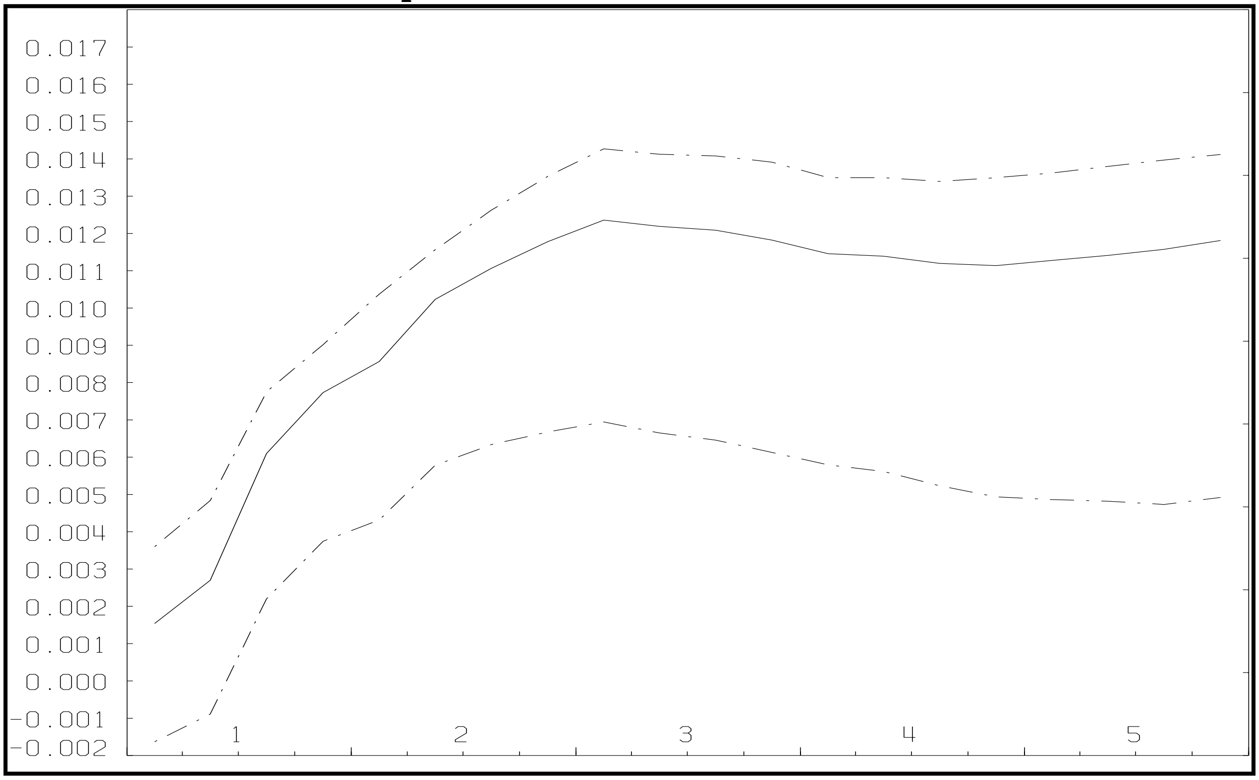


CHART 2: Response of real GDP in the trivariate case



The important message of Charts 1 and 2 is that permanent shocks are characterized by a statistically significant diffusion process, i.e., permanent shocks have a richer dynamic than the random walk.[7] As mentioned above, this could be due to factors such as adjustment costs on capital and labour, learning, habit formation, or time to build. Note that nothing constrains our estimation procedure to find such a diffusion process, i.e., this exercise can be seen as a test of the assumption that potential output follows a random-walk process. One implication of the rejection of that assumption is that methods that do not take into account the diffusion process of permanent shocks could miss an important part of potential output.

