The Canadian Debt-Strategy Model

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- *The government's objective in the management of its domestic debt portfolio is to raise stable, low-cost funding for its operational needs. The Bank of Canada provides analysis and advice to the government regarding the management of this portfolio.*
- *To assist in this challenging task, a mathematical model was constructed by Bank staff for the consideration and comparison of alternative choices for financing this debt portfolio.*
- *A debt strategy depends on several factors, many of them unknown to, or not under the control of, the debt manager. Examples include the future behaviour of interest rates, the macroeconomy, and the government's fiscal policy.*
- *This article describes the basic features of the model and provides a detailed example of its key inputs and outputs.*

In its role as fiscal agent to the government, the Bank of Canada provides analysis and advice on decisions about the government's domestic debt portfolio. Debt-management decisions depend on assumptions about future interest rates, macroeconomic outcomes, and fiscal policy, yet when a debt-strategy decision is taken, none of these factors can be known with certainty. Moreover, the government has various financing options (i.e., treasury bills, nominal bonds, and inflation-linked bonds) to meet its objectives of minimizing debt-service charges while simultaneously ensuring a prudent risk profile and well-functioning government securities markets. Bank of Canada staff have therefore developed a mathematical model to assist in the decision-making process. This article describes the key aspects of the debt manager's challenge and the principal assumptions incorporated in the debt-strategy model, illustrated with specific results.

The Debt Manager's Challenge

The debt managers who are responsible for the government's financing strategy have the complex task of choosing a strategy that minimizes the cost of the debt portfolio within certain risk limits.[1] In any given year, a government must borrow to finance any excess of government expenditures over revenues as well as any maturing debt issued in previous periods. This borrowing requirement thus depends on past decisions regarding debt issuance and on the government's current surplus or deficit position. The government's position, in turn, depends on the general performance of the macroeconomy and on fiscal policy. The debt manager's challenge is to select a strategy for financing this borrowing requirement that meets the government's policy objectives.

1. In the Canadian context, this means that they determine the relative mix of nominal versus inflation-linked debt and the maturity composition of the debt stock.

This is not unlike the challenge faced by households, which must determine the proportion of mortgage, credit-card, and vendor-based debt to use in financing their borrowing needs stemming from any consumption or investment that exceeds income. The relative proportion of each type of debt should be selected to ensure the lowest possible financing costs, yet cost minimization is not the sole objective of the household. For example, the household may decide to lock in its mortgage for a lengthy (10-year) time horizon at an interest rate above that immediately offered by a floating-rate mortgage to ensure greater certainty in its financing costs. Elimination of uncertainty is essentially reducing risk. Thus, we can see that borrowing objectives, even for a household, relate to both cost and risk. Indeed, a tenet of financial economics is that there is invariably a cost associated with reducing risk.[2]

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Practically speaking, the situation is more complicated for a government than it is for a household. The incremental complexities include:

- The amount of borrowing required by a government is enormous. This implies that substantial changes in the amount borrowed in different financing options can affect market conditions and thus debt-service charges.
- The financing options available to the government are broader and more complex than those faced by a household, implying exposure to a range of interest rates.
- The interaction between government revenues and expenditures—the government's fiscal policy—is also more involved. Unlike a household, whose income typically stems from salary, government income is a complicated function of various taxation policies and macroeconomic conditions, while its expenditure programs also depend importantly on macroeconomic conditions.
- Another difference is that government borrowing is a repeated activity. It is consequently advantageous for the government to nurture deep and liquid markets for its debt in order to lower its costs over time rather than exploiting temporary market movements to minimize short-term costs.

2. In finance, the typical trade-off is between risk and return. These concepts apply equally to risk and cost. An increase in cost, for example, is equivalent to a lower return.

Describing the Model

Models, which are essentially mathematical representations of real-world phenomena, are frequently constructed in both the physical and social sciences. They help to understand and solve practical challenges by permitting the consideration and comparison of alternative choices. For the debt manager, this is the comparison of the cost and risk characteristics of alternative financing strategies. Although the comparisons provided by the model are quite valuable, they are not a replacement for judgment in decision making. An experienced debt manager has accumulated general situational knowledge, intuition, and judgment regarding alternative financing strategies and combines these qualitative factors with the model's quantitative output to select a debt-management strategy.

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The construction of any model requires an explicit description of the principal elements of the challenge. Often there are multiple ways to describe a given element, requiring the use of assumptions. It is important to understand that all models require assumptions. Moreover, in any model, the tension between complication and simplification must be addressed. The debt-strategy model attempts to balance these challenges in describing the following four aspects of the debt-manager's challenge:

(i) the financing strategy

(ii) uncertainty about the future

(iii) debt and fiscal mechanics, and

(iv) the set of policy objectives.

The remainder of this section addresses each of these aspects in turn to provide an overview of the general approach.

The financing strategy

The central component of the debt-strategy model is the financing strategy. Mathematically, a financing strategy is defined as a set of weights, summing to one, that describe the proportion of new issuance in each of the government's financing options, which currently include 3-, 6-, and 12-month treasury bills; 2-, 5-, 10-, and 30-year nominal coupon bonds; and 30-year inflation-linked bonds.[3] The model therefore includes eight separate weights or financing options, and the quantitative descriptions of the allocations to each permit an understanding of how different decisions affect the risk and cost characteristics of the portfolio. The financing strategy is also assumed to be constant through time, i.e., the allocations do not vary from year to year in search of short-term cost reductions. This foundation of the model permits a mathematically precise and succinct definition of a government financing strategy.[4] It also reflects the reality that the Canadian government does not typically alter its financing strategy dramatically from one year to the next.

Introducing uncertainty

If the debt manager knew the future path of the Canadian economy and interest rates with complete certainty, it would be relatively straightforward to determine the most advantageous financing strategy for the government. An absence of uncertainty implies an absence of risk; without risk, one would merely select the least-expensive financing strategy. Since this is not the case, the debt manager needs an approach that incorporates future macroeconomic and interest rate uncertainty in an organized manner. The debt-strategy model therefore assumes that the random evolution of the Canadian macroeconomy and interest rates can be summarized by a reduced-form statistical model whose parameters are estimated from historical data.[5]

The statistical model is not a single model, but rather a collection of approaches, since reliance on a single description of future uncertainty exposes the analysis to a misspecification of this statistical component.[6] Therefore, a wide range of alternative statistical models that each summarize the uncertainty policy-makers face in a slightly different manner have been implemented and evaluated. In each case, the macroeconomy is described by the output gap, inflation, and a monetary policy interest rate. Interest rates are assumed to depend on these macroeconomic quantities and a collection of term-structure-related variables. Differences in the models stem from alternative descriptions of the interaction between the key macroeconomic variables and interest rates, as well as the basic structure of the interest rate aspect of the statistical model.

Chart 1 summarizes the average evolution of inflation, the output gap, the monetary policy rate, and interest rates associated with 10,000 simulations from one of the statistical models estimated with monthly data from January 1994 to August 2007.[7] The solid line in each quadrant denotes the average outcome, while the dashed lines build a 95 per cent confidence interval around this average value. In short, our statistical model describes how key aspects of the debt-manager's challenge move randomly, and thus uncertainly, forward.

Debt and fiscal mechanics

The third component of the debt-management model uses these statistical inputs to illustrate the effect of a given financing strategy on key indicators. This requires a description of the mechanics of government debt and fiscal management. A large part of the debt-strategy model consists of mathematical expressions

3. See Boisvert and Harvey (1998) for more information on Canadian treasury bills; Côté et al. (1996) for more information on Canadian inflation-linked bonds; and Branion (1995) for a more detailed discussion of Canadian nominal coupon bonds.

4. More detail on this aspect of the model is found in Bolder (2003).

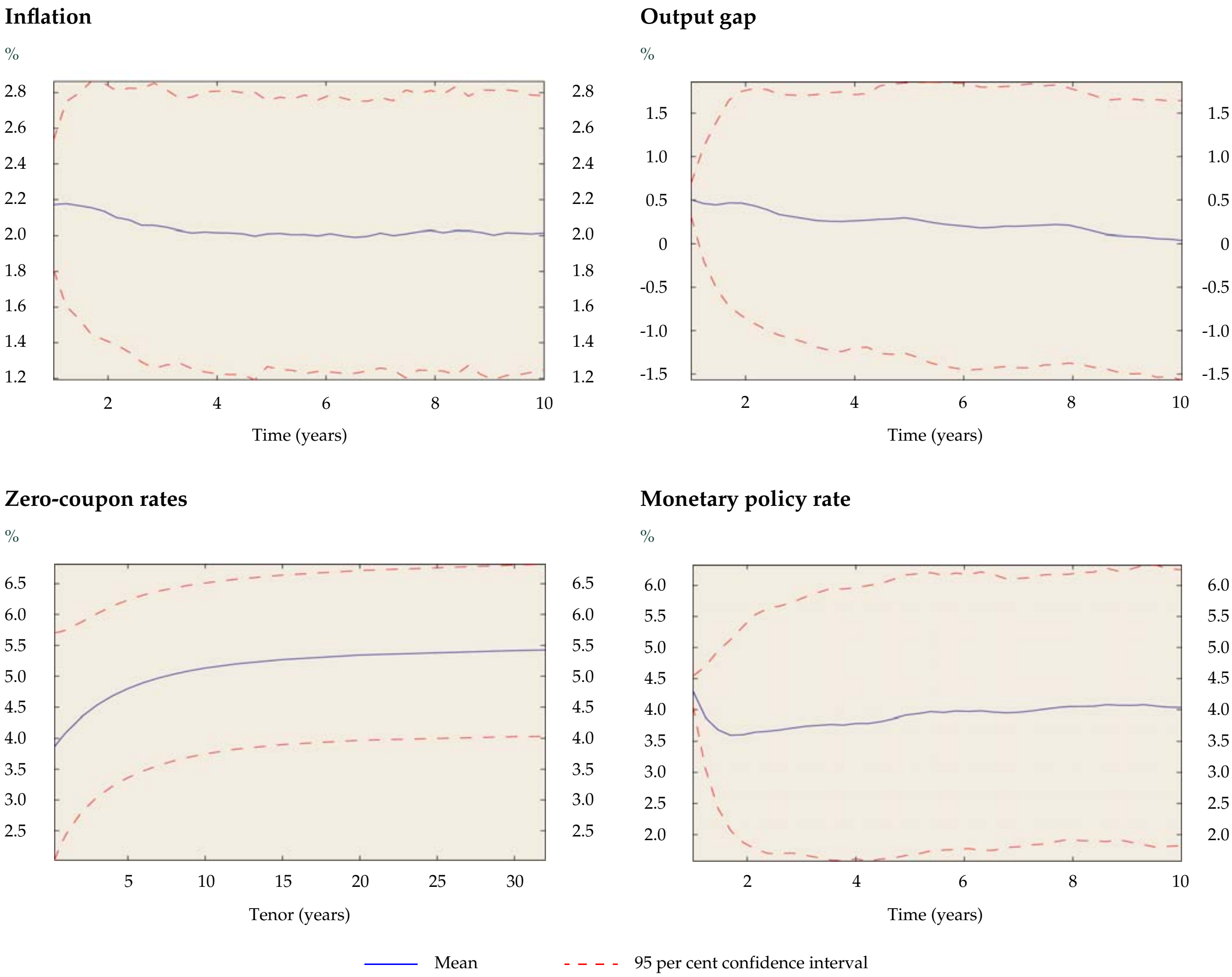
5. A sequence of Bank of Canada working papers describe the structure of the statistical model in substantial detail. See, for example, Bolder (2001, 2002, 2006) and Bolder and Liu (2007).