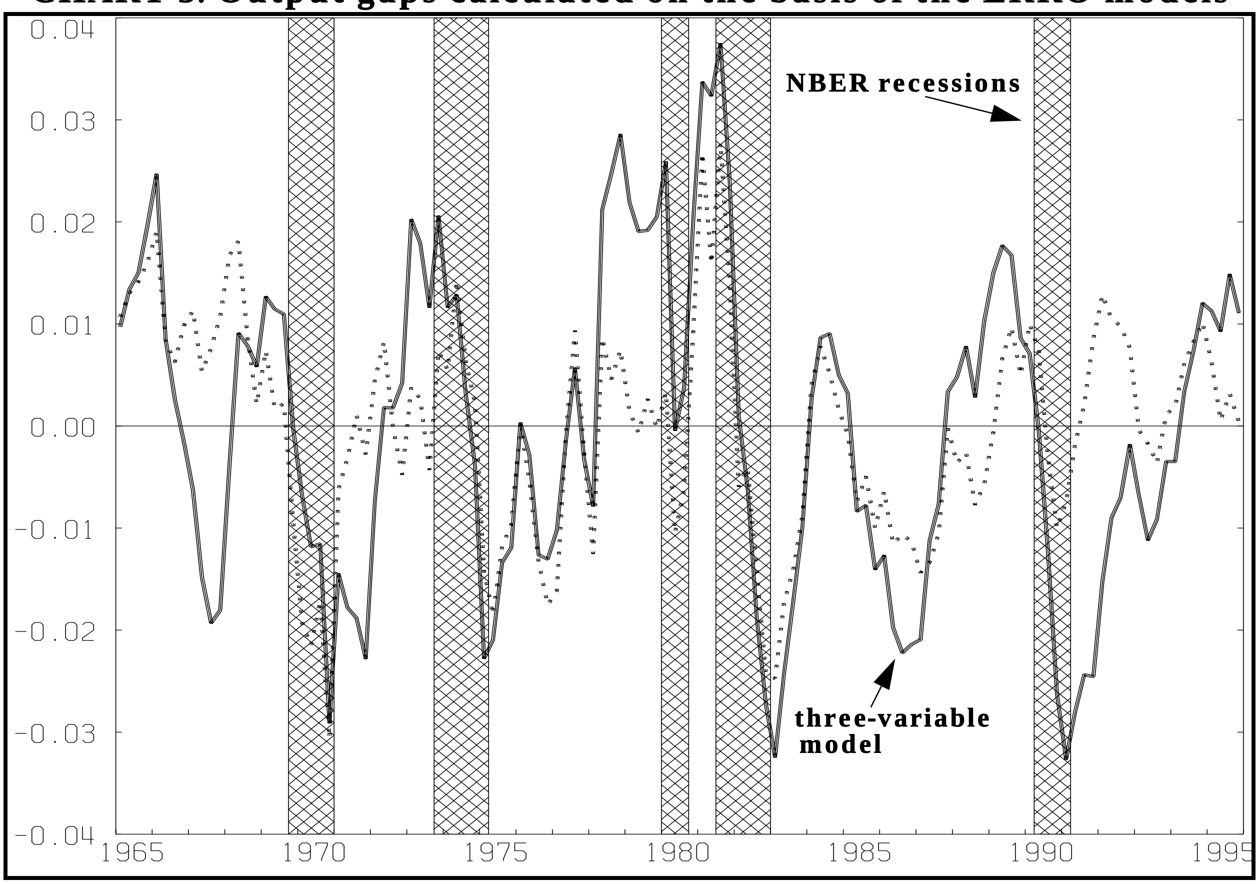
As mentioned in Section 2, models in which the permanent component of output is a random walk imply that the economy is below (above) potential in the transition period following a permanent positive (negative) shock to output. To the extent that the transition primarily reflects factors associated with an adjustment in the supply side of the economy, assuming that potential

7. Blanchard and Quah (1989) and Gali (1992), among others, report similar results.

output follows a random walk can be misleading. It could, in particular, provide misleading signals about the extent of inflationary pressures in the economy.

Chart 3 shows the output gaps calculated on the basis of the LRRO methodology in the bivariate and trivariate cases. (The NBER periods of recession are also shown on the chart.) The reported output gaps are clearly different. Indeed, the correlation between these series is only 0.66 (see Table 1). This suggests that interest rates are informative with respect to the transitory component of output. It is interesting to note that, while the bivariate model implies that the 1990 recession was driven mainly by permanent shocks, the trivariate application suggests that there was a large output gap at that time.

CHART 3: Output gaps calculated on the basis of the LRRO models



We tried money and inflation as the third variable, but interest rates was the only variable that brought a significant change to the results when compared with the bivariate case. Indeed, using interest rates with inflation or money gives results that are very similar to those obtained via using interest rates with $y - c$. Preliminary estimates also suggest that the difference between real GDP and real consumption of non-durables and services is not stationary in other countries (e.g., Canada, Germany, the United Kingdom). Thus, it may be difficult to generalize the LRRO specification in this paper to countries other than the United States. Again, preliminary estimates suggest that using output with interest rates and another nominal variable is more promising for these other countries.

Chart 4 shows the output gaps calculated on the basis of the LRRO, the MBN and the CO approaches. In the cases of the LRRO and MBN approaches, we present trivariate applications. The LRRO and MBN applications are based on the same estimated VARs so that only different identifying assumptions distinguish the two. For the MBN application, we simply assume that potential output corresponds to the accumulation of the long-run effect of reduced form shocks plus the drift in output. The application of the CO approach assumes that the output gap corresponds to the difference between the log of real GDP and the log of real consumption of non-durables and services. This supposes that the permanent-income hypothesis is a good approximation of the real DGP for consumption. As mentioned in Section 2, if the permanent-income hypothesis were not true, it is not clear to what the CO decomposition would correspond.

Table 1 shows that the cross-correlations between the output gaps calculated on the basis of the different approaches are relatively small.[8] The fact that the LRRO approach allows for a gradual diffusion of permanent shocks into potential output accounts for part of this. Nevertheless, the correlation between the output gaps identified on the basis of the CO and MBN methodologies is rather small, indicating that consumption may not be a random walk. This is consistent with results reported in Watson (1993), showing that the spectrum of the first difference of consumption has a peak at business-cycle frequencies. In Section 2, we noted that if consumption followed a random-walk process, the CO and MBN methodologies would give identical results.

8. The standard errors presented in parenthesis in the table are generated using the GMM method. This method does not fully take into account the uncertainty of the estimated VARs.