6. More formally, the ability to use alternative approaches helps to guard against what is termed "model risk" in our policy recommendations. Recent work–see Bolder and Romanyuk (2008)–also considers alternative statistical techniques for combining these various models into a single approach.

7. None of the charts in this article represent the actual data used in debt-strategy analysis, but illustrate a stylized analysis to describe the government's basic approach. The zero-coupon rates in the bottom left-hand quadrant of Chart 1 represent average borrowing rates for different terms to maturity (i.e., tenors) across the simulation horizon. This relationship is often referred to as the "term structure of interest rates."

Chart 1

The Debt-Strategy Model: Critical Inputs



that describe how the debt stock matures, how the government's annual borrowing requirement is computed, how the maturing debt and new borrowing requirement are financed, how debt charges are computed, and how these outcomes affect the size and composition of the debt stock. For a given financing strategy and a single realization of future macroeconomic and interest rate outcomes, each of these quantities can be computed.

Since this is the heart of the model, it merits a bit more description. We need three basic inputs to run the model. First, we require the existing federal debt stock: the amount of treasury bills, nominal bonds, and inflation-linked bonds.[8] Second, we need a sequence of future macroeconomic and interest rate outcomes from the statistical model. Finally, we need a financing strategy. From the debt stock, we determine a sequence of known maturities into the future.

8. Retail debt, non-market debt, and foreign-denominated debt are also included in the model, but only in a deterministic manner. The incremental complexity of modelling these relatively small parts of the government's debt stock is not offset by the incremental benefit of including them. Note that a separate modelling framework is used for decisions on the government's foreign exchange reserves, which is the source of the Canadian government's foreign-denominated debt stock.

In the first period, we compute the government's funding requirement (i.e., surplus or deficit), which will depend on the state of the macroeconomy in that period. Adding in the maturing debt from previous periods provides us with the amount of debt that must be issued in the first period. The financing strategy determines how this amount will be issued and the implications for the debt stock. Once the amount and composition of issuance is determined, we compute the debt-service charges for the first period, which will depend on current and past interest rates. This sequence of steps is repeated for the second period, although it is slightly more complicated, since the outcome of the second period will depend on that of the first period. The sequence is repeated iteratively for each period across the time horizon. Chart 2 provides a schematic overview of the algorithm.

These repeated calculations assess the performance of a given financing strategy under an enormous number of possible macroeconomic and interest rate outcomes.

Thus far, we have only described the computation of debt quantities for a single realization of the statistical model. This provides little or no insight into the uncertainty faced by the debt manager. The solution is to repeat the previous analysis, for the same fixed financing strategy, many thousands of times to construct a statistical distribution of debt quantities associated with a given financing strategy. Conceptually, these repeated calculations (known as stochastic, or Monte Carlo simulations) assess the performance of a given financing strategy under an enormous number of possible future macroeconomic and interest rate outcomes, which are consistent within specific models. Comparisons among different financing strategies are essentially comparisons between different aspects of these distributions. To illustrate, we provide a simple example of the debt-strategy stochastic-simulation model. We took a recent actual debt portfolio and applied a financing strategy composed of equal amounts in each of the available financing instruments.[9] Chart 3 provides an overview—across the 10,000 simulations summarized in Chart 1—of the debt-service charges, the government's surplus or deficit position, the size of the debt stock, and government revenues less expenditures over the course of a 10-year

9. This is the so-called 1/N approach.

Chart 2

The Debt-Strategy Model: Stochastic-Simulation Algorithm

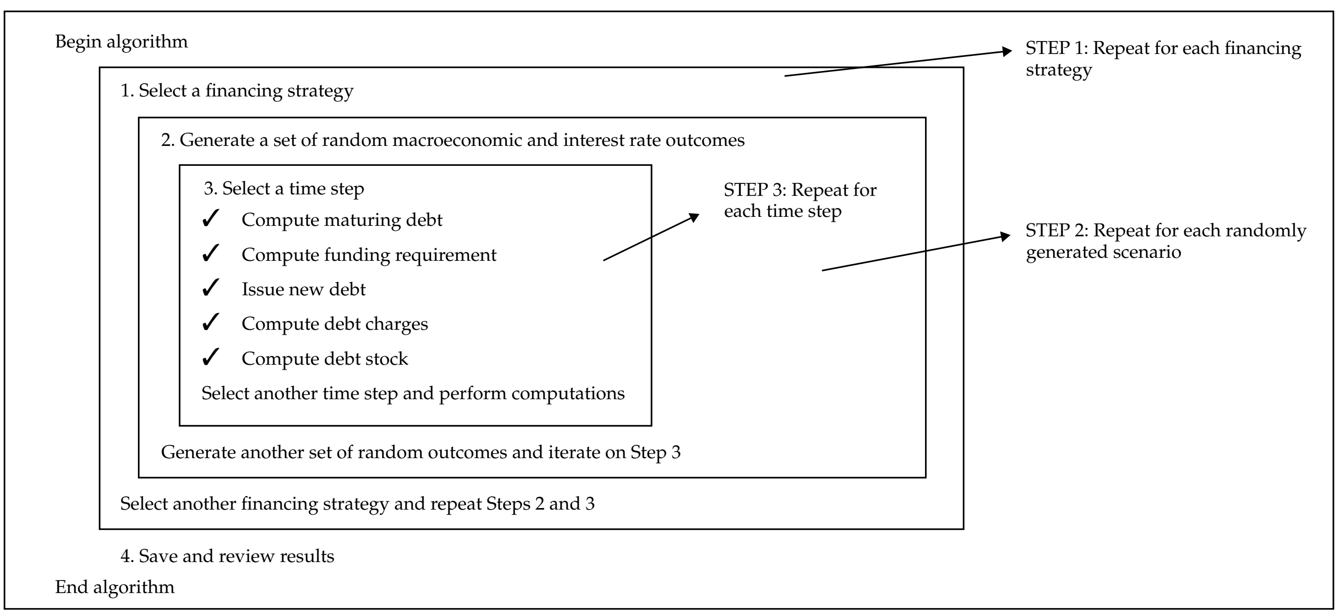
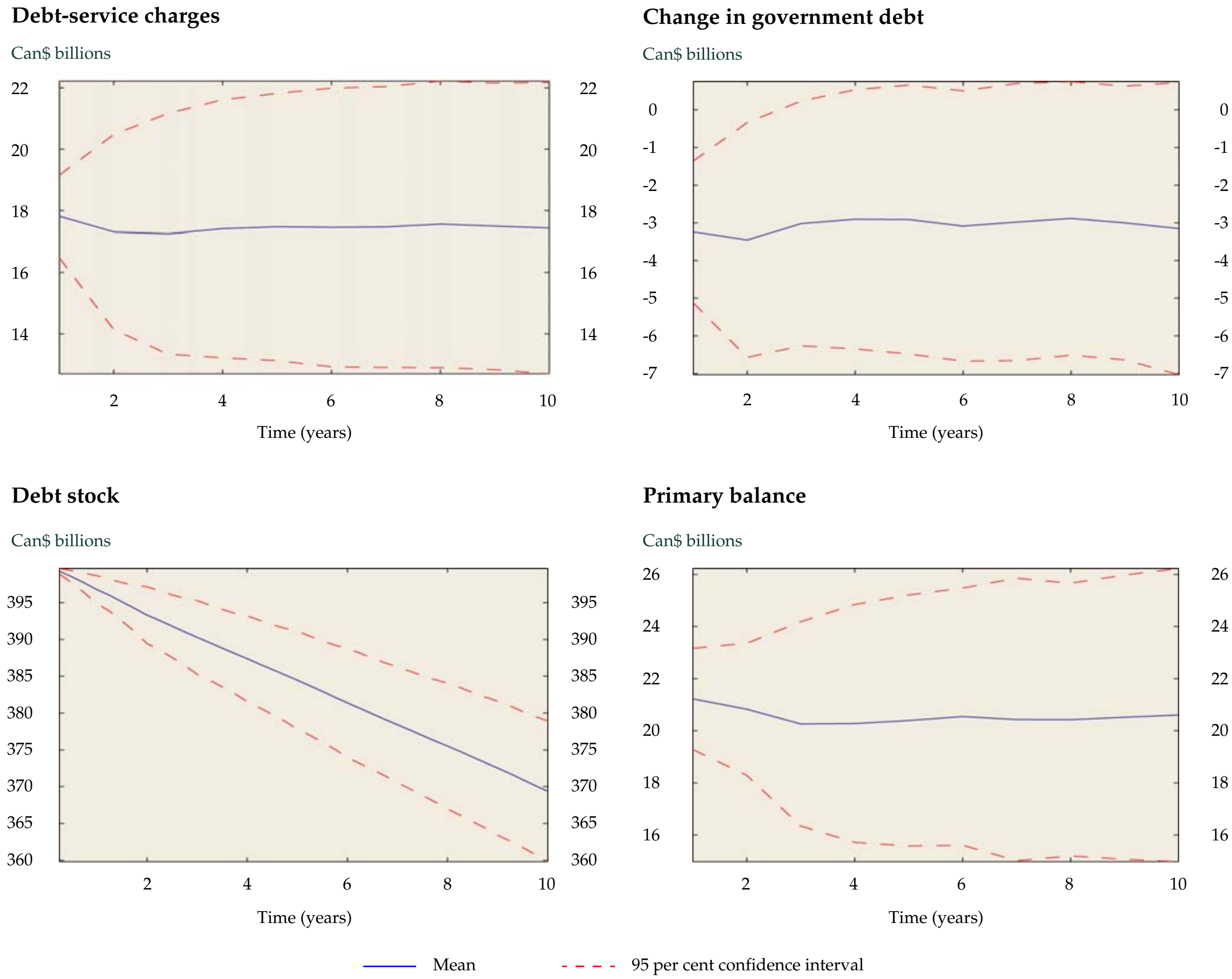


Chart 3

The Debt-Strategy Model: Critical Outputs



time horizon. Observe that each quantity is surrounded by a 95 per cent confidence interval that describes the uncertainty about their future evolution. The model can provide much more detailed information about these specific aspects of the debt manager's challenge, but Chart 3 simply provides a general sense of the model's principal outputs to illustrate how, for any choice of financing strategy, it supplies a rich description of the key elements of the government's debt strategy and a quantification of their relative uncertainty.[10]

The government's deficit or surplus position, what might be termed "fiscal mechanics," depends on a

10. In particular, the model computes portfolio summary measures such as the fixed-debt ratio, the average-term-to-maturity, and the duration, as well as issuance amounts in each financing option. The model also includes a number of different measures of cost and risk associated with a given strategy.

number of elements: the government's fiscal policy, its financing strategy, and the evolution of the macroeconomy. In the model, we ignore the non-cash items that generally contribute to a difference between the cash-based borrowing requirement and the government's budgetary position so that the change in the government's debt stock equals the government's annual deficit for that year. We assume a specific relationship between government revenues and non-interest expenditures (often referred to as the government's "primary balance") and the macroeconomy. Using this relationship, government requirements are computed as debt-service charges minus the primary balance.