CHART 4: Output gaps calculated on the basis of the LRRO, CO, and MBN approaches

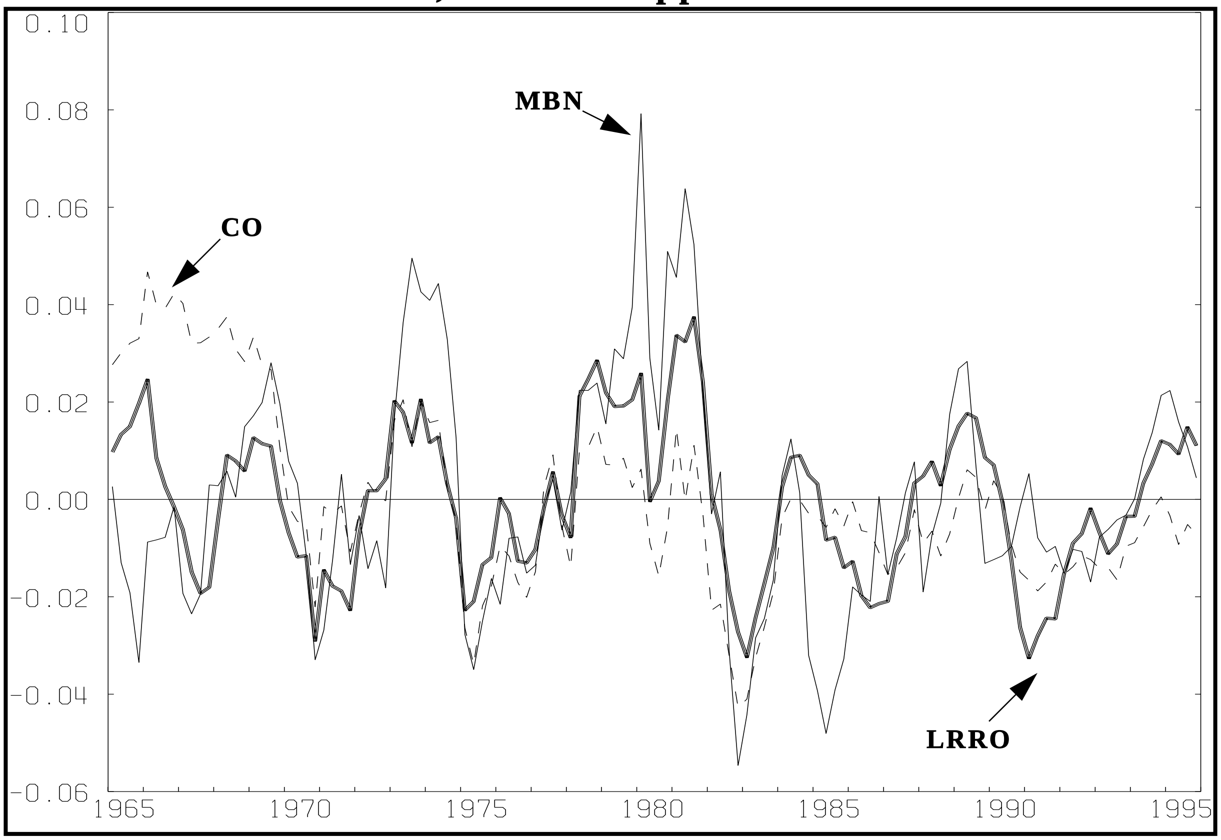
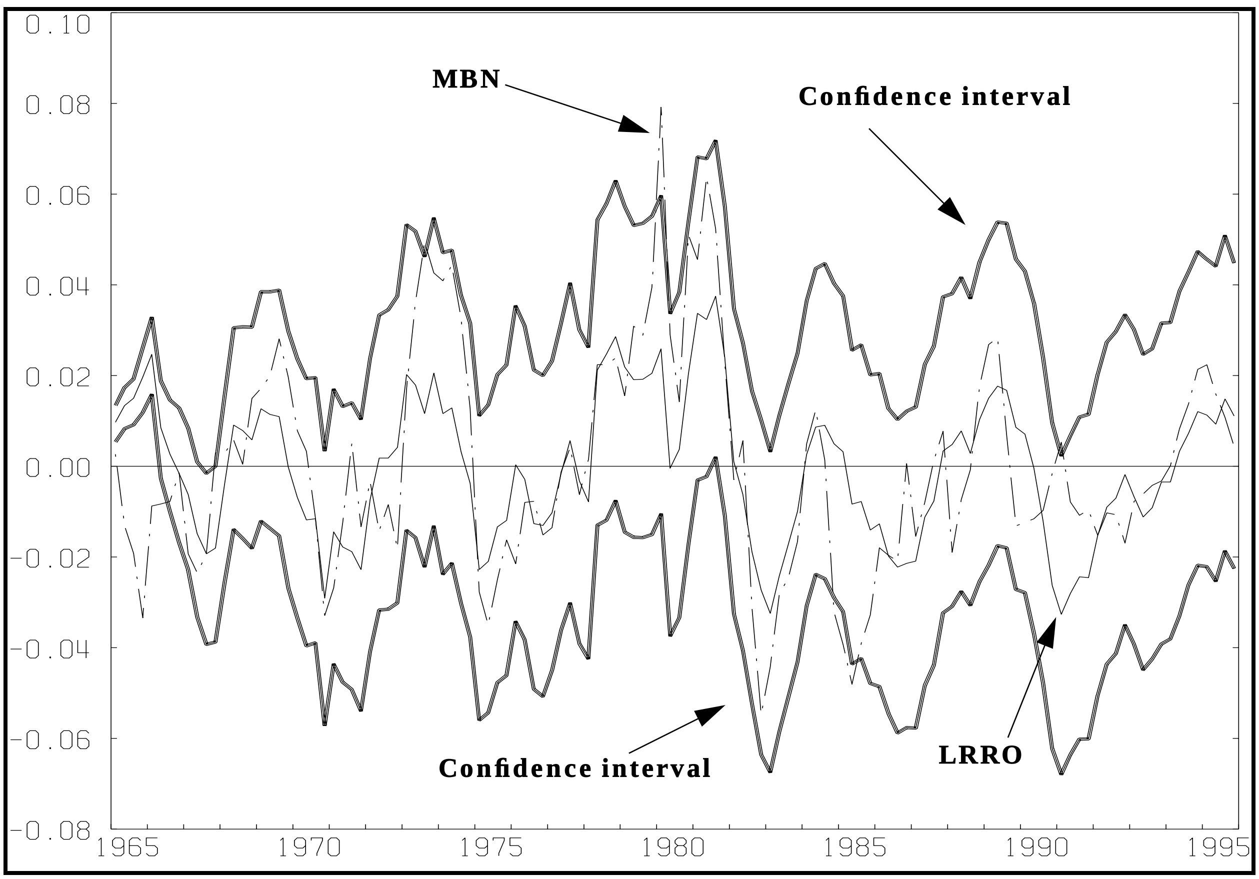


TABLE 1: Correlations between the output gaps
(Standard errors are shown in parenthesis)