The model also permits the government to target its annual debt paydown, implying that it will usually generate a surplus sufficient to meet the target. Chart 3 demonstrates surplus and deficit results for an assumed Can$3 billion debt paydown. On average, the Can$3 billion paydown is met, but the actual paydown is sometimes greater or less than this amount. A final assumption regarding debt and fiscal mechanics is that we do not place *a priori* bounds on the financing strategies; the evolution of the macroeconomy does not depend on the government's financing strategy.[11] We could, for example, consider an extreme financing strategy composed entirely of 3-month treasury bills. With a debt stock of Can$400 billion, this would amount to approximately Can$1.2 trillion of annual treasury bill issuance. To avoid such extreme outcomes, which may be hard for the market to digest, the model includes a price adjustment for excessively large or small issuance. We assume that if issuance falls within the benchmark target ranges defined by the government, it can be issued at prevailing market prices described by the statistical model. If issuance falls below or rises above these target ranges, then the financing cost generally increases in a non-linear fashion.[12] This is essentially a function that penalizes excessively small or large issuance in a given financing instrument and therefore more realistically characterizes the potential cost associated with extreme financing strategies.[13]

Policy objectives

The final aspect of the debt model relates to the government's policy objectives. Operationally, this refers to how we define risk. Debt management has traditionally attempted to evaluate the trade-off between the levels of, and the uncertainty about, debt-service charges.[14] Uncertainty about future financing costs is therefore how we typically define risk in this setting. We can also define risk in another way. The selection of a portfolio that minimizes budgetary uncertainty while also considering the level and volatility of debt costs is useful in considering debt management in the context of fiscal policy. Greater budgetary certainty would allow for a smoother tax profile and a larger proportion of permanent, as opposed to temporary, expenditure initiatives. Both notions of risk are factors in the government decision-making process.

Uncertainty about future financing costs is how we typically define risk in this setting.

Policy objectives are incorporated into the analysis in three steps. First, we define a set of policy objectives for the debt-strategy decision. Second, we determine what measure, from the debt-management model, best describes the attainment of each objective. Finally, we consider a wide range of financing strategies and select the one that best achieves the measures related to the policy objectives. In a simple example where the government wishes only to minimize the cost of debt

11. The financing strategy is assumed to depend on the macroeconomy, but we make the simplifying assumption that the macroeconomy does not depend on the financing strategy.

12. For some instruments, such as 3-month treasury bills and inflation-linked bonds, the borrowing costs could actually fall as issuance decreases because of inelastic demand for these instruments and the lack of acceptable substitutes.

13. Determining the parameters for this penalty function is far from obvious. The idea is not to be precisely correct, but rather, generally reasonable. We currently use conservative values determined through consensus discussion, examination of past experiences where applicable, and consideration of government securities markets in other countries.

14. The generally upward-sloping nature of the yield curve implies that, on average, nominal short-term debt is less expensive. Since nominal short-term interest rates are more volatile than their long-term counterparts, we typically have to be prepared to accept higher uncertainty for lower nominal debt charges. This relationship is less obvious when considering inflation-linked debt.

issuance, a reasonable measure of the policy objective would be the average debt charges over the simulation horizon. We could then examine a large set of financing strategies and select the one with the smallest average debt charges over the simulation horizon. Since the government's actual policy objectives are much richer than suggested by this, finding the specific strategy that best meets a set of policy objectives can be considerably more complicated. For this reason, an optimization module was developed that provides significant flexibility in defining the government policy objectives in various forms.[15] For example, we might want to minimize the cost of the debt portfolio with constraints on the volatility of debt-service charges and the amount issued in various financing options. While it is possible to use the debt-strategy model without the optimization module, the module is a useful tool, given the complexity of the challenge.

In the next section, we will examine how we use the debt-strategy model in conjunction with the optimization module to provide insight into the debt manager's challenge of minimizing cost with some restriction, or constraint, on the amount of portfolio risk.

Using the Model

The main objectives of the Canadian government with respect to the domestic-debt portfolio "is to raise stable and low-cost funding to meet the operational needs of the government. An associated objective is to maintain a well-functioning Government of Canada securities market" (Department of Finance Canada 2007).

Risk is the mirror image of stability. Requiring the level of risk to be *less* than some amount is equivalent to requiring the level of stability to be *greater* than some amount. The concepts of cost and risk are therefore clear from this quotation, but they need to be made operational for modelling purposes. Given that cost and risk have a variety of dimensions, there are several possible specifications for the government's operational objectives for debt management.

Defining cost and risk

Dealing with portfolio cost is relatively straightforward. In this analysis, we use as our measure of cost the average annual debt-service charges as a percentage of the total debt stock over the 10-year simulation horizon. A key advantage of expressing cost as a percentage is that it remains stable even when the size of the debt changes over the period of analysis.[16] Cost can be represented in other ways, but the model results are generally robust to the choice of measure. The definition of risk, however, is somewhat more complex. Two issues must be addressed. First, as previously discussed, it depends on the policy objectives of the government. In particular, how is risk defined—in terms of the volatility of the debt-service charge or of budgetary outcomes? Because of the impact of debt-service charges on the government's budget, these perspectives are related. Second, it is necessary to consider what type of risk most concerns the debt manager: the average volatility of the debt-service-charges, or a more extreme notion of debt-service-charge uncertainty.

It is necessary to consider what type of risk most concerns the debt manager.

These are not easy questions. We will not attempt to answer them here, but instead will provide results representing each of four different possibilities. We will examine the differences in optimal portfolios where the government seeks to minimize debt-service charges with constraints on average and extreme debt-service charges and budgetary risk. These average and extreme risk measures are different aspects of the statistical distributions generated by the debt-strategy model. In this analysis, the average measure of the debt-service charge and of budgetary risk is the conditional standard deviation of the debt-service charge and budgetary distributions outlined in Chart 3. The conditional standard deviation (often called conditional volatility) summarizes the average 1-year uncertainty regarding the debt-service charge and budgetary risk over the entire 10-year horizon, i.e., the average risk characteristics of a financing strategy

15. The optimization module is mathematically involved and beyond the scope of this article. Interested readers are directed to Bolder and Rubin (2007).

16. Given that we assume a positive debt paydown each year, the domestic-debt stock is decreasing, on average.

under normal circumstances.[17] Thus, this analysis defines average debt-service charge and budgetary risk as the conditional debt-service charge and budgetary volatility over the 10-year horizon, respectively.

Extreme outcomes are also of interest to debt managers. To understand them, we must examine the tails of the previously illustrated distributions.[18] The principal measure we use is Cost-at-Risk (CaR). There are two different types of CaR: absolute and relative. Absolute CaR is the worst-case debt-service charge expected with a certain degree of probability. Or, as a percentile measure, a 95 per cent CaR indicates that, for 95 per cent of the time, the government will not pay more than this amount in debt-service charges. Relative CaR, the measure we will use in our extreme analysis, is the difference between absolute CaR and the mean of the distribution. If we use the mean of the debt-service charge (or budgetary) distribution for planning, then relative CaR tells you that with, for example, 95 per cent probability, your worst-case outcome will not exceed your planned outcome by more than this amount. We use the same concepts for Budget-at-Risk (BaR). Thus, a 95 per cent absolute BaR indicates that, for 95 per cent of the time, the government's actual deficit or surplus position will not be worse than this amount. We thus define extreme debt-service charge and budgetary risk as the average relative CaR and relative BaR over the 10-year time horizon, respectively.

Optimal portfolios

We can now illustrate the results of using the optimization module to identify optimal portfolios. In this analysis, we define optimal by assuming that the government wishes to minimize the percentage cost of the debt over the next 10 years—already defined as *low cost*—subject to a single risk constraint. We explore four risk constraints: average debt-service-charge and budgetary risk and extreme debt-service charge and budgetary risk. We also examine constraint levels for each of these risk definitions and compare the results.

Before presenting the results, some clarification is necessary. First, there are a variety of stochastic models from which to choose. In this analysis, we use the Diebold and Li (2003) approach applied to the Nelson and Siegel (1987) model, as described in Bolder (2006). We have forced the long-term inflationary mean to be 2 per cent and the long-term output gap to be zero. Second, the financing strategies include 3-, 6-, and 12-month treasury bills, 2-, 5-, 10-, and 30-year nominal bonds, and 30-year inflation-linked bonds. Finally, we assume an annual debt-paydown target of Can$3 billion.

Each quadrant in Chart 4 represents one of our definitions of risk. Conceptually, the optimizer identifies the portfolio weights that provide the lowest possible debt-service charges while respecting the risk constraint. In each quadrant, the horizontal axis represents the resulting levels of risk; the further to the left we move, the lower the risk. The vertical axis denotes the proportion of the portfolio allocated to each of the eight financing options; these portfolio weights denote the financing strategy that produces a given level of risk. In all cases, the weights sum to one. The focus of this analysis is to understand how the composition of the portfolio changes as we relax the risk constraint.

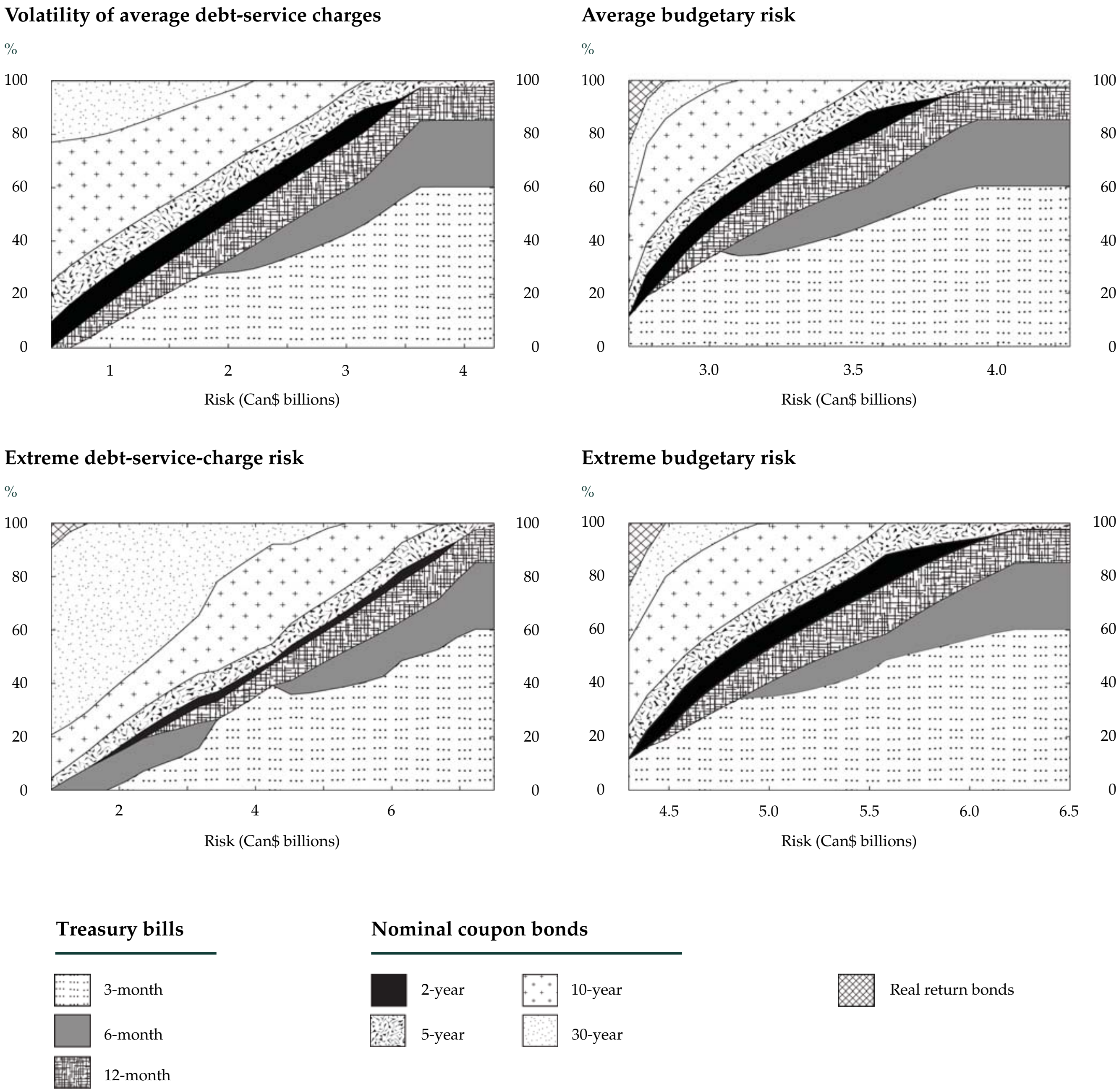
The top left-hand quadrant in Chart 4 provides the optimal financing strategies for the volatility of the average debt-service charge, ranging from Can$500 million to Can$4.25 billion. The easiest way to read this quadrant is from left to right on the horizontal axis. To achieve the lowest level of the average debt-service-charge risk (i.e., a constraint of Can$500 million), the portfolio weights require the following allocations to nominal bonds: about 23 per cent 30-year, 52 per cent 10-year, 15 per cent 5-year, and 10 per cent 2-year. As the risk constraint is relaxed, however, the 30- and 10-year bonds are replaced with 2-year bonds and treasury bills. Beyond a risk level of approximately Can$3.5 billion, the portfolio weights settle down at about 90 per cent treasury bills, with the majority allocated to 3- and 6-month tenors, as well as a small allocation to 5-year bonds. This trend reflects two empirical facts: Long-term debt is refinanced much less frequently than short-term debt; and short-term interest rates are generally more volatile than long-term rates. More frequent refinancing of short-term debt therefore exposes the government to greater variability in their debt-service charges. Consequently, financing strategies with larger proportions of long-term debt are typically more stable.