	LRRO (2 variables)	LRRO (3 variables)	CO	MBN (2 variables)	MBN (3 variables)
LRRO (2 variables)	1	—	—	—	—
LRRO (3 variables)	0.66 (0.01)	1	—	—	—
CO	0.68 (0.01)	0.54 (0.12)	1	—	—
MBN (2 variables)	0.60 (0.10)	0.45 (0.14)	0.42 (0.12)	1	—
MBN (3 variables)	0.44 (0.12)	0.69 (0.08)	0.33 (0.16)	0.64 (0.07)	1

Although the correlation between the different output gaps appears to be relatively small, it might still be relatively difficult to discriminate among the different measures of the output gap for specific points in time. The monetary authority is, of course, interested in whether it is possible to discriminate among different methodologies for the estimation of the current output gap and in knowing how precise this estimation is. To answer that question, Chart 5 presents the estimation of the three-variable LRRO output gap together with its own 90 per cent confidence interval and the output gap consistent with the three-variable MBN methodology.

CHART 5: Uncertainty surrounding the estimation of the output gap



The main message of Chart 5, which would apply to other estimates of the output gap reviewed in this paper, is that there is a substantial amount of uncertainty surrounding the estimation of the output gap. Staiger, Stock and Watson (1996), using a different methodology, reach a similar conclusion concerning the estimation of the NAIRU. This uncertainty should probably be taken into account by policymakers who use the output gap to guide their decisions. It may be possible to reduce the uncertainty by taking into account other indicators or inflationary pressures, such as money growth and measures of wage pressures. Chart 5 also shows that it is difficult to distinguish statistically among the different methodologies when estimating the output gap for specific points in time. Note that one advantage of statistically based methods like the ones discussed in this paper is that, unlike mechanical filters, they reflect at least some of the uncertainty.

3.2 Spectra analysis

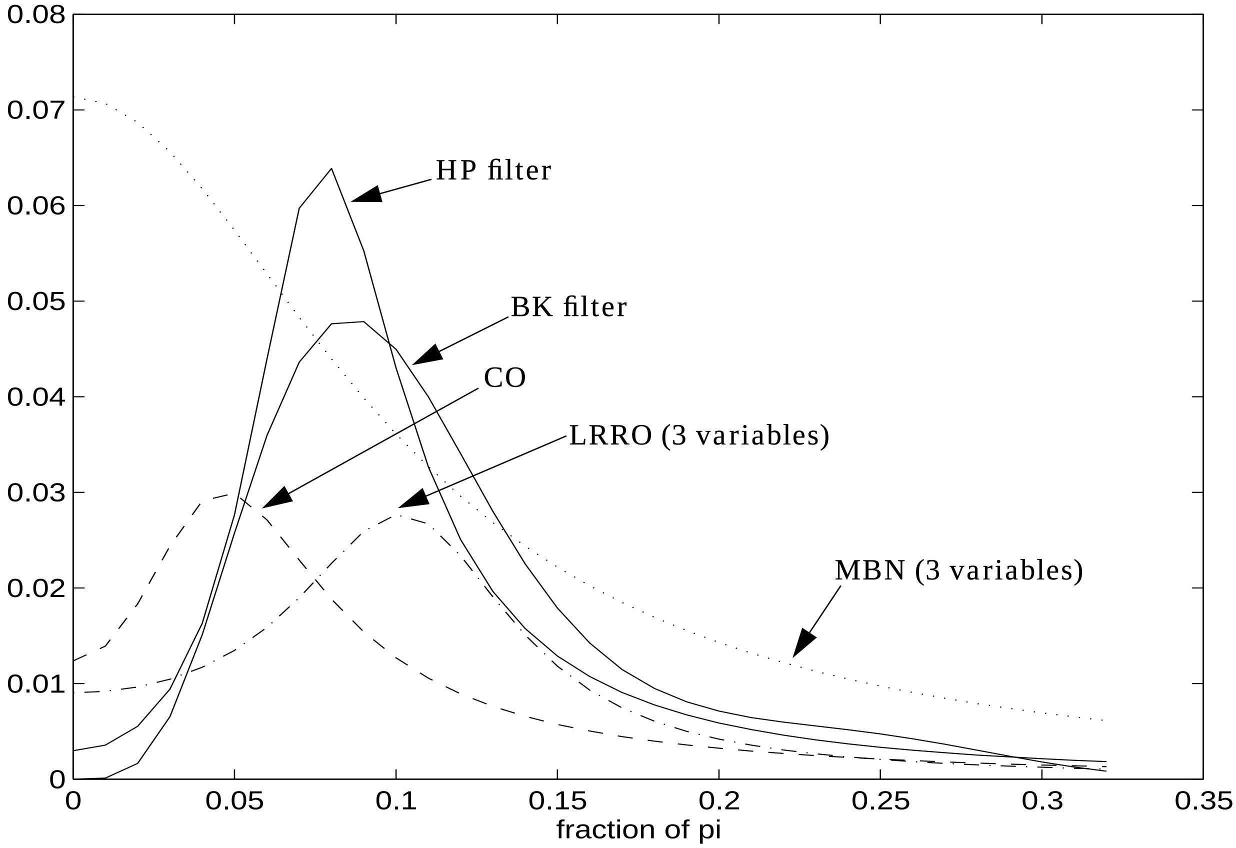
Chart 6 shows the estimated spectra of the CO, MBN (trivariate case) and LRRO (trivariate case) output gaps plus those resulting from the application of two mechanical filters: the Hodrick-Prescott filter (HP) and the band-pass filter (BK) proposed by Baxter and King (1995). Loosely speaking, the spectrum of a series is that series expressed as the integral of random periodic components that are mutually orthogonal. The total area below the spectrum corresponds to the variance of the series. The height of the spectrum at a given frequency shows to what extent cycles of a certain length contribute to the variance of a series. Consequently, the spectrum is an interesting tool for studying the properties of a time series in the frequency domain. We use a parametric estimator of the spectra for which ARMA processes were fitted.[9]