17. A formal definition of conditional volatility and how it is computed is found in Bolder (2003).

18. The tail of a distribution describes those outcomes that are far from the mean and generally occur with low probability.

Chart 4

Optimization Module: Optimal Portfolio Weights



The bottom left-hand quadrant in Chart 4 provides the same analysis for extreme debt-service-charge volatility. The trend, whereby lower levels of risk include larger proportions of longer-term debt, is also evident. Nevertheless, the proportion of long-term debt required to attain the lowest extreme risk level is substantially higher. To obtain Can$1 billion in extreme debt-service-charge risk, approximately 70 per cent of the portfolio must be allocated to 30-year nominal bonds, 16 per cent to 10-year nominal bonds, and the remainder to 30-year inflation-linked bonds and 5-year nominal bonds. This suggests that the optimal debt portfolio can vary significantly, depending on whether we attempt to avoid average or extreme definitions of risk. The remaining quadrants in Chart 4 illustrate the optimal portfolio allocations for average and extreme budgetary risk. These results are, again, quite different from those found when focusing on debt-service-charge notions of risk. The top right-hand quadrant addresses the average budgetary definition of risk. The lowest risk portfolio, with a risk of Can$2.7 billion, allocates approximately 75 per cent of the portfolio more or less equally among 10-year nominal, 30-year inflation-linked, and 30-year nominal debt, with the remainder approximately evenly split between 3-month treasury bills and 5-year nominal bonds.

The optimal debt portfolio can vary significantly, depending on whether we attempt to avoid average or extreme definitions of risk.

As the risk constraint is relaxed, the optimal portfolio weights shift to include significant proportions of 10-year bonds and treasury bills. Beyond approximately Can$3.75 billion, however, the portfolio requires 5 per cent in 5-year bonds, with the remaining 95 per cent in treasury bills—most of this, about 60 per cent, is allocated to 3-month treasury bills. Interestingly, the portfolio allocations are essentially the same for both the average and extreme definitions of budgetary risk.

It is evident from this analysis that alternative definitions of risk lead to alternative portfolio allocations. A focus on budgetary risk, for example, involves substantially larger allocations to inflation-linked bonds and treasury bills relative to the definition of debt-service-charge risk. This difference relates to the relationship among government revenues, expenditures, and debt-service charges. Remember that the government's budgetary position is defined as the government's primary balance less debt-service charges. The uncertainty of the government's budgetary balance depends upon the variability of its fiscal policy (i.e., revenues less expenditures), the variability of debt-service charges, and the interaction between these two quantities. Clearly, the notion of budgetary risk simultaneously subsumes and is more complicated than that of debt-service-charge risk.

Inflation-linked bonds and treasury bills play a larger role in controlling budgetary risk because they generate debt-service charges with the necessary interaction between the government's fiscal policy and the financing strategy to obtain the lowest level of budgetary risk.[19] Note, however, that the debt manager's capacity to influence budgetary risk is significantly less than their ability to affect debt-service charges. This is evident when we examine the relative minimum levels of risk attainable in the definitions of debt-service charge and budgetary risk. This should not be surprising, since, as previously mentioned, budgetary volatility depends on uncertainty in government revenues and expenditures, which are largely beyond the debt manager's control.