The spectra resulting from the LRRO, HP, and BK output gaps have their peak at business-cycle frequencies as defined by Burns and Mitchell (1946), i.e., frequencies corresponding to cycles lasting between 6 and 32 quarters. Indeed, the peaks in the spectrum of the HP, BK and LRRO gaps correspond to cycles lasting around 20 quarters. Note that the spectrum of the two-variable LRRO gap (not shown on the chart) is similar to that of the three-variable case. The spectrum of the CO gap has its peak at frequencies corresponding to 34 quarters, i.e., just outside business-cycle frequencies, and the peak of the MNB gap is at frequency zero. Indeed, the spectrum of the later has the typical Granger shape — see Granger (1966). The spectrum of the gap resulting from the two-variable MBN application (not shown on the graph) has the same shape as the three-variable

9. For an introduction to spectral analysis see Hamilton (1994). The order of these processes was determined on the basis of the Akaike criteria.

case, although with a lower peak and a smaller total variance. The latter result is not surprising, since it is well known that the MBN methodology gives a transitory component whose importance increases with the number of series used to identify it.

CHART 6: Spectra of the output gaps



Cogley and Nason (1995) and Harvey and Jaeger (1993) show that the HP filter amplifies business-cycle frequencies when compared with the first-difference of integrated or highly persistent time series. Guay and St-Amant (1996) extend this result to the BK filter but also note that, when compared with macroeconomic series in level terms, the HP and BK filters minimize the importance of business-cycle frequencies. Chart 6 shows that, when assessed against the transitory components (output gaps) obtained from the LRRO and CO methodologies, transitory components obtained from mechanical filters tend to amplify the importance of business-cycle frequencies. It is difficult to compare the spectra of MBN output gaps with the others.

4 Conclusions

In this paper, we compared different techniques that are used to measure potential output. We started with a brief explanation of why we think that mechanical filters such as the Hodrick-Prescott filter and the band-pass filter proposed by Baxter and King (1995) perform poorly in accomplishing this task. We then compared the LRRO approach based on long-run restrictions with two alternative multivariate approaches: the one proposed by Cochrane (1994) and the multivariate Beveridge-Nelson methodology. We argued that one advantage of the approach based on long-run restrictions is that it allows for estimated transitional dynamics following permanent shocks.

The applications considered in this paper are based on simple two-variable and three-variable VARs. These simple VARs are useful to illustrate the methodologies. We find evidence that there is a statistically significant gradual diffusion process associated with permanent shocks and that the output gap series estimated on the basis of LRRO approach, Cochrane's approach, and the multivariate Beveridge-Nelson methodology are different in the time and frequency domains. We note, in particular, that only the output gap associated with the LRRO approach has a peak at business-cycle frequencies as defined by Burns and Mitchell (1946), i.e., cycles lasting between 6 and 32 quarters. However, the estimates are imprecise for specific points in time and it appears difficult to distinguish between these methodologies in that context. This later result is consistent with the conclusions of Staiger, Stock and Watson (1996).

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