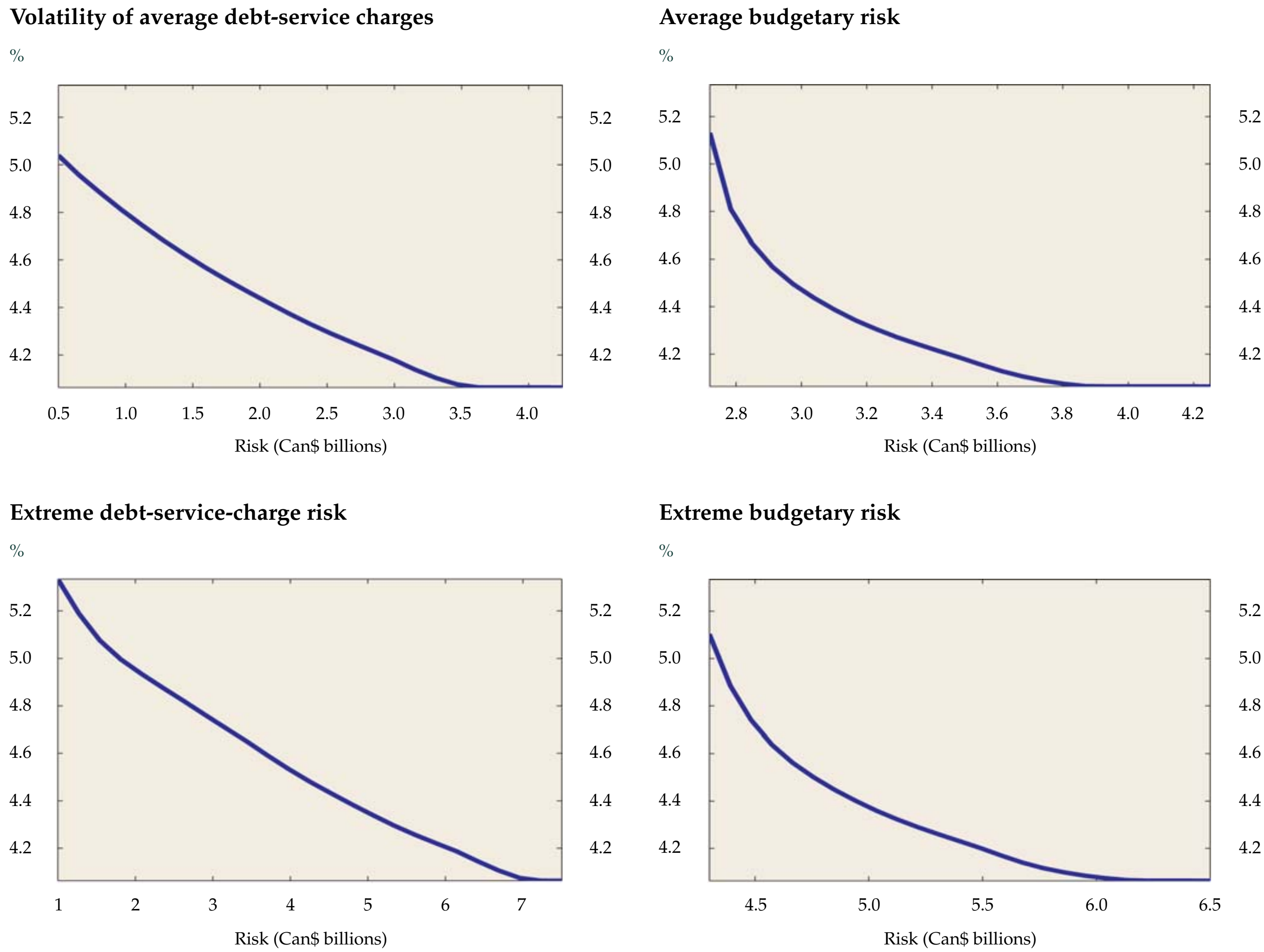
Risk and cost trade-offs

In addition to the portfolio composition for different definitions and levels of risk, we also need to know the cost characteristics associated with each (Chart 5). Each quadrant in Chart 5 corresponds to one in Chart 4, and the horizontal axis is identical for each quadrant in the two charts. In Chart 5 however, the vertical axis represents the expected cost of the portfolio, in percentage terms, relative to the level of risk. The stylized results in Chart 5 depend importantly on the assumptions made about the average shape of the term structure of interest rates over the analysis horizon.

19. The idea is that portfolios with larger proportions of treasury bills and inflation-linked debt have debt-service charges that are typically larger (smaller) when the primary balance is also large (small) and acts like a partial hedge to offset budgetary uncertainty. This situation arises from the (typically) positive relationship between short-term interest rates, inflation, and output.

Chart 5

Optimization Module: Risk-Cost Trade-Offs



The shape of the curves in Chart 5 is consistent with what is predicted in finance theory; i.e., the reduction of risk in the government's debt portfolio is not free. It comes at the price of increased debt-service charges. These are merely illustrative results, however. While the basic results remain the same, the slope of the curves will vary, depending on which statistical model is selected and the time period used to estimate the model's parameters.

We can still draw some basic conclusions from these risk-cost trade-offs. In the top left-hand quadrant of Chart 5, as the risk constraint for the average debt-service charge is eased—from Can$500 million to Can$4.25 billion—the expected cost falls by approximately 100 basis points.[20] A similar pattern is found for each of the four alternative definitions of risk, with a few differences. First, the differential between the cost of the lowest and highest risk portfolio is approximately 100 basis points for all of the risk definitions except extreme debt-charge risk, where the differential is almost 130 basis points. This suggests that eliminating extreme debt-service-charge risk is relatively expensive. Second, the shape of the risk-cost trade-off is relatively linear for average and extreme debt-service-charge risk, whereas the curve displays substantially more curvature for average and extreme budgetary risk. It appears, therefore, that once budgetary risk falls below a certain threshold, small risk reductions are achieved with substantial increases in cost.

In summary, Charts 4 and 5 illustrate how the optimal portfolio weights and expected costs evolve for different definitions of risk at alternative constraint levels. While the results vary as the constraint changes, there is a trend towards an increase in the amount of longer-tenure instruments to reduce risk. Moreover, in all cases, this reduction in risk is accompanied by a commensurate increase in expected debt-service charges.

20. For a debt stock of Can$400 billion, this represents approximately Can$4 billion of annual cost reduction.

There is a trend towards an increase in the amount of longer-tenure instruments to reduce risk.

Conclusion

To meet the government's financing needs, including its objectives of ensuring low-cost and stable financing, the debt manager must select a financing strategy in the face of substantial uncertainty about the future. Given the complexity of this task, a mathematical model was developed to help debt managers better understand the implications of various financing choices. This article provides a brief overview of the structure of Canada's debt-strategy model and demonstrates how it is actually used. In particular, the optimal portfolio weights and the risk-cost trade-offs are identified for alternative definitions and levels of risk. Different definitions of risk generate different results. This is an important result and should underscore the role of the debt-strategy model: It is not a substitute for intuition, experience, and judgment but is a tool to assist in understanding and organizing this multi-faceted decision.

